

Title: Random Sequences of Simple Rearrangements
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Abstract: It is often useful to approximate the symmetric decreasing rearrangement by a sequence of simpler rearrangements, such as polarization (two-point rearrangement), Steiner symmetrization, or the Schwarz rounding process. In this talk, I will discuss recent results with Marc Fortier on random polarizations.

We derive conditions under which random sequences of polarizations converge almost surely to the symmetric decreasing rearrangement. The parameters for the polarizations are independent random variables whose distributions may be far from uniform. For the special case of i.i.d. uniformly distributed polarizations on the sphere, we show that the distance from the symmetric decreasing rearrangement typically decreases with $1/n$ in the number of steps. We also show almost sure convergence for i.i.d. sequences of polarizations chosen at random from small sets.

These statements about polarization allow us to improve the existing convergence results for Steiner symmetrization. In particular, we show that full rotational symmetry can be achieved by Steiner symmetrization along a random sequence chosen from finite sets of directions that satisfy an explicit non-degeneracy condition. Finally, we construct examples for dense sequences of directions such that the corresponding Steiner symmetrizations do not converge.