

Nurturing Mathematical Talent:
Views from Top Finishers in the William Lowell Putnam
Mathematical Competition*

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Executive Summary

Interviews with 21 of the top finishers on the 2004 Putnam exam, which was taken by nearly 4,000 undergraduates in the United States and Canada, highlighted several prominent deficiencies in U.S. mathematics education. The increasing number of international students who are outperforming U.S. students points to a lack of opportunities for U.S. students to develop high-level problem-solving skills. Students who do well in middle school and high school competitions and on the Putnam tend to come from just a few parts of the United States, and a select group of middle school and high school teachers produce many of the country's most accomplished problem solvers. A substantial expansion of opportunities for middle school and high school students to learn higher-level mathematics and problem-solving skills would produce a much broader base of mathematical competency in the United States.

The middle school years are particularly important. Student attitudes toward mathematics worsen significantly during this period, and gender and ethnic imbalances arise that persist and intensify in subsequent years. Professional development focused on the nation's 124,000 teachers of middle school mathematics could yield major benefits for large numbers of students.

In high school, inflexible and poorly coordinated curricula keep many mathematically able students from achieving their potential. Many students pursue their interests through extracurricular activities such as math clubs and competitions, but the geographic concentration of the top finishers in competitions suggest that many students are not being reached. Web sites and written resources targeted at these students can supplement the standard high school curriculum, but skilled and knowledgeable high school teachers are still needed to introduce students to these resources and to guide their extracurricular explorations.

Inflexible curricula and uninspired teaching also can be problems at the college level, but accomplished problem solvers often find ways to circumvent these difficulties. Introductory mathematics classes designed for mathematically talented students can reduce dissatisfaction among first-year students. Opportunities to become involved in research can develop abilities that young people can apply no matter what profession they enter.

Many groups in the United States would benefit if the experiences of students who enjoy mathematics and are skilled at problem solving were enhanced. A forum to exchange ideas and coordinate initiatives affecting these students could make significant contributions to U.S. mathematics education.

Introduction

In the spring of 2005, I traveled to six leading universities -- Duke, the Massachusetts Institute of Technology, Princeton, Harvard, Stanford, and Berkeley -- and interviewed 21 students who were among the top finishers in the 2004 William Lowell Putnam Mathematical Competition. The Putnam exam has been offered since 1938 to undergraduates in U.S. and Canadian colleges and universities. It consists of 12 difficult mathematical problems to be solved over the course of six hours (three hours in the

morning and three in the afternoon, with a two-hour break for lunch). The top five finishers on each year's exam are honored as Putnam Fellows, the top 25 finishers receive monetary prizes, and the following 40 to 50 finishers receive honorable mentions. Also, three-person teams from each institution compete for additional prizes, and the top finisher among the female competitors receives the Elizabeth Lowell Putnam prize.*

I had two broad objectives in talking with these highly proficient problem solvers. The first was to discover the educational, social, and personal factors that had encouraged and had hindered these students as they became interested in mathematics and worked to develop their mathematical abilities. The second was to explore possible ways of interesting much larger numbers of young people in mathematics and in high-level problem solving.

Edited transcripts of the interviews accompany this summary report. Here I discuss the common themes that arose during my conversations. First I consider the increasing number of international students who are attending U.S. colleges and universities and excelling on the Putnam. Then I examine some of the most important influences on the U.S. students in middle school, high school, and college, with a brief look at what both the U.S. students and the international students plan to do after college. A final section draws some general conclusions about possible initiatives.

The Growing Representation of International Students Among the Top Putnam Finishers

Exact figures for the percentages of international students taking the Putnam in any given year or over time are not available. But of the top 26 finishers on the 2004 test, fewer than half attended U.S. schools. Of the 21 students I interviewed, 9 received their precollege education outside the United States. The percentages of international students finishing with high scores on the Putnam are much higher than has been the case for most of the exam's 65-year history, according to long-time Putnam observers.

