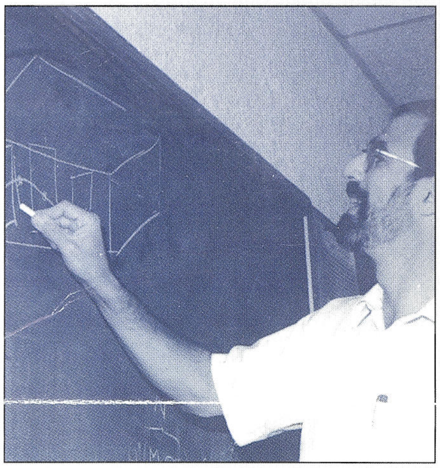
The EMISSARY

December 1998

MATHEMATICAL SCIENCES RESEARCH INSTITUTE

Notes from the Director

David Eisenbud



I've now been at MSRI through a whole revolution of the earth around the sun. I've gotten a good look at the Institute and have learned a lot about its rhythms—when I stand up in public to explain them, I even feel that I know what I'm talking about! It's a wonderful and impressive view mathematically (the programs are strong, the lectures interesting), socially (many visitors are enormously involved in

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Inside This Issue

Mathematics and the Media

By Allyn Jackson

Senior Writer, American Mathematical Society
Journalist-in-Residence, MSRI

The September 14, 1998 issue of the political magazine *The Nation* carried on its back cover a full page ad from Barnes and Noble bookstore touting the popular mathematics books it offers, including three on Paul Erdös. Last year's hit movie *Goodwill Hunting* featured a mathematical prodigy as protagonist; this year there is the film π (which bills itself as a "mathematical thriller"). Last year public television stations aired the BBC program *The Proof*, which told the gripping story of Andrew Wiles' proof of Fermat's Last Theorem. There has been a good deal of coverage of mathematics in the press: in tracking this coverage in a small number of newspapers and magazines, I found 50 articles about mathematics published between April 1997 and April 1998.

Mathematics is receiving media attention like never before. And yet there remains among mathematicians the conviction that public awareness and appreciation of mathematics is lacking. In early October, MSRI held a conference entitled "Mathematics and the Media" which was intended to bring those who work in the media into contact with mathematicians. According to Robert Osserman, MSRI director of special projects and one of the main planners of the event, the idea was to address two problems: first, getting

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You are warmly invited to join us and other friends at the

Mathematical Sciences Institutes Reception

During the AMS-MAA Joint Meetings
In Salon A of the Grand Ballroom
at the Marriott Rivercenter San Antonio

On Wednesday, January 13, 1999
5:30 – 7:30 PM

Jointly Sponsored by:

The Mathematical Sciences Research Institute
The Fields Institute for Research in Mathematical Sciences
Institute for Mathematics and its Applications
National Institute for Statistical Sciences
Discrete Mathematics and Theoretical Computer Science Center

Director's Message
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the seminars, workshops, and outreach, and very enthusiastic about their stays), and of course, geographically (if you've been to MSRI on a clear day, you'll know what I mean.)

It was a year with highly varied programs: Stochastic Analysis, with lots of Itô integrals and applications from finance to oceanography; Harmonic Analysis, opportunely caught in a moment of intense activity, consolidating recent gains; and Model Theory of Fields, where astonishing new applications to number theory, real algebraic geometry, and other fields bid fair to convince even the hardened skeptics that the field has plenty to offer classical mathematics. Now we're in full swing with programs on Symbolic Computation and on Foundations of Computational Mathematics, of which you will read more elsewhere in this newsletter.

We had a first-rate group of postdocs — but people who have been around MSRI longer than I tell me that that's always been the case!

It was a year with many interesting happenings at the Institute:

- Hot Topics: The first Hot Topics workshop covered the recent work of Voevodsky and Suslin on K-theory and the homotopy of algebraic varieties. You can see and hear Voevodsky's beautiful and highly accessible series of lectures for yourself in streaming video, as described below! By the time you read this, the topic for the next of these workshops (to be held in the late Spring of 2000) will have been chosen.

