MSRI
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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: A. Secelean Email/Phone: asecelean2@math.uiuc.edu

Speaker's Name: Alek Vainshtein

Talk Title: Cluster algebras and Poisson geometry

Date: 09/05/12 Time: 2:00 am/pm (circle one)

List 6-12 key words for the talk: cluster algebra, Poisson geometry, Poisson bracket.

Please summarize the lecture in 5 or fewer sentances: This lecture relates cluster algebras to Poisson geometry.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☐ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☐ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - Computer Presentations: Obtain a copy of their presentation
  - Overhead: Obtain a copy or use the originals and scan them
  - Blackboard: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - Handouts: Obtain copies of and scan all handouts
- ☐ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☐ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☐ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

<u>Cluster Algebras and Poisson Geometry</u> #3 9/5/12

Alek Vainshtein

① Perfect Planar Networks and cluster structures
② Perfect network in an annulus
③ applications

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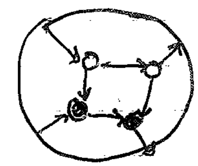
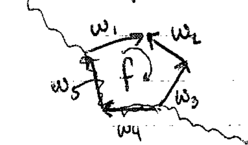
① <u>Perfect Planar Networks & cluster structures</u>



(i) <u>face weights</u>



$$w_f = \frac{w_1 w_4 w_5}{w_2 w_3}$$

Invariant under gauge group action at vertices
invariant under path reversal

Space of face weights: $\mathcal{F}_N$

$$\begin{tikzcd} \mathcal{E}_N \arrow[r, "\beta_N"] \arrow[d] & M_d \\ \mathcal{F}_N \arrow[ur] & \end{tikzcd}$$

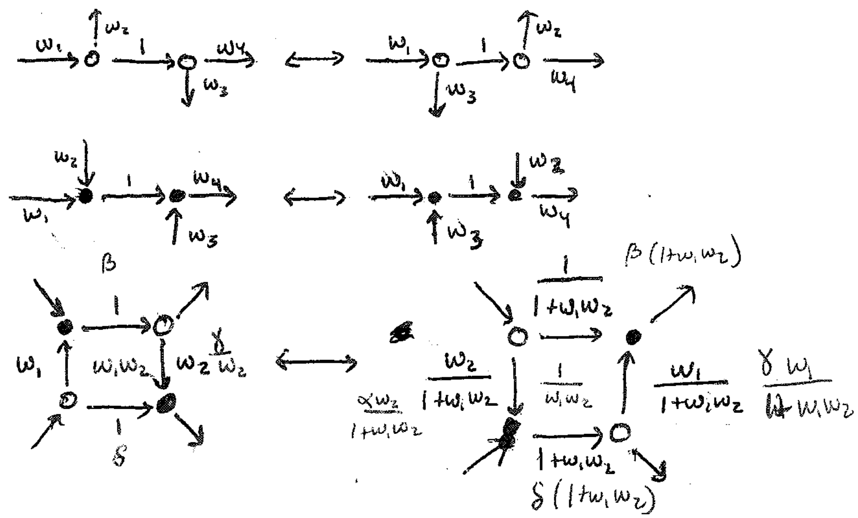
<u>Claim</u> β_N and δ_N factor through $\mathcal{F}_N$

(ii) equivalent transformations of networks

I.

II.

III.



$w_1 w_2 \longmapsto \frac{1}{w_1 w_2} \qquad \frac{\alpha}{w_1} \longmapsto \frac{\alpha}{w_1} \cdot \frac{w_1 w_2}{1+w_1 w_2}$

$\beta \longmapsto \beta(1+w_1 w_2)$

Recall how $\tau_i \longmapsto \frac{1}{\tau_i}$

$\tau_j \longmapsto \tau_j (1+\tau_j)^*$

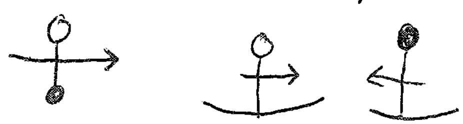
$\tau_j \longmapsto \tau_j \left(\frac{\tau_i}{1+\tau_i}\right)^*$