A key factor behind this increased representation of international students is a substantial rise in the number of mathematically skilled students who are coming to the United States for college. "The numbers of foreign students in U.S. universities in the past were tiny," said Joseph Gallian of the University of Minnesota in Duluth, who writes often about the Putnam. "But in the last 10 or 15 years there's been literally a flood of foreign students." The collapse of the Soviet Bloc and the opening up of China have enabled many more students from countries with rich mathematical traditions to attend colleges in the United States. In addition, some colleges and universities have begun to target these students with scholarships and other inducements. Duke, for example, has offered full scholarships to a number of mathematically accomplished applicants, including students from other countries.

The expansion of the International Mathematical Olympiad (IMO) -- from about 150 competitors in 1981 to about 500 in 2005 -- also has given more high school students

* Of the 21 students I interviewed, two were Putnam Fellows, six were among the next 21 finishers, nine received honorable mentions, three who were not among the top 70 were on a team that finished among the top ten teams, four were women, and one fourth-year student was no longer eligible to take the exam but had been a Putnam Fellow for the previous four years.

in other countries the mathematical skills to be recognized in an applicant pool and excel in college. “It’s no coincidence that the gold medal winners in the IMO are getting full-ride scholarships,” said Gallian. Many of the foreign IMO participants who come to the United States for college compete in the Putnam, just as do many U.S. IMO participants. At the same time, more students overall are taking the Putnam -- from about 1,000 in 1960, to about 2,000 in 1980, to almost 4,000 today. As a result, the best U.S. students now face many more competitors than was the case in the past. Gallian, for example, has urged that the number of Putnam Fellows be increased from five to ten because of the much higher levels of competition today.

Other factors also seem to be behind the declining percentages of U.S. students among the top Putnam finishers. The educational opportunities available to precollege students in other countries appear to be greater than those available to U.S. students. To take one very rough indicator of these differences, Romania (population 22 million) and Bulgaria (population 7 million) together produced as many Putnam Fellows this year as did the United States (population 295 million). Mathematics education in other countries often emphasizes high-level problem solving much more than does mathematics education in the United States. In Shanghai High School, according to Duke’s Lingren Zhang, students take “more advanced math, like calculus, and also more intense math, like the problems we did in the Putnam. There is a problem-solving part of the class.” Ana Caraiani, a sophomore at Princeton who attended high school in Romania, said, “compared to the American system we do calculus, analysis, linear algebra -- things that people here do only in college. Also, there were some interesting problems that required the same sort of skills as Olympiad problems required.”

Schools in other countries also seem to have larger numbers of teachers who are knowledgeable and enthusiastic about problem solving and about mathematical competitions. “Every grade [in high school] had at least one teacher who was really into contests,” said Duke’s Nikifor Bliznashki of his school in Bulgaria. “They had to be passionate about the problems themselves to make us passionate about them. In some grades there were even two or three teachers doing contest math with students.”

Generating similar levels of enthusiasm and commitment among U.S. middle and high school teachers is one of the great challenges facing U.S. mathematics education.

Middle School: The Critical Years

Over and over, the students I interviewed identified their middle school years as the critical period that sparked their interest in mathematics. According to Sam Vandervelde, a recent mathematics Ph.D. graduate from the University of Chicago who has taught mathematics at both the high school and middle school levels, “If people are interested in presenting mathematics to students in the hope of capturing their imagination, I believe that those efforts should be directed toward middle schools. Middle school students have the time to explore these subjects on their own. At that age they’re ready to think about deeper and more interesting mathematics. And they tend to be more malleable than high school students, who often have decided that they’re on a particular track.”

The middle school years are a unique period for students. They typically enter sixth or seventh grade with a relatively fluid disposition toward academic subjects, friends, and social settings. (I'm including students who attend junior high schools as middle schoolers.) By the time they enter high school in the ninth or tenth grade, many have developed strong preferences for particular academic subjects and have begun to think of themselves as good in particular subjects and not good in others. In mathematics in particular, middle school is often the period when students begin to turn away from the subject. In a longitudinal study of 1,301 middle school students and mathematics teachers, University of Michigan researchers Carol Midgley, Harriet Feldlaufer, and Jacquelynne Eccles found significant declines in middle school students' perceptions of the value, importance, and usefulness of math.* By the end of middle school, many U.S. students seem to have decided that they are intrinsically unskilled at mathematics and do not like the subject.

