Oct 18, 2011

11.00 – 12.00 noon

Olivier Guédon

Sparsity and Non-Euclidean Embeddings.

Notes for page 28

The concentration of measure ineq comes from a Martingale ineq in (2).

Notes for page 36

For latest "explicit" construction, refer to Bourgain, Dilworth, ~~Ford~~, Konyagin, Kutzarova.

where they break the obstruction at $\sqrt{n}$ and get $n^{(\text{larger than } \frac{1}{2})}$, but still far from $\frac{n}{\log n}$ obtained from random methods.

Notes for page 40

Replace RIP with property (ϕ_1).

(Maybe easier to get compared to RIP if using log-concave random matrices)

Notes for page 45

Goal is to find an operator with

① $\frac{\alpha}{\beta} \sim$ constant

② $\frac{1}{\kappa} \sim m$
(kappa)

③ $m \sim$ proportional to n.

Notes for page 49

- (1) Compare with (1) on page 29, but only look at sparse vectors (which live in a space of dimension m.)
- Classical Union bound:

$\alpha \in \text{sparse}_p(m)$ means $\left\{\begin{array}{l}\alpha \text{ is } m\text{-sparse} \\ \text{and } \alpha \in \text{unit sphere of } \ell_p.\end{array}\right.$

Notes for page 53

Got our wish 1 and 3 as in "Notes for page 45."

Notes for page 55

Note, we have erased the first m largest coordinates of x.
So for m-sparse vectors, we don't ask anyth about the lower bound.
(ie the lower bound is only meaningful when x is not m-sparse)

Notes for page 56

Second $\leq$ comes from Hölder's ineq (so $\kappa = \frac{1}{m}$)

First $\leq$ comes from Compressed sensing techniques, once the first m largest coordinates are erased.

Notes for page 58

Embedding $\ell_p^n \hookrightarrow \ell_1$ is now good:

$$\frac{\alpha}{\beta} = \text{const}, \quad m = \frac{1}{\kappa}, \text{ and } \quad \frac{m}{n} \sim \frac{\log(1+\eta)}{\eta}$$

Notes for page 62

Mixture of random and deterministic methods was used by

① Kashin in his original paper.

② Gluskin, that Ban-Maz compactum has diam $\geq n$.

③ Szarek on basis constant.

These can also be done with full random methods.

But for [Friedland-Guédon] here, mixture of random and deterministic is important.

Notes for page 70

(1) $\frac{8}{5} |Th_{I_1}|_1 = \frac{8}{5} \left| T\left(\sum_{k=2}^{M} h_{I_k} \right) \right|$

because $h \in \ker T$, so $Th_{I_1} = -T(h_{I_2} + \cdots + h_{I_M})$

(2) $|h_{I_1}|_p \leq \frac{8}{5} \sum_{2}^{n} |Th_{I_k}|_1 \underset{(\mathcal{P}_1)}{\leq} \frac{11}{5} \sum_{k=2}^{M} |h_{I_k}|_p$

$\{$ each h_{I_k} is also m-sparse

(3) Last line, use $(\mathcal{P}_2)$ instead of Hölder's ineq as in Comp Sensing.

Notes for page 71

typo

$$\left| \sum_{k=2}^{M} h_{I_k} \right|_p^p = \left(\sum_{k=2}^{M} |h_{I_k}|^p \right) \quad \text{without } \tfrac{1}{p}$$

Notes for pages 74, 75

$N = (1+\eta)n$

so $\operatorname{codim} S = N - \underbrace{\operatorname{rank}(W)}_{n} = N - n = \eta n = k$

Notes for page 80

T: enconder

Ty: is prescribed

Want to reconstruct y

s: size of the sparsity of the vector.