- Special conferences: As usual, after the major programs wound down, and sometimes appearing in the cracks while they were still in session, we offered a variety of conferences such as the week-long events on Low-Dimensional Topology and on Advances in Applied and Computational Mathematics. We also held 3-day conferences on topics less familiar to core mathematicians: Cryptography, Genomics, and Parallel Symbolic Computing. We plan to continue to reach out to other sciences and technologies that use (or might use) mathematics to provide new approaches to answering problems and establishing connections. We don't intend to abandon fundamental mathematics in any sense, but we do take a broad definition of what's fundamental. An openness to ideas from other fields has always been an enriching part of the mathematical sciences, and mathematics, in return, has allowed fantastic development in the fields of science that use it most.

- Streaming Video: We began broadcasting lectures in streaming video. This technology allows us to connect distant mathematicians to the lectures within our walls, including the widely accessible introductory workshops for each program and many special talks. We've also been publishing tapes given to us by others in this way, and we're committed to making the technology available to any department or institute that would like to try it for themselves. (If you haven't watched one of our lectures yet, go to www.msri.org and click on "Video" to get started...)

- Journalist in Residence: We have started a Journalist in Residence program, initially supported by a grant from the Gabriella and Paul Rosenbaum Foundation, with the hope of promoting better media exposure for mathematics and of helping mathematicians and writers/journalists establish effective contacts. The first holder of the position was K. C. Cole of the Los Angeles Times. An article inspired by our conference on Low Dimensional Topology last summer, published in the October 29 Science Section of the *LA Times*, was one of the results. In residence earlier this Fall was our second writer/journalist, Allyn Jackson, from the American Mathematical Society, and Brian Hayes from *American Scientist* will be here for the Spring semester and the Random Matrix Models program. We held a successful and varied conference on Math and the Media (supported by Sun Microsystems) to help get the program started. The event got national attention - did you happen to catch Ira Flatow's "Science Friday" broadcast by National Public Radio, October 9, with three members of that conference on the air?

- We've been through a large part of the re-competition process, which will be completed at a National Science Board meeting in the spring. We've received tremendous support from mathematicians around the country, from the University of California, and from many new affiliates, such as Hewlett-Packard Labs and Sun Microsystems. Needless to say, this has been very important for MSRI.

- If you are reading this far away from MSRI, perhaps the first of our innovations that you'll experience yourself will be a reception at the Joint Winter Meetings of the AMS, MAA, and SIAM (see the announcement on page 1). This year, we invited several other Institutes to join us, to make it easier for mathematicians to keep up with what's going on (and to think about plans for future sabbaticals.) Look for us in the Meetings program, and do come!

David Eisenbud

Mathematics and the Media
(continued from page 1)

mathematics mentioned at all in the press, and, second, getting it mentioned accurately. The first problem may be solving itself; the second problem is much harder.

The conference was in many ways eye-opening. First, let's dispel the myth that science writers aren't interested in mathematics. At the conference, they said loud and clear that they are interested. But, as Sharon Begley of *Newsweek* put it, "I can't do a story if I don't know it exists."

Journalists simply aren't finding out about what's going on in mathematics. At the same time, they are bombarded with information about developments in other fields of science. According to one of the conference speakers, Curt Suplee, a science writer and editor at the *Washington Post*, 75% of science coverage in the major national newspapers derives from 5 sources: *Journal of the American Medical Association*, *New England Journal of Medicine*, *Science*, *Nature*, and the Hubble Space Telescope. All of them have large public relations operations feeding stories to the press. As a result, science reporting tends to overemphasize biology, medicine, and space research. Science writers know they are not covering science evenly, and they resent the dominance of these few major public relations machines. But the fact is that these machines make the work of a science writer enormously easier. There simply isn't that kind of support to help them find out about and write stories on mathematics. The AMS might seem the natural home for a public relations effort in mathematics, and in fact it has a very small one that sends out occasional press releases and answers press calls. But right now the Society simply does not have enough staff for a full-blown public relations operation.

With television, the problems are different. Television programs are complicated and expensive to produce: *Life by the Numbers*, a six-part series about mathematics aired on public television last spring, took several years of development and production, at a total cost of about $4 million. Keith Devlin, a mathematician at St. Mary's College, was deeply involved in the development of *Life by the Numbers* and wrote a companion book for the series. During the conference he revealed the dispiriting news that, despite the high quality of the series, many television station managers decided that because it was about mathematics, no one would watch it, and they ran the series in the wee hours of the morning. This scheduling cut into the already small audience, which for a typical *Nova* program is 3.4 million; compare this to 28 million for the network show *Frasier*. As Joel Schneider of Children's Television Workshop commented, maybe it would have been more cost-effective to promote the cause of mathematics by buying air time during one of the popular network programs. Radio is a simpler, cheaper medium than television, and there is evidence it can be successfully used to popularize mathematics. Devlin has also worked on a number of radio programs, and he played a delightful segment from one of them, a BBC radio program on Fermat's Last Theorem. During the conference, National Public Radio's *Science Friday* program devoted an hour to mathematics and featured as guests three of the conference speakers.

