Functions Rearranged

In this problem we will rearrange five functions to see what happens

\[ A(x) = 3x + 4 \]
\[ B(x) = x^2 \]
\[ C(x) = 4x - 1 \]
\[ D(x) = 3x^2 \]
\[ E(x) = 7x + 3 \]

The Main Question

1. You get to use each function exactly once, one after the other, with a starting input of 2. In what order should you use the functions to get the largest possible final result?

Hints

2. If you have only \( B \) and \( D \) to work with, what order is best?

3. If you have only \( A \) and \( C \)?

4. If you have only \( A \), \( C \), and \( E \)?

5. If you have only \( A \) and \( B \)?

6. If you have only \( C \) and \( D \)?

7. Now can you use what you’ve learned here to go back and answer the main question?

Extensions

8. If you want the smallest possible final output, is it just the same functions in the reverse order?

9. What if the input is not 2 and you want the largest possible output. Is it always the same order?

10. What if the input is not 2 and you want the smallest possible output. Is it always the reverse of the order that would give the largest?

11. What if you can use each function at most once? Is there ever a circumstance where you’d be able to make a bigger number by leaving one function out?

Thanks to Bill Graham’s “Mind Games” for writing much of this problem set and to Irving Lubliner for the ideas behind several of these problems.