MSRI
Mathematical Sciences Research Institute

17 Gauss Way Berkeley, CA 94720-5070 p: 510.642.0143 f: 510.642.8609 www.msri.org

NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Justin Hilburn **Email/Phone:** jhilburn@uoregon.edu

Speaker's Name: Ivan Mirkovic

Talk Title: Loop Grassmannians from the Perspective of Local Spaces

Date: 11 / 18 / 2014 **Time:** 3 : 30 **am pm (circle one)**

List 6-12 key words for the talk: Local space, factorization space, Affine Grassmannian, Beilinson-Drinfeld Grassmannian, Zastava Space, Vertex Algebra

Please summarize the lecture in 5 or fewer sentences:
Local spaces are a generalization of the factorization spaces of Beilinson-Drinfeld. Mirkovich showed how Local spaces can be used to reconstruct the BD Grassmannian of a reductive group from the BD Grassmannian of its maximal torus.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

☑ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.

☑ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.

- **Computer Presentations**: Obtain a copy of their presentation
- **Overhead**: Obtain a copy or use the originals and scan them
- **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
- **Handouts**: Obtain copies of and scan all handouts

☑ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.

☑ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)

☑ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

Mirkovic – Local Spaces

simplification of Factorization spaces als Beilinson–Drinfeld

X scheme$/k$ $\quad \mathcal{H}_X$ Hilbert scheme of pts on X

Def A <u>local space over X</u> is a space

$$\begin{tikzcd} Z \arrow[r] & \mathcal{H}_X \\ Z_D \arrow[u, hook] \arrow[r] & D \arrow[u, phantom, "\cup" rotate=90] \end{tikzcd}$$

fiber

~~such that~~ along with a coherent system of isomorphisms $c_{D',D''} : Z_{D'} \times Z_{D''} \cong Z_{D' \sqcup D''}$ whenever D' and D'' are disjoint.

Variants 1) $Z \to \mathcal{H}_X$ is a weak local space if the $c_{D',D''}$ are ~~[illegible]~~ embeddings.

2) An I-colored local space over X is a space $Z \to \mathcal{H}_{X \times I}$ with isomorphisms $c_{D',D''}$ whenever D' and D'' are disjoint (after forgetting color).

Ex

Tautological Bundle

$$\begin{tikzcd} Z_X \arrow[d] & D \arrow[l, hook'] \arrow[d] \\ \mathcal{H}_X & D \arrow[l, phantom, "\ni"] \end{tikzcd}$$

fiber

local in additive sense

fiber over D

$$Gr(Z_X)_D := Gr(D) := \mathcal{H}_D$$

Ex $Z \hookrightarrow \mathcal{H}_{X \times I}$ weak local space

$$\begin{matrix} Z \\ \cup\!| \\ Z^{loc} \end{matrix}$$

Interrupted by Frenkel:

D discrete $\qquad Gr(D) = 2^D$

$D = 2 \cdot p \qquad Gr(D) = \emptyset,\ \underset{\overset{\shortparallel}{D}}{Gr_1(D)},\ D$

$$(Z_{loc})_D = \prod_{i \in I} \prod_{a \in D_i} Z_{a,i} \qquad D = \sum_{i \in I} D_i$$

$\nwarrow$ particles

Z_{loc} = closure in Z of Z_{loc}^{reg}

Def A local v.b. on a local space $Z \to \mathcal{H}_X$.

V v.b. over Z

$$V|_{Z_{D' \amalg D''}} = V|_{Z_{D'}} \boxtimes V|_{Z_{D''}}$$

Taking projective space $\mathbb{P}(V)$ gives a weak local space over X.

Segre Embedding

Can construct $\mathbb{P}_{loc}(V)$.

Another thing you can do is take subspace $Z'_{ai} \subseteq Z_{ai}$ and generate a smaller local space $Z^s \subseteq Z$.