In addition to providing a forum in which mathematicians and journalists could exchange ideas, the Math and Media conference also featured some talks by mathematicians. For example, Peter Sarnak of Princeton University presented a lecture about a mysterious connection between the Riemann hypothesis in number theory and random matrix models that arise in physics. Linking prime numbers with fundamental ideas from physics, this story has the potential to inspire the same kind of awe and wonder as do stories about cosmology, which so often land on the front page. However, as the discussion after Sarnak's talk revealed, even the most fascinating stories in mathematics can get lost when the technical level of the presentation is too high for the listener.

One of the highlights of the conference was a performance by the writer and actor Josh Kornbluth of his monologue "The Mathematics of Change". He has performed the monologue in theaters in San Francisco and New York as

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Mathematics and the Media
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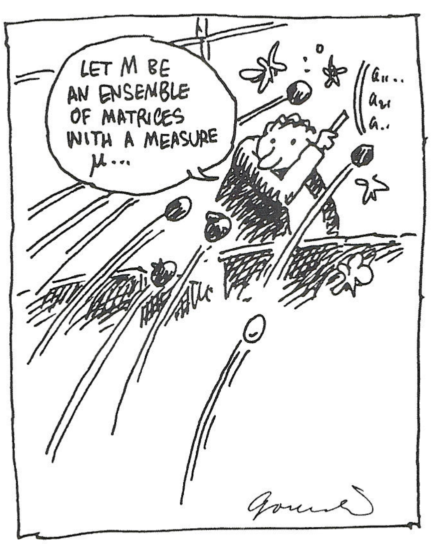
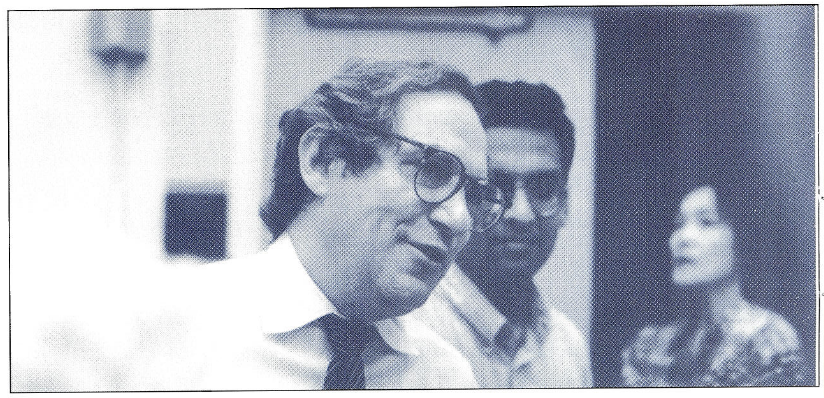
well as on college campuses around the country. Portraying about a dozen different characters during the 90-minute performance, Kornbluth related the hilarious and poignant story of his own transformation from the kid who knew everything in math class, to the Princeton math major who "hits the wall" when confronted with the notion of limits in calculus class.

The Math and Media conference ended without an answer to one of the main questions it raised: What do mathematicians hope to accomplish by influencing media coverage of their subject? If the hope is that increased media coverage will translate into increased financial support for mathematics, that hope might be misplaced. Consider the example of NASA, whose highly successful public relations organization captivated the nation with full-color footage of space exploration but whose budget has shrunk dramatically in recent years. Is celebrity the goal? It is hard to imagine that many mathematicians yearn for the harsh and fickle limelight accorded to celebrities today. Perhaps the aim is simpler: To edify the public about an important part of human culture. This is the most exalted and difficult goal of all. What it requires is a new orientation for media coverage of mathematics, one that makes a place for all the important developments in mathematics, not just the most easily explainable. It also requires mathematicians to think deeply about how to describe in plain terms why these developments are important. The media, with their newfound attention to mathematics, may well be ready to listen.