Social forces can be an important influence on this process. During middle school, many students begin to define themselves as people interested in particular subjects. They may or may not be able to find friends who share that interest. Often they have to endure social pressures from middle school classmates who are making similar but different decisions. In middle school, said Duke's Oaz Nir, "it's a little bit more nerdy to do math, and some people might make fun of you. But in high school, you can find other people who have the same interests as you -- at least I did."

These social pressures can be particularly acute for girls. Girls enter middle school with the same average proficiency and interest in mathematics as boys, but by the end of middle school they already have begun to drift away from higher-level mathematics. This trend is not so easy to see in enrollment figures, though boys continue to take somewhat more advanced mathematics classes in high school than do girls. But girls definitely are less well represented in extracurricular mathematical activities and in mathematics competitions. Surveys and other tests suggest that girls begin to see mathematics, and particularly extracurricular mathematical activities, as stereotypically male activities.†

The 2004 Putnam exam had an unusual number of top female finishers -- four of the top fifteen participants were women. This marks by far the most women who have finished in the top levels of the exam and comes after several years of growing female participation in the Putnam. Several of the students I interviewed expressed the belief that increasing numbers of women would enter and perform well in the competition because of the examples being set by current top finishers. "It [competition] somehow seems like a male thing to do," said Andrei Negut of Princeton. "I think it's just a cultural thing that girls don't participate more."

The influence of culture on female participation and success in mathematics competitions is apparent from the differences between countries. Of the top four female finishers on the Putnam this year, just one received all her elementary schooling in the United States -- and she was home schooled. Of the three female Putnam Fellows in the

* Carol Midgley, Harriet Feldlaufer, and Jacquelynne Eccles (1989) Student/teacher relations and attitudes toward mathematics before and after the transition to junior high school. *Child Development* 60:981-992.

† Brian Nosek, Mahzarin Banaji, and Anthony Greenwald (2002) Math = male, me = female, therefore math = not me. *Journal of Personality and Social Psychology* 83:44-59.

history of the competition, two were educated in Romania. Though no country has equalized participation between men and women, some are much more successful than others in involving female competitors in mathematics contests.

The United States has been making some progress on other measures of female participation in mathematics. Women received one third of the approximately 1,000 mathematical sciences Ph.D.s awarded in the United States in 2003-2004, up from about 15 percent in the early 1980s.* However, at all levels in the United States, from middle school through college, more boys than girls enter mathematics competitions, and boys tend to earn higher scores than females.

These observations raise the question of the extent to which participation in mathematics competitions, among both girls and boys, can be seen as representing an interest in advanced mathematics in general. Teachers report that most students who are interested in mathematics engage in a competition at some point. But they may do so only once and thereafter find other ways to pursue their mathematical interests. The interviews I conducted were, by design, focused on students who had not only engaged in competitions but had excelled in those competitions. Whether competitions attract most or just some of the students who are attracted to mathematics is an interesting question that has not yet been investigated. Certainly many prominent mathematicians never participated in competitions. By the same token, many accomplished problem solvers enter professions other than mathematics and apply their skills in those fields.

Middle school also appears to be where ethnic imbalances are first established in extracurricular mathematics activities. In many cities, Asian-Americans are overrepresented in these activities from the earliest stages of middle school while disadvantaged groups, including African Americans, Hispanics, and Native Americans, are underrepresented. These ethnic imbalances remain in place throughout high school, college, and beyond. Most students are acutely aware of these ethnic preferences. In some schools, even European American students shy away from math clubs, claiming that mathematics is “an Asian thing.”

All three of the trends discussed above -- a hardening of students' attitudes toward mathematics in middle school, a greater loss among girls compared to boys of interest in higher-level math, and the overrepresentation and underrepresentation of particular ethnic groups in higher-level mathematics -- can be difficult to document using existing data sources. But anyone who has taught middle school mathematics or coached a middle school mathematics team has been exposed to at least some if not all of these trends.

Beyond the social forces at play during the middle school years are more institutional factors. Many middle school students do not have readily available opportunities to be exposed to more sophisticated mathematical concepts and high-level problem solving. Most of the students I interviewed had been influenced by a particularly important teacher in middle school who introduced them to mathematics that was beyond the standard middle school curriculum. Harvard's Inna Zakharevich described her middle school teacher as “the best math teacher I've ever had. . . . When he taught math classes, he didn't just teach people how to do the problem, he talked more about how to think about the problem, and from that followed how to solve the problem.”

