- Thursday, May 03, 2012
- Speaker: Omer Angel
- Title: Linearly reinforced random walks
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1. Model:
   - $G = (V, E)$: an undirected graph.
   - $a_e > 0$: initial weight of edge $e \in E$.
   - Every time the random walker visits an edge, the weight of the edge increases by 1. ("linear reinforcement" random walk)
2. Observations:
   - LRRW is partially exchangeable
   - (Diaconius-Freedman) recurrence $\Rightarrow$ RWRE, i.e., the law $\mathbb{P}$ of the RRW $= \int P^\omega \, \mathrm{d}\mu(\omega)$, where μ is a law of the environment.
   - (Merkl-Rolles) the same equivalence for not recurrent LRRW+ explicit expression of μ ("magic formula").
   - (Pemantle) transient/recurrent results on trees.
   - (Merkl-Rolles) recurrent on stretched $\mathbb{Z}^2$.
3. **Theorem** In any graph with bounded degrees, the LRRW is recurrent for small enough initial weights $a_e < a$, where a depends on the maximal degree.
   Remark: Sabot-Tarres have a different proof of this theorem.

   Theorem For $s < 1/3$, $\mathbb{E}(\omega_e)^s \le (C\sqrt{a})^{\mathrm{dist}(e,v_o)}$, where $a = \max_e a_e$ and C depends on s and the maximal degree.

   Theorem On any non-amenable graph with degrees bounded by K, LRRW is transient if $a_e > a_0$ for $a_0 = a_0(\iota, K)$, where ι is the Cheeger constant.