Collaboration with San Francisco State University

By Sheldon Axler, Chair, Department of Mathematics

The Mathematics Department of San Francisco State University, one of our most recent Sponsoring Institutions, has received a three-year grant from the National Science Foundation to establish a collaboration with MSRI. SFSU is one of the country's largest centers of education for minority students. This collaboration will include, each semester, an advanced undergraduate topics course based on one of the emphasis programs at MSRI that semester. This will begin in the Spring of 1999, based on the program on Random Matrix Models and their Applications. The topics course will be taught by SFSU faculty, featuring monthly talks by leading mathematicians who are visiting MSRI to participate in its program. The SFSU faculty member who teaches the topics course will be a participant in the MSRI program, conducting research in the area during the semester. In addition, the project will support four SFSU graduate students to attend talks and workshops at MSRI.

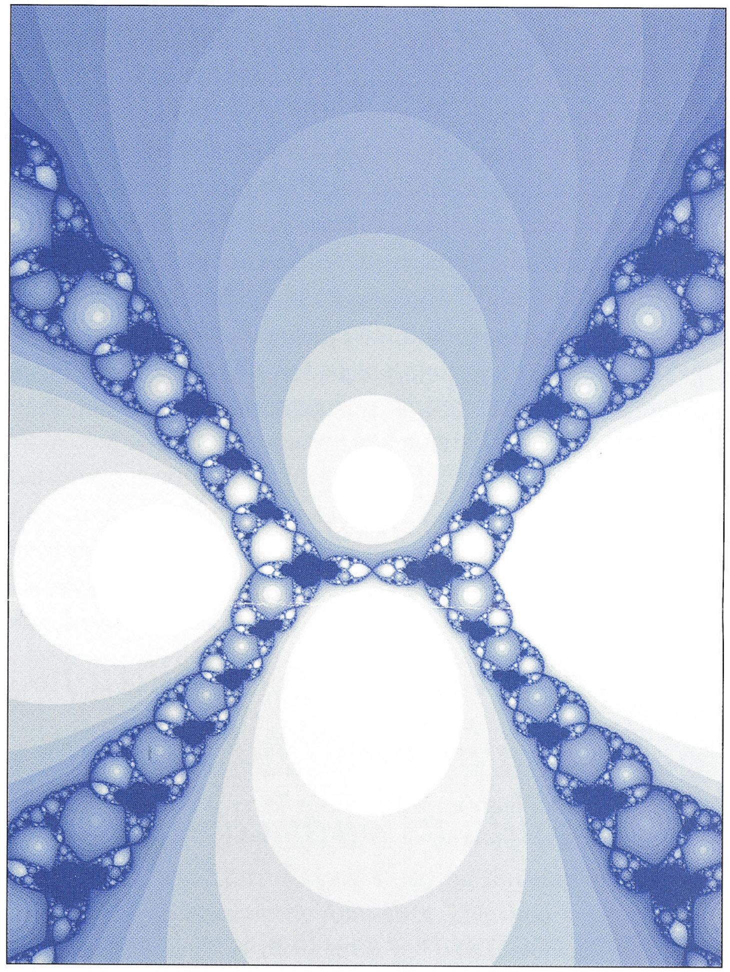
Funds from the grant will be used to provide SFSU with the equipment it needs to access MSRI's streaming video library, so that its students can view talks given at MSRI. In addition, the course lectures will be videotaped and posted as streaming video on the MSRI website. Because research mathematicians rarely pitch their work to undergraduates, as they will in the SFSU-MSRI collaboration course, these lectures will be of significant value to the entire mathematical community. In addition, the lectures will also be made available on CD-ROM for dissemination to other universities.

Foundations of Computational Mathematics

August to December 1998

Organizers: Felipe Cucker (Hong Kong), Arieh Iserles (Cambridge), Tien Yien Li (Michigan), Mike Overton (New York), Jim Renegar (Cornell), Mike Shub (IBM,Yorktown Heights), Steve Smale (Hong Kong) and Andrew Stuart (Stanford)