* Ellen E. Kirkman, James W. Maxwell, and Colleen A. Rose (2005) 2004 Annual Survey of the Mathematical Sciences. *Notices of the American Mathematical Society* 52:236-251.

Yet these teachers are highly concentrated geographically. In the past, many of the top U.S. middle school competitive problem solvers have come from a handful of places -- the Boston area; the San Francisco Bay and greater Los Angeles areas; New York City and its suburbs; the suburbs of Washington, D.C.; Madison, Wisconsin; South Bend, Indiana; and Sugar Land, Texas, outside Houston. Middle school teachers who routinely produce nationally successful middle school problem solvers are so few that they are known (to each other at least) by name and reputation. In large parts of the country, students appear not to have access to the resources needed to excel at the middle school level.

Professional development activities focused on middle school teachers could make a substantial difference in the lives of students. There were approximately 124,000 middle school mathematics teachers in public schools in the United States in the year 2000 (the last year for which data are available). But only 66 percent were certified to teach mathematics, much less knowledgeable enough about mathematics to lead their students in higher-level activities outside the classroom. The percentages of qualified teachers are even lower in high-minority or low-income schools.

By the same token, reaching relatively small numbers of teachers could benefit large numbers of students. Even a single teacher in a middle school can change the culture of that school by providing students with an opportunity to develop their mathematical interests. In the San Francisco Bay area, for example, Sam Vandervelde is organizing a program called The Teacher's Circle that brings teachers to a one-week program at the end of the summer and monthly dinners during the school year to learn engaging mathematics and observe participatory methods of classroom presentation.

Any examination of middle school mathematics in this country has to take into account the principal program that interests middle school students in competitive mathematics, MATHCOUNTS. MATHCOUNTS was created in 1983 by the National Society of Professional Engineers, the National Council of Teachers of Mathematics, and the CNA Insurance Company as a way of highlighting mathematical proficiency in the same way that the national spelling bee highlights memorization and linguistic knowledge. Over the years, the program has begun to address some of the broader issues of middle school mathematics education. The handbooks distributed to registered schools each year contain a wealth of valuable problems that can be used in any middle school mathematics classroom to build conceptual understanding, and the MATHCOUNTS web site offers resources for middle school teachers. But the focus of the program remains the chapter, state, and national competitions designed to identify mathematically proficient problem solvers.

MATHCOUNTS has begun to pursue opportunities for its material to be used more intensively in schools. For example, in 2004-2005, every middle school and elementary school with a sixth grade in the District of Columbia registered for the competition and received MATHCOUNTS handbooks, and MATHCOUNTS volunteers worked with the District school system to hold workshops for participating teachers. Similar expansions of the program elsewhere would be challenging but could have large payoffs. There are approximately 15,000 public middle schools in the United States, plus several thousand private schools that serve middle school students. Of those, more than 6,000 schools registered for MATHCOUNTS in 2004-2005. But only about 80 percent of schools re-register from one year to the next, which means that use of the materials

may be cursory at best, and in most cases just a small percentage of students within each school receive and benefit from the materials.

High School Experiences: Consolidation or Disillusionment

For most of the students I interviewed, high school was a period when their interests in mathematics and in mathematics education were consolidated, not a period when their interests changed substantially. Many began to make their own decisions about which opportunities to pursue in mathematics, drawing from a broader array of possibilities than in middle school. All pursued their interests in mathematics far beyond the standard high school curriculum.

Many mathematically talented high school students engage in an active search for learning opportunities. “Almost all high schools students can find opportunities to perform,” said Richard Rusczyk, the founder of the Art of Problem Solving web site (www.artofproblemsolving.com). “If students sit down and google ‘math contest,’ they can find as many opportunities as they want. But even at the best schools, students don’t necessarily get opportunities to learn advanced mathematics. They tend to learn that math on their own or from each other.”

