Problems and Activities

The following problems are from *Saint Mary's College Mathematics Contest Problems: For Junior and Senior High School* by Brother Alfred Brousseau, Palo Alto: Creative Publications, 1972.

77. By what rule based on the digits in a number is it possible to tell whether a number in base 7 is even?

92. Given the sequence 1, 2, 4, 5, 7, 8, 10, 11, 13, 14, ... where every third integer is missing, find the sum of the first hundred terms in the sequence.

98. A pyramid is formed in the following fashion: the first layer is made up of 169 equal cubes arranged in the form of a square; the second layer is symmetrically placed and has 11 cubes on a side; the third layer has 9 cubes on a side; etc. The final layer has but one cube. This whole structure is painted (but not the bottom). When it is taken apart, how many cubes have no faces, 1 face, 2 faces, 4 faces, 5 faces, 6 faces painted?

116. Cards are given numbers: 1 for ace, 2 for 2, ..., 11 for jack, 12 for queen and 13 for king. Let one player secretly count out three piles of cards as follows: Starting with any card, such as 7, put down cards counting 7, 8, 9, 10, 11, 12, 13. Turn the pile face down; do likewise for two other piles. Now turn two piles over so that the other player can see the starting cards. This player now counts out ten cards plus a number of cards equal to the sum of the numbers shown on the two piles. Prove that the number of cards remaining in the pack gives the number on the bottom of the unturned pile.

129. Show how to make six equilateral triangles with nine toothpicks of equal length, the sides of the triangles all being the length of one toothpick.

144. A company named TOPIC has an advertising display with the letters of its name "TOPIC". Colors are to be used for each letter, but the colors may be repeated. On a certain day the colors might be red, green, green, blue, red. The company wishes to have a different color scheme for each of the 365 days of the year. What is the minimum number of colors that can be used for this purpose?

158. A computer is programmed to scan the digits of the successive integers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, ... To this point it has scanned 21 digits. It continues until it has scanned 6,000,000,000,000 digits. At what integer has it arrived?

177. Using five 5's and the operations $+$, $-$, $\times$, $\div$, $\sqrt{\ }$, and factorial (!), represent the numbers up to 30 inclusive by using five 5's in each case.

On the next page is an activity from the Julia Robinson Mathematics Festival in 2008 and 2009. You can find other JRMF activities online at www.msri.org/specials/festival/festival_activities.html.