Solving systems of equations is among the oldest and thorniest problems of mathematics. For centuries mathematicians have alternated between 'negative' and 'positive' results. An example of a negative result is the famous theorem of Abel and Galois, who proved that there exist polynomials of degree 5 in one variable whose zeros cannot be expressed by means of the four basic arithmetic operations and root extractions. A positive result of tremendous applicability is Newton's method. It associates to a function f (e.g. one of the polynomials just mentioned) another function N_f with the following property: if z is a point close enough to a root ζ of f then the sequence defined by $z_0 = z$, $z_{i+1} = N_f(z_i)$, converges to ζ. Moreover, in general, this convergence is very fast: the distance between ζ and z_i is at most that between ζ and z_0 divided by $2^{2^{i-1}}$. In more recent years mathematicians have focused on questions such as what exactly "close enough" or "in general" mean in the description above. Can one decide whether a point z is close enough? Will the sequence starting with a specific z_0 converge to a root of f? In the accompanying picture bad initial points (those for which the sequence fails to converge to a root) for the polynomial $f(z) = (z^2-1)(z^2+ 0.16)$ are the darkest.



Note the complicated pattern arising from this set of bad points. The clear implication is that the question whether an initial point is 'good' might well be an undecidable problem. Yet, to formally prove such an assertion requires the development of a formal computational model and a deep understanding of the geometry of decidable sets. It has been accomplished very recently.

The situation described above is in a sense very simple. In general, one needs to deal with functions in several variables and occasionally inequalities must be considered as well. Such problems are ubiquitous in optimization, where the goal is to find x maximizing (or minimizing) a function f subject to inequality constraints. Moreover, in a realistic computing environment it is important to consider the effects produced by round-off and other sources of error.

The goal of the FoCM program at MSRI is to develop a better understanding of how mathematics underpins numerical calculations. The program addresses the four following issues and the interplay among them:

Complexity: What is the 'cost' inherent in a computational problem that no algorithm can circumvent?

Optimization: How to find the best value of a function (or a functional) subject to constraints?

Homotopy: How does the knowledge of the solution of a 'nearby' problem assist us to compute the problem in hand?

Geometric integration: How to compute approximate solutions that share qualitative properties with the true solution of the problem?

All these issues share two important characteristics. Their methodology requires deep and demanding pure mathematics, while their understanding is vital to the development of a new generation of powerful and reliable algorithms in scientific computing.

Symbolic Computation in Geometry and Analysis

by Marie-Françoise Roy and Michael F. Singer

A unifying theme of this program has been *the calculation of properties of solutions of systems of equations*; solutions of polynomial equations and functions satisfying differential equations. A special feature of the research done by participants in this program has been the interaction with symbolic computation, that is, the ability to do exact calculations with such equations on a computer. Symbolic computation has been both the beneficiary and inspiration for many results in mathematics and no area has seen this more than the study of polynomial and differential equations. We will describe our program by giving a general overview of the kinds of problems being considered and then give three specific examples of research carried out during the semester.

A special feature of the research in the program is the ability to do exact calculations with polynomial and differential equtions on a computer.

When studying systems of equations, several general questions naturally arise. The first is to *describe the size and geometry of solutions.* For example one would like to determine if a system of equations is consistent, that is, has any solutions and, if so, count them if there are only a finite number or measure their dimension if there are an infinite number. The Euclidean Algorithm allows us to do this for polynomial equations in one variable and Gaussian Elimination allows us to do this for linear equations in several variables. Groebner bases and the Buchberger algorithm generalize these two techniques and allow one to answer these questions for complex solutions of general polynomial equations in several variables. Analogous techniques allow one to answer these questions for systems of linear partial differential equations and active research is generalizing this to systems of nonlinear differential equations. Finding real solutions of polynomial equations or inequalities with optimal complexity depends on tools from differential geometry to reduce to one variable and then uses improvements of real root counting, that is generalizations of Descarte's Law of Signs and Sturm Sequences. One can ask for more refined information about solution sets such as: how can one decompose these sets into indecomposable pieces or, for polynomial equations, determine the number of connected components. Algorithms to answer these questions have been developed.

Another question being attacked is the problem of *parametrizing solutions or expressing them in terms of special functions.* For example, there are techniques which allow one to decide if the solution set of a system of polynomial equations can be parameterized by rational functions and if so, find such a parametrization. For linear differential equations, a Galois theory gives necessary and sufficient group theoretic conditions for such an equation to be solvable in terms of quadratures. Furthermore these conditions lead to algorithms to effectively do this.

A third question is to understand how one can *take advantage of the form of the equations to describe properties of their solutions.* For example, techniques have been developed to use symmetries of the equations to reduce their complexity, that is reduce the equations to simpler equations. Another example is our ability now to measure the sparseness of a system of polynomial equations and relate this to the size of the solution set.