Several of the students I interviewed expressed frustration with the slow pace of high school mathematics. “The standard high school math curriculum is, in some sense, fairly pathetic,” said Timothy Abbott of MIT. “Much of the time your high school math class is repeating itself or repeating some previous class. It’s very unlikely that someone is going to be interested in something if they never feel like they’re doing anything new.”

Other students were frustrated by the difficulties they encountered in trying to find alternatives to the standard high school mathematics classes. As Stanford’s Andrew Lutomirski said of his high school mathematics program, “It wasn’t an official acceleration program. It was accomplished more by bugging people until they gave in.”

A major problem with high school mathematics, according to Susan Wildstrom, the MAA’s Governor at Large for High School Teachers, is that it’s taught “as a series of topics unrelated to each other or anything else.” As in earlier grades, most U.S. students learn high school mathematics largely as a set of procedures with rules about when to apply which procedures. “Most of the capable students are bored to death,” said Wildstrom.

On the rare occasion when I encountered an alternative to traditional high school instruction, it was a breath of fresh air. For example, MIT’s David Vincent enthusiastically described his high school mathematics classes at Philips Exeter Academy:

All of the mathematics classes there are taught around a big wooden oval table. Classes tend to be about 12 people, plus the teacher. The system that Exeter uses is called the Harkness system. Students have about eight homework problems, all word problems. They’re out of a book that the department wrote. The table is surrounded by blackboards. During class, each student, one at a time, goes to the blackboard and presents a solution to a problem. It’s designed so that you learn from each other, basically. Of course, the teacher has to help

students sometimes because there are problems that no one can solve, or there may be a cleaner solution. It's a great system.

One surprising thing about my conversations was the lack of social pressures associated with doing mathematics reported by the students. Most seemed to find a group of friends with whom they could share their interests and enthusiasm, and they weren't unduly persecuted by other students. "I had a very small group of friends, about a half dozen people who I'd known since kindergarten, and they were my social group," said Daniel Kane of MIT. "I would socialize with these people, and if anyone else was saying things about us, I wouldn't notice." This is not to say that social pressures do not exist. On the contrary, these pressures seem to be very effective at quenching the enthusiasm of many students for mathematics. (One interesting hypothesis that I heard from a reviewer of this paper is that the occasional association of Asperger's Syndrome with mathematical prowess relates not to any neurological connection but to a tendency to overlook social conventions.) But the students who excel in competitions typically have found ways to avoid or ignore these pressures.

All of the students I interviewed took advantage of opportunities offered outside the traditional high school curriculum. For example, all of the U.S. students had taken the exams offered by the American Mathematics Competitions (AMC); students in other countries took comparable exams. More than 400,000 students take the AMC 10 or AMC 12 every year, which corresponds to about 1 in every 35 high school students. (In addition, about 150,000 middle schoolers take the AMC 8 each year.) Students who do well on the AMC 8, 10, or 12 are invited to take the American Invitational Mathematics Examination (AIME), which is given to about 10,000 students per year (approximately 1 of every 1,400 high school students). The 250 top finishers on the AIME qualify for the U.S.A. Mathematical Olympiad. Several dozen students who do well on the USAMO are invited to the Mathematical Olympiad Summer Program (colloquially known as MOP from an earlier acronym; attendees often are called Moppers), and the top six students in that program represent the United States at the International Mathematical Olympiad.

All of the U.S. students I interviewed had taken the AIME, most had taken the USAMO and had attended one or more sessions of MOP, and some had been on U.S. IMO teams. In addition, some attended other mathematically oriented summer programs or participated in extracurricular training sessions (such as mathematical circles) during the school year. All cited these experiences as critical to their mathematical development.

Nevertheless, it seems clear from the geographical distribution of the students who do well on the USAMO and attend MOP that opportunities to excel are not evenly distributed across the United States. So many students at MOP come from either the Boston area or the San Francisco Bay area that Moppers refer to the two regions as "Calimass." The geographic distribution of this year's top 12 finishers on the USAMO is fairly typical. Two were from California (both from the Bay Area), two were from New Jersey, two were from Canada, and the others were from New Hampshire, Massachusetts, Maryland, Virginia, and Illinois. Many of the top finishers are from high schools that have produced USAMO winners in the past. "Students with tremendous promise are being neglected," said Jerry Alexanderson, the associate director of the Putnam competition since 1975. "The assumption has been that good students will take care of

themselves. But that has not been my experience. Some will, but many will drop by the wayside.”

