

## NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Justin Hilburn Email/Phone: jhilburn@uoregon.edu

Speaker's Name: Ivan Mirkovic

Talk Title: Loop Grassmannians from the Perspective of Local Spaces

Date: 11 / 18 / 2014 Time: 3 : 30 am **pm** (circle one)

List 6-12 key words for the talk: Local space, factorization space, Affine  
Grassmannian, Beilinson-Drinfeld Grassmannian,  
Zastava Space, Vertex Algebra

Please summarize the lecture in 5 or fewer sentences: Local spaces are a generalization of the factorization spaces of Beilinson-Drinfeld  
Mirkovich showed how Local spaces can be used to reconstruct the BD  
Grassmannian of a reductive group from the BD Grassmannian of its maximal  
torus.

## CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3<sup>rd</sup> floor.
  - **Computer Presentations:** Obtain a copy of their presentation
  - **Overhead:** Obtain a copy or use the originals and scan them
  - **Blackboard:** Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - **Handouts:** Obtain copies of and scan all handouts
- For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list.  
(YYYY.MM.DD.TIME.SpeakerLastName)
- Email the re-named files to [notes@msri.org](mailto:notes@msri.org) with the workshop name and your name in the subject line.

Mirkovic - Local Spaces

Simplification of Factorization spaces as Beilinson-Drinfeld

$X$  scheme ( $k$  Hilbert scheme of pts on  $X$ )

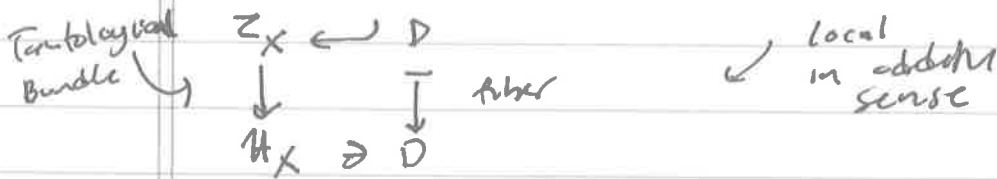
Def A local space over  $X$  is a space

$$\begin{array}{ccc} Z & \longrightarrow & \mathbb{A}^1_X \\ \uparrow & & \cup \\ \text{fiber} \curvearrowright Z_D & \longrightarrow & D \end{array}$$

~~such that~~ along with a coherent system of isomorphisms  $\varphi_{D', D''} : Z_{D'} \times Z_{D''} \cong Z_{D' \cup D''}$  whenever  $D'$  and  $D''$  are disjoint.

- Variants
- 1)  $Z \rightarrow \mathbb{A}^1_X$  is a weak local space if the  $\varphi_{D', D''}$  are ~~embeddings~~ embeddings.
  - 2) An  $I$ -colored local space over  $X$  is a space  $Z \rightarrow \mathbb{A}^1_{X \times I}$  with isomorphisms  $\varphi_{D', D''}$  whenever  $D'$  and  $D''$  are disjoint (after forgetting color).

Ex



$\text{fiber over } D \curvearrowright Gr(Z_X)_D := Gr(D) := \mathbb{A}^1_D$

Ex  $Z \hookrightarrow \mathbb{A}^1_{X \times I}$  weak local space

$$\begin{array}{l} U \\ Z^{loc} \end{array}$$

Interpreted by Frenkel:

$$D \text{ discrete} \quad Gr(D) = Z^D$$

$$D = \sum_{i \in I} p_i \quad Gr(D) = \emptyset, Gr_i(D), D$$

$\Downarrow$   
D

$$(Z_{loc})_D = \prod_{i \in I} \prod_{a \in D_i} Z_{a,i} \quad \leftarrow \text{particles}$$

$$D = \sum_{i \in I} p_i$$

$Z_{loc}$  = done in  $Z$  of  $Z_{loc}^{mg}$

Def A local v.b. on a local space  $Z \rightarrow \mathbb{A}^1_X$ .

$V$  v.b. over  $Z$

$$V|_{Z_{D'} \cup D''} = V|_{Z_{D'}} \otimes V|_{Z_{D''}}$$

taking projective space  $P(V)$  gives a weak local space over  $X$ .

Segre Embedding  $\rightarrow$  can construct  $P_{loc}(V)$ .

Another thing you can do is take subspace  $Z'_{a_i} \subseteq Z_{a_i}$  and generate a smaller local space  $Z' \subseteq Z$ .