Finally, we have been studying the *interaction of these methods with numerical techniques as well as applications to other fields.* Homotopy methods deal with the problem of deforming complicated systems of equations into simpler systems, solving the simpler systems and then deforming these solutions into solutions of the original system. Groebner basis techniques allow one to make a judicious choice of the simpler systems and the way one deforms the original system to this simpler system before one uses numerical methods to deform the solutions of the simpler system to solutions of the general system. Ideas from Groebner bases have been used in integer programming and signal processing as well. Techniques developed to understand real solutions of polynomial systems as well as Groebner basis techniques have been used in robotics and mechanism design.

Most importantly, the above tools have been implemented in commercially available general symbolic software packages such as Maple and Mathematica as well as several special purpose packages. Many of the participants in the program are involved in developing the underlying mathematics and algorithms as well as improving efficiency and implementing these tools.

We now turn to some specific examples of research in this area.

We start by discussing enumerative aspects of real algebraic geometry. According to Schubert, "enumerative geometry counts the number of geometric figures having specified position with respect to general fixed figures". In traditional algebraic geometry, this is naturally interpreted as the number of complex solutions of specific polynomial systems. Finding whether all solutions are real is a very natural question though. Real enumerative geometry identifies the cases where there are configurations where all solutions are real. For problems of enumerating linear subspaces meeting general fixed subspaces, Shapiro and Shapiro conjecture that if the fixed subspaces osculate a real rational normal curve, then all the incident subspaces are real. Part of their conjecture has been proven this autumn at MSRI. The software developed for polynomial system solving (GB by Jean-Charles Faugère, Real Solving by Fabrice Rouillier, PHC by Jan Verschelde) confirmed instances of it in the spectacular cases of 462 four-planes meeting 12 three-planes in R^7, and 1430 8-planes meeting 16 2-planes in R^{10}. Inspired by these computations, Frank Sottile has shown for any m, p there exist mp such osculating m-planes such that all incident p-planes are real.

Consider now a linear ordinary differential equation of degree n with coefficients that are rational functions. As mentioned above, a Galois theory allows one to associate a group to any such equation which measures properties of the differential equation. In particular, it is a finite group if and only if all solutions of the equation are algebraic functions. In this case the Galois group has an analytic interpretation. Given n linearly independent analytic solutions defined in the neighborhood of some nonsingular point of the equation, they can be analytically continued along any closed loop in the Riemann Sphere minus the singular points of the equation. When one returns to the original point, one still has a fundamental set of solutions which differ from the original set by multiplication by an invertible $n \times n$ constant matrix. The set of matrices gotten in this way will form a group called the monodromy group. In general this will be a subgroup of the Galois group but when the Galois group is finite, these two groups coincide. Having only algebraic solutions also puts restrictions on the singular points of the equation. At such a point the ith coefficient can have at worst a pole of order $n-i$ and an equation whose coefficients satisfy this condition at all of its singular points is called a Fuchsian equation. One can now ask the inverse problem: Given a finite group, does there exist a Fuchsian equation having this as its Galois/monodromy group? At MSRI this semester, Marius van der Put and Felix Ulmer have finished a paper that gives a constructive solution to this problem. For example, they can show that given a finite group $G \subset GL(n,Q)$ with generators g_0, g_1, g_∞ satisfying $g_0 g_1 g_\infty = 1$, one can produce a scalar Fuchsian differential equation L with singular points only at $0,1,\infty$ and of order n, such that g_0, g_1, g_∞ are the monodromy matrices for loops around $0,1,\infty$. Their algorithm is quite effective at least for $n = 2, 3$ and they have used it to find many new examples of such equations.