As with the top MATHCOUNTS students, the concentration of students on the USAMO points toward the importance of a few influential teachers at particular schools. Given that there are 134,000 mathematics teachers in U.S. public schools, and thousands more in private schools, great potential exists to interest more teachers in working with students on mathematics that goes beyond the traditional high school curriculum. Yet some question even whether the well-known high school teachers who have produced many mathematically proficient students in the past are being replaced. “The old line teachers are retiring,” said Wildstrom, “and as they do, the teachers coming out of education programs tend to regard teaching as a job and not a calling. When they leave school at the end of the day, they’re going to their other lives.”

New forms of communication may offer a way to reach high school students who are not now being well served. The Art of Problem Solving web site developed by Richard Rusczyk has more than 10,000 registered users, and 20 to 40 new students are registering each day. The median age of the registrants is 14, so half are elementary and middle school students and half are high school and college students. Furthermore, Rusczyk believes that the web site has barely begun to reach its full potential. “We’re not reaching more than 5 to 10 percent of the kids who would be interested if they knew about it,” he said.

Mathematics in College and Beyond

Inflexible curricula and uninspired teaching also were problems in college for some of the students I interviewed, though most had found ways to work around the difficulties. For example, some students had trouble adjusting to the lecture-intensive styles of collegiate teaching. “I really didn’t enjoy the 200-level course I took,” said Ana Caraiani of Princeton. “At the same time I was taking this art course, and that was so exciting. Everything was so new, and you could take a thesis and try to prove it, whereas in the multivariable course it was all follow the proofs as your professor explained them at the board and then plug in answers. I didn’t like it.” However, the difficulties generally subsided as students moved into more advanced classes and began working on mathematics in a way more reminiscent of their experiences with competitive mathematics. “Once I took a 300-level course it was better,” said Caraiani. “The course was more interesting, and you had more challenging assignments and exams. When the material becomes more advanced, you can see that the person is actually working in that field and interested in that field, and that makes it more appealing.”

Several students praised first-year mathematics classes that they felt had been designed specifically for undergraduates with their backgrounds and interest, such as Mathematics 55 at Harvard and the honors mathematics sequence at Stanford. “The first two quarters [of the advanced honors sequence] are pretty much whatever that professor wants to teach,” said Andrew Lutomirski of Stanford. “I’m sure there’s some structure, because people learn similar things. We had professor Eliashberg, who’s excellent. It was kind of insane, it moved really fast.” Though the numbers of students who would be served by such classes would be small at many colleges, the enthusiasm for these classes

points toward a way of reducing dissatisfaction with mathematics among first-year college students.

These highly capable mathematics students also took advantage of opportunities to enroll in graduate courses right away as undergraduates. “Right from the get go, they let you take graduate courses here, so I’ve taken some really interesting things,” said Duke’s Oaz Nir. Many of the older students I interviewed also have undertaken independent research projects in college and report favorably on these experiences. “Competition is not really appropriate for learning material past a certain point,” said Gabriel Carroll of Harvard. “Research is really what is relevant.”

As in high school, social pressures did not seem to be a prominent concern for these students. “MIT has so many more interesting people than high school,” said Daniel Kane. “Basically my social group went from maybe a dozen people to my entire dorm. Here I can talk math at some reasonable level and expect most people to understand what I’m saying. I can have fun, nerdy, technical conversations with almost anyone.”

Most of the students I interviewed said that they don’t prepare for the Putnam, though the accuracy of those claims with competitive students such as these can be difficult to ascertain. Yet even the two Putnam Fellows I talked with seemed sincere in their claims that they don’t prepare for the test.