Ex Every factorization space is a local $Z \to \mathcal{H}_{X \times I}$ along with an idempotency structure:

$$D \le D' \le nD \implies Z_{D'} \xrightarrow{\sim} Z_D$$

Implies that
$$\begin{tikzcd} & 1_{\mathcal{H}_X \times I} \arrow[d, leftrightarrow] \\ Z \arrow[r] \arrow[ur] & \mathrm{Ran}_{X \times I} \end{tikzcd}$$

Given a weak factorization space ($Z_D \hookrightarrow Z_{D''}$)
can get a factorization space
$$\mathcal{Z}_S = \lim_{\mathrm{supp}(D) = S} Z_D$$
$$\downarrow$$
$$S$$

Can think of these as factorization space
filtered by local° spaces.
(Heuristically – related to Hodge filtration)

Geometric Class Field Theory
Homology in algebraic geometry

Recall In topology $H_0 Y \cong$ (Dold–Thom) connective group ~~object~~ object in Top generated by X

$$\|$$

$H^0_c(Y)$ P.D. $\Rightarrow \omega_Y = \mathbb{C}_Y[d_Y]$

~~Algebraic ... sheaves with transfers.~~

construct universal local group over X
which will be denoted by $\mathbb{Z}_X^{loc}$.

Idea: (a) Y finite scheme

$$A_Y = \mathrm{Hom}(\mathrm{Maps}(Y, \mathbb{G}_m), \mathbb{G}_m)$$

Claim A_Y is group scheme generated by Y.

(b)
$$\begin{array}{c} A(Z_X) := Z^{loc}_X \\ \downarrow \\ \mathcal{H}_X \end{array}$$

Claim is local. + weakly factorizable

$Z^{loc}_X \Rightarrow$ Get factorization space $\widetilde{Z}^{loc}_X$

This will be poor man's homology.

$$Z^{loc}_a = \mathbb{Z}$$

could do everything with Y ind-finite

$$\underset{\hat{a}}{\overset{Z^{loc}_d}{\vee}} = Z^{loc}_{\hat{a}} = \underset{A_d}{\overset{G(\mathbb{G}_m)}{\|}} = \lim_{\rightarrow} A_{d_n} = \lim_{\mathrm{Cartier}} \mathbb{Z}[\epsilon]$$

Connective Something generated by disc is

$\mathbb{D} = \mathcal{H}_d$ invert center point to get $G(\mathbb{G}_m)$

~~understand general~~

$$\mathcal{Z}^{loc}_X \qquad (\mathcal{Z}^{loc}_X)_a = \Lambda_a = \mathbb{Z} \;\; S = Gr(\{0\})$$

$$\begin{matrix} \cup \\ S^{loc}_X \\ \shortparallel \\ Gr(\mathcal{O}_X) \end{matrix}$$

can construct Zastava space corresponding to Quadratic form.

2 projections !!

$$\begin{tikzcd} Gr(\mathcal{Z}_X) \arrow[r, "q"] \arrow[d, "p"] & \mathcal{H}_{C\times I} \\ \mathcal{H}_{X\times I} & \end{tikzcd} \qquad \begin{tikzcd} S \subset D \arrow[r] \arrow[d] & D \\ S & \end{tikzcd}$$

$\mathcal{L}$ local line bundle on $\mathcal{H}_{C\times I}$

$$r = q_* p_* \mathcal{L}$$

$$r_D = \Gamma(Gr(D), \mathcal{L}) \quad \swarrow \text{local sys.}$$

$$\mathbb{P}_{loc}(r) = Z(\mathcal{L})$$

In general can take $Q \rightsquigarrow \mathcal{L}(Q) \rightsquigarrow Z(Q)$
$\downarrow$
$\mathcal{H}_{C\times I}$

The points of $\mathbb{P}_{loc}(r)$

are $\mathbb{P}_{loc}(r)_{a,i} = \mathbb{P}(\Gamma(Gr(a), \mathcal{L}))$
$\shortparallel$
$\mathbb{P}^1$

$Z(Q) \supseteq Gr(\mathcal{I}_X)$ by Kodaira like embedding.

In higher dimensions need sheaves that can be twisted at point

From $Z(Q)$ can construct $G(Q)_q = \mathbb{Z}[I] \times_{\mathbb{N}[I]} \tilde{Z}(Q)_q$ ← does a $\mathbb{N}[I]$ action

To make sense of this $I \rightsquigarrow Q$

Q gets us from T to reductive gp G

$G(Q)^T = G(T)$ $\quad \mathcal{O}_{G(Q)}(1)$ comes from $\mathcal{L}(Q)$

$G(Q) \hookrightarrow \mathbb{P}_{loc}(\Gamma(G(T), \cancel{\ } \mathcal{O}_{G(Q)}(1))^*)$