Another active area of research in Symbolic Computation has been the study of A-hypergeometric functions. In the 1980's Gel'fand, Kapranov and Zelevinsky proposed a general theory of hypergeometric functions in several variables. They are solutions of an overdetermined system of linear PDEs, related to finite dimensional representations of a complex torus, and associated with an integer matrix A and an exponent vector β. The study of A-hypergeometric functions is intimately connected with the geometric and combinatorial properties of the integral convex polytope P spanned by the columns of the matrix A. For example, Gelfand, Kapranov and Zelevinsky constructed series solutions (for non-resonant exponents) associated with regular triangulations of P. Very recently, Saito, Sturmfels, and Takayama extended the construction of series solutions to all exponents. Hypergeometric functions are related to polynomial systems by the fact that powers of roots of univariate polynomials and local multidimensional residues are A-hypergeometric when viewed as functions of the coefficients. Their sum over all roots yields rational functions which may be effectively computed. In their ongoing project, Cattani, Dickenstein, and Sturmfels are studying the classification of rational A-hypergeometric functions. The theory of oriented matroids provides the context for the parameterization of the Laurent polynomial solutions by certain bounded regions in an oriented hyperplane arrangement. On the other hand, general rational solutions have series expansions whose coefficients are of independent interest. This approach yields an interesting combinatorial description of rational hypergeometric functions even in the classical univariate case. Moreover, rational hypergeometric functions may be explicitly described in terms of geometric objects, namely residues in toric varieties, while the number of independent rational solutions is related to algebraic properties of the toric ideal associated with A.

We feel that the events of this semester underscore the remarkable and beneficial connections between analysis, geometry and symbolic computation.

The authors would like to thank Eduardo Cattani, Michel Coste, Alicia Dickenstein, Marius van der Put, Frank Sottile, and Jan Verschelde for their assistance in preparing this article.

Foundations of Computational Mathematics

Geometric integration: computation of differential equations on manifolds

by Arieh Iserles

Traditionally, differential equations have been computed subject to an underlying (and mostly unspoken) assumption that the solution evolves in a Euclidean space. In recent years there is a growing recognition, though, that in many important instances the right configuration space for numerical work is a *differentiable manifold.* The philosophy underpinning this outlook is that, ideally, a numerical solution should reflect qualitative properties of a mathematical structure that it endeavours to approximate. Such properties can be often expressed conveniently in the terminology of differential topology, e.g.

Conservation laws, whereby the solution evolves on a given manifold;

Lie symmetries and other differential invariants that evolve on the tangent bundle of a manifold; and

Invariants that evolve on the cotangent bundle of a manifold, in particular *symplectic forms.*

Numerical methods that address themselves to the conservation of invariants and geometric structure are known collectively as *geometric integration,* a subject that has emerged in the last few years and is a focus of a lively attention, at present centered on the special semester at MSRI.

Techniques of geometric integration originated in different mathematical backgrounds. Some workers in the subject are motivated by specific problems in application areas (weather forecasting, solid mechanics, molecular dynamics, celestial mechanics), others are numerical analysts wishing to change the emphasis of their subject, yet others are pure mathematicians with an interest in computation. The value of bringing together this diverse throng for a period of intensive discussion is truly immense and, beyond any individual result, the main benefit of the geometric integration component of the "Foundations of Computational Mathematics" programme is likely to be the formulation of coherent mathematical description of the subject and the identification of broad algorithmic techniques that will carry it forward in the next decades.

Three such techniques, which underlie a number of diverse algorithms and applications, are:

Splittings: In its most basic manifestation, this approach consists of replacing a complicated flow $y(t) = \varphi_t y(0)$ by $\hat{y}(t) = \psi^{[1]} \psi^{[2]} \ldots \psi^{[s]}_t y(0)$ where each constituent flow $\psi^{[i]}_t$ is easy to approximate (or perhaps even evaluate exactly) and so that the product maintains the correct geometry (as well, of course, as representing a sufficiently small perturbation of the original flow ψ_t). An early success of splittings has been in symplectic integration of Hamiltonian flows, but recently the technique was extended to more general equations.

Lie-group methods: Many invariants can be formulated in the terminology of differential geometry as flows on homogeneous spaces (i.e., on manifolds that are subjected to a transitive Lie group action). It is possible to show that, as long as we can discretize equations in a Lie group G, the method can be extended to an arbitrary homogeneous space acted upon by G. Several exciting Lie-group methods have emerged recently. Their common denominator is that the equation is pushed to the underlying Lie algebra, approximated there and the outcome is pulled back to the group.

Adaptivity and Lie symmetries: Effective solution of time-evolving partial differential equations requires adaptive approaches to the discretization of spatial variables. Such equations often display important Lie symmetries, the most familiar example being self-similarity. A recent development is the use of adaptivity to recover symmetry.

The future of the subject will, in all likelihood, hinge on the attempt to weave techniques of this kind into a seamless theory, and this has been the subject of intense activity at MSRI in the last two months.