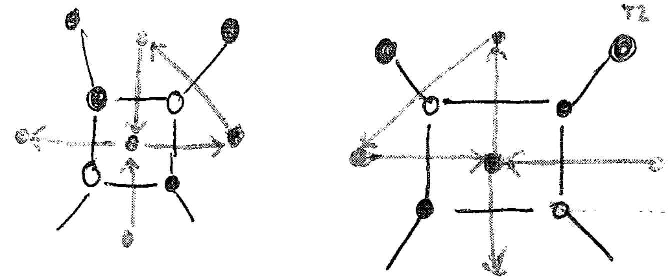
(iii) directed dual graph

vertices $\longrightarrow$ faces of N

edges $\longrightarrow$ ~~edge~~ bicolored edges of N

orientation rule:

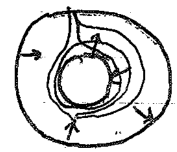




(like quiver transformations)

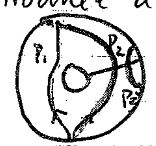
$\{w_f, w_{f'}\} = \left(\sum_{e: f \to f'} p_e - \sum_{e: f' \to f} p_e\right) w_f w_{f'}$

② Perfect Networks in an annulus.



treat nonisotopic paths differently!

Introduce a cut



$w_{P_1} = \pm \prod_{e \in P_1} w_e$

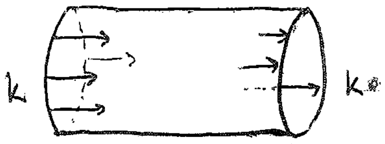
$w_{P_2} = \pm \lambda \prod_{e \in P_2} w_e$

$w_{P_3} = \pm \frac{1}{\lambda} \prod_{e \in P_3} w_e$

Claim: boundary measurement is a rational function in edge weights and λ

Proof is inductive.

(Talaska for disk)

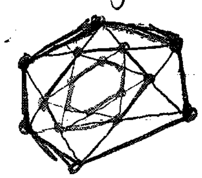


The bracket obtained from N is a trigonometric R-matrix bracket on $Mat_{k,k}$.

concatenation $\equiv$ matrix multiplication

(3) Applications:

Pentagram map



T_n $\qquad \mathbb{R}P^2$

$$\cdots \underbrace{v_i, v_{i+1}, \cdots \to v_{i+n}}_{A}$$

T_n is a completely integrable system:

(1) There is an invariant (under transformation) Poisson structure

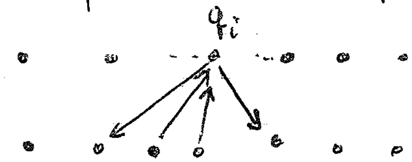
(2) There are many integrals in involution

Coordinates (Glick):

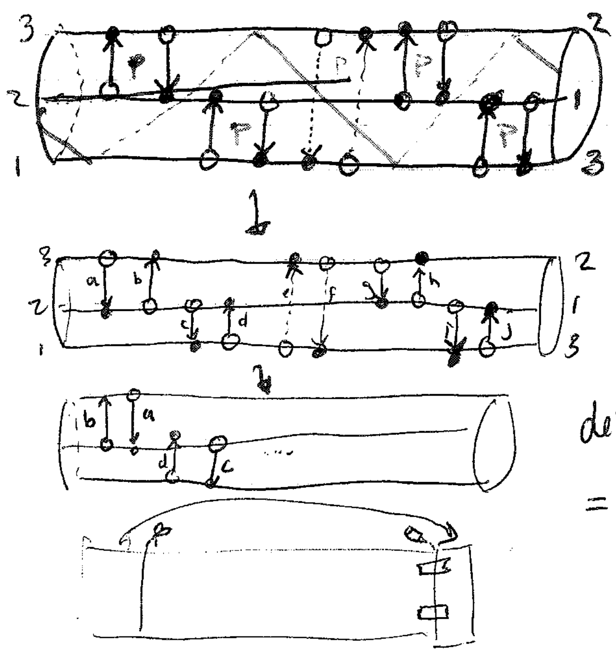
$p_1, \cdots, p_n \qquad q_1, \cdots, q_n$

$$p_i' = \frac{1}{p_i}, \quad q_i' = q_i \frac{(1+p_{i-2})(1+p_{i+1})p_{i-1}p_i}{(1+p_{i-1})(1+p_i)}$$ subscripts mod n

build a quiver with p_i, q_i vertices.



Sits on a torus!



$M(\lambda)$ remains the same

$\det(M(\lambda) - zI)$

$= \sum_i' I_{ij} z^i \lambda^j$

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