Ex Every factorization space is a local  $Z \rightarrow \mathbb{A}^1_{X \times Z}$  along with an idempotent map structure:

$$D \subseteq D' \subseteq \eta D \quad \Rightarrow \quad Z_{D'} \xrightarrow{\sim} Z_D$$

Implicit, fact  $Z \rightarrow \text{Res}_{X \times I}^{X \times I}$

Given a weak factorization space  $(Z_D \hookrightarrow Z_D^{loc})$   
 can get a factorization space

$$\Sigma_S = \lim_{\text{app}(D) = S} Z_D$$

$$\downarrow$$

$$S$$

Can think of these as factorization space filtered by local spaces.  
 (Heuristically - related to Godez filtration)

Geometric and Field Theory  
 Homology in Algebraic geometry

Recall In topology  $H_0(Y) \cong$  <sup>0th-hom</sup> ~~direct~~ <sup>commutative group</sup> ~~in top~~ <sup>generated</sup> by  $X$

$$\parallel$$

$H_c^0(Y)$  P.D.  $\Rightarrow \omega_c = \mathbb{C}_Y[dY]$

~~Algebraic geometry: sheaves in topology.~~

construct universal local group over  $X$   
 which will be denoted by  $\mathbb{D}_X^{loc}$ .

Idea: (a)  $Y$  finite scheme

$$A_Y = \text{Hom}(\text{Maps}(Y, \mathbb{G}_m), \mathbb{G}_m)$$

Claim  $A_Y$  is group scheme generated by  $Y$ .

$$(b) \quad A(\mathbb{Z}_X) := \mathbb{Z}_X^{\text{loc}} \\ \downarrow \\ H_X$$

Claim  ~~$A(\mathbb{Z}_X)$~~  is local + weakly factorizable

$$\mathbb{Z}_X^{\text{loc}} \Rightarrow \text{Get factorization space } \mathbb{Z}_X^{\text{loc}}$$

This will be poor man's homology.

$$\mathbb{Z}_a^{\text{loc}} = \mathbb{Z}$$

could do everything with  $Y$  ind-thing

$$\mathbb{Z}_d^{\text{loc}} = \mathbb{Z}_d^{\text{loc}} = G(\mathbb{G}_m) = \lim_{\substack{\rightarrow \\ A_d}} A_{d_n} = \lim_{\text{Cantor}} \mathbb{Z}[E]$$

Commutative semigroup generated by disc is

$$\mathbb{Z} = H_d \text{ insert center point to get } G(\mathbb{G}_m)$$

~~understand general~~

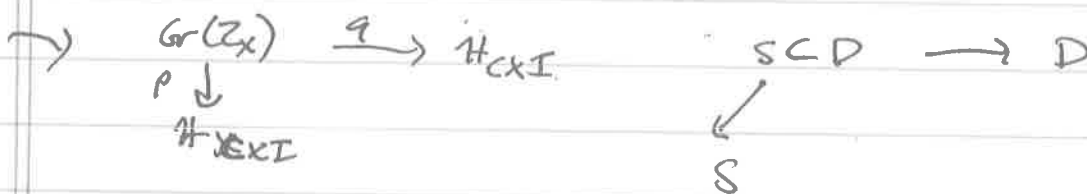
$$\mathbb{Z}_X^{loc}$$

$$(\mathbb{Z}_X^{loc})_q = \Lambda_q = \mathbb{Z} \oplus S = Gr(\mathbb{Z}^2)$$

$$\begin{matrix} U \\ S_X^{loc} \\ \cup \\ Gr(\mathbb{Z}_X^{loc}) \end{matrix}$$

can construct Zeta space corresponding to quadratic form.

2 processes !!



$\mathcal{L}$  local line bundle on  $H_{CXI}$

$$r = 9 + p_{\mathcal{L}} \mathcal{L}$$

$$r_D = \Gamma(Gr(D), \mathcal{L})$$

local 218



$$P_{loc}(r) = Z(\mathcal{L})$$

In general on take  $\mathbb{Q} \rightarrow \mathcal{L}(\mathbb{Q}) \rightarrow Z(\mathbb{Q})$   
 $\downarrow$   
 $H_{CXI}$

The points of  $P_{loc}(r)$

$$P_{loc}(r)_{\text{all}} = \mathbb{P}(\Gamma(Gr(\mathbb{Q}), \mathcal{L}))$$

$$\cup$$

$$\mathbb{P}^6$$

$Z(Q) \cong \mathcal{O}_X(Z_X)$  by Kodaira like embedding.

In higher dimensions need sheaves that can be twisted at point

From  $Z(Q)$  can construct  $G(Q)_a = Z(\mathbb{F}) \times_{\mathbb{N}[\mathbb{F}]}$   $\tilde{Z}(Q)_a$

To make series of this  $I \rightarrow Q$

$Q$  gets us from  $T$  to reductive gp  $G$

↑  
dim  
 $\mathbb{N}[\mathbb{F}]$   
action

$$G(Q)^T = G(T)$$

$\mathcal{O}_{G(Q)}(-1)$  comes from  $Z(Q)$

$$G(Q) \hookrightarrow \text{Pinc}(\Gamma(G(T), \mathcal{O}_{G(T)}^*(1)))$$