Geometric integration does not inhabit a separate universe of its own and it is vital in early days of the subject to explore its interconnectedness with other areas of mathematical computation. The activity benefited from interaction with the wide range of other mathematicians resident at MSRI. This has led to fascinating new insights and ideas about the role of nonlinear dynamical systems, complexity, homotopy and, perhaps most importantly, symbolic analysis in the context of geometric integration.

Workshop on Symbolic Computation in Geometry and Analysis

October 12-16, 1998

by Hugo Rossi

As part of the Fall 1998 program on Symbolic Computation in Geometry and Analysis, MSRI hosted a weeklong workshop on Symbolic Computation, October 12-16, 1998. The workshop was organized by Eberhard Becker, Y. N. Lakshman, Michael Singer and Peter Stiller. In their announcement they described the workshop's scope in this way:

"The aim of the workshop is to stimulate interaction between the long-term participants in the SCGA program and other members of the Symbolic Computation and general mathematical community who have not been able to come for an extended stay. It is hoped that holding such a workshop halfway through the special semester will increase the impact that the special semester will make on the mathematical community as well as allow for that community to influence the direction of research at MSRI."

This in fact describes very well the purpose of all the topical workshops at MSRI.

The computer was presented sometimes as a valuable tool, at others as a partner in research.

The workshop was focused on the use of computation in a wide range of theoretical mathematics, as well as the interaction of numerical and symbolic computation. The computer was presented sometimes as a valuable tool, at others as a partner in basic research. Doron Zeilberger speaks of his computer, Shalosh B. Ekhad, as an enthusiastic collaborator and coauthor of articles. In the first few minutes of his talk, Doron, in his inimitable and sometimes caustic, but always humorous style, described his sense of this workshop. Here are some excerpts:

"I want to congratulate MSRI and the organizers for such a stimulating and interesting conference. I have learned so much already and I am sure I will learn even more in the remaining two days. All the talks were excellent - with no exception! Lakshman's talk beautifully showed the future of interfacing numerical and symbolic computation, getting rid of the obsession of mathematicians with exact answers. Many times it is impossible to get exact answers, and numerical people knew it. But interfacing both cultures – the numeric and symbolic – was, I think, a big breakthrough. David Eisenbud's talk, although not strictly symbolic, was stimulating too – it is amazing how he can master the blackboard so well and deliver as much if not more, in as much clarity and facility, as most transparencies talks. I was able to find out what the so-called MRC conjecture is; I never knew anything about it. And finally I know what the twisted cubic is – unlike David Bayer who has been studying it all his life. Now I believe I know what it is – although not at the same depth as Dave Bayer. Marc Chardin's talk was over my head, but still I got the message that it is very deep and interesting stuff. Reinhard Laubenbacher's talk told us that symbolic algebra and symbolic computation, computer algebra and combinatorics together will solve the traffic problems of LA and Saddam

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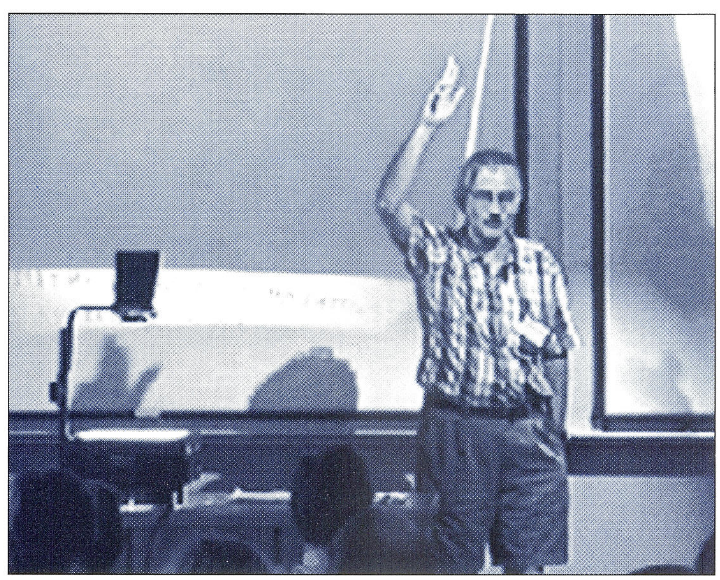


Workshop on Symbolic Computation
(continued from page 9)

