

Introduction to Frobenius splitting #3

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We discuss an interesting obstruction to F-regularity called the test ideal. This is a "characteristic p analog" of the multiplier ideal, an important tool in algebraic geometry which was the subject of introductory talks by Rob Lazarsfeld at the special year in commutative algebra 10 years ago at MSRI. At that time, little was understood in prime characteristic, but now a very satisfying theory is established. We hope to explain some of the basics of this theory, giving the definition and main properties of the test ideal. As an application, we prove the following theorem: if P is a radical ideal in a regular ring of characteristic p and dimension d , then the symbolic powers $P^{(nd)}$ are contained in P^n for all n . This was proved in characteristic zero using multiplier ideals, and now the analogous proof, using test ideals instead of multiplier ideals, gives the characteristic zero case.