The plans these students have for their postcollegiate years are extremely varied. Some entered college determined to go to graduate school and haven’t wavered in that intention. “I always had this plan that I was going to go to college, I was going to get a Ph.D., and I was going to become a professor,” said Princeton’s Ana Caraiani. “I haven’t had any doubts about that.” Others have veered away from a commitment to mathematics toward other subjects. “As it turns out, I really don’t want to be a mathematician,” said Steven Byrnes of Harvard. “That’s why I’m taking a lot of physics courses. But I still find the math courses to be enjoyable.” A few have returned to mathematics from other subjects: “As a freshman I was intending to study engineering instead of math,” said Nikifor Bliznashki of Duke. “But after one year of taking courses designed for engineering majors, which involves engineering classes, physics, things like that -- and stupid math -- I felt as if I missed math a lot. So I switched, and now I’m doing math.” Some remain unsure about what they will do. For example, Gabriel Carroll, one of the most accomplished Putnam Fellows in the history of the exam, is taking next year to teach English in China. “Sometimes people need to do something different for a while and get some perspective,” he said. “I didn’t think that that was going to happen to me, but it did.”

A major question is how many of the accomplished international students will remain in the United States. Most have provisions in their visas that require them to return to their home countries eventually, but many clearly would like to stay in the United States. “I don’t know where I’d want to be in my mid-50s, but until my mid-40s I’d want to stay here,” said Princeton’s Suehyun Kwon, who grew up in Korea.

Conclusions

As has been the case with the other top problem-solvers I’ve met over the years, the 21 students I interviewed for this project were accomplished, well-rounded, articulate

young people with a strong sense of themselves as problem solvers and scholars. They are people who can be expected to make a difference in whatever profession they enter.

One question that must be asked is whether U.S. schools are producing enough of these kinds of students. The signs are not positive. The influx of top international students into U.S. colleges and universities is a promising development for the United States, especially since many of these students plan to stay in the United States after graduate school if they can. But relative declines of U.S. students pursuing advanced education in mathematics and some areas of science -- along with a more general sense that careers in mathematics or science have lost some of the luster they had in the past -- are worrisome trends.

Beyond the overall numbers, the students I interviewed tend to see themselves as a minority with interests that distinguish them from most of their peers. They have come to terms with this distinction -- some seem to enjoy it. But they are out of the mainstream, despite the many ways in which their abilities could benefit society.

Efforts to increase the number of U.S. students who are interested in mathematics and skilled in problem-solving must start at young ages. I focused in this summary report on middle school because attitudes toward mathematics both worsen and sharpen during this period. But some attitudes, as well as a base of knowledge and skill, are developed during elementary school, and many (perhaps a large majority) of elementary school teachers are not prepared to build the attitudes, knowledge, and skills that students need.

At the middle school and high school levels, the components of a successful extracurricular mathematics program are well known. Knowledgeable and enthusiastic teachers are essential. Parents make major contributions to many such programs, especially in the earlier grades. Schools focused on the sciences, technology, or mathematics can pull together larger numbers of students interested in mathematics, but any school can become a mathematical powerhouse by establishing a tradition of participation and achievement.

New methods of information dissemination and teaching offer intriguing possibilities for a generation that has spent more time on the World Wide Web than reading books. The educators who will be most familiar with new technologies and capabilities typically will be younger than the average mathematics teacher. Ways should be found of involving these individuals in efforts to improve mathematics education in the United States.

The increased number of females participating in mathematics competitions also offers an opportunity to heighten the interest of girls in mathematics. The more visible these young female mathematicians are, the greater their influence will be. The severe lack of underrepresented minorities in mathematics similarly deserves much greater attention.

Many groups in the United States have an interest in enhancing the experiences of mathematically talented students, including business and government leaders, elementary and secondary school teachers, college faculty, and new generations of students. Today these groups have very few opportunities to discuss their common interests in the state of U.S. mathematics education. A forum to exchange ideas and coordinate initiatives, perhaps sponsored by the Mathematical Association of America or the American Mathematical Society in conjunction with the National Council of Teachers of

Mathematics, could serve a catalytic role in fostering achievement in mathematics and problem solving.

Mathematics education is in the midst of great changes. Many elementary and secondary school teachers will retire in the next decade and must be replaced. The participation of groups who have been underrepresented in mathematics in the past is increasing. Far more international students are coming to the United States for college, and many of them will remain here and contribute to our economy, educational system, and society. Periods of change also can be periods of great opportunity if goals are set and steadfast efforts are made to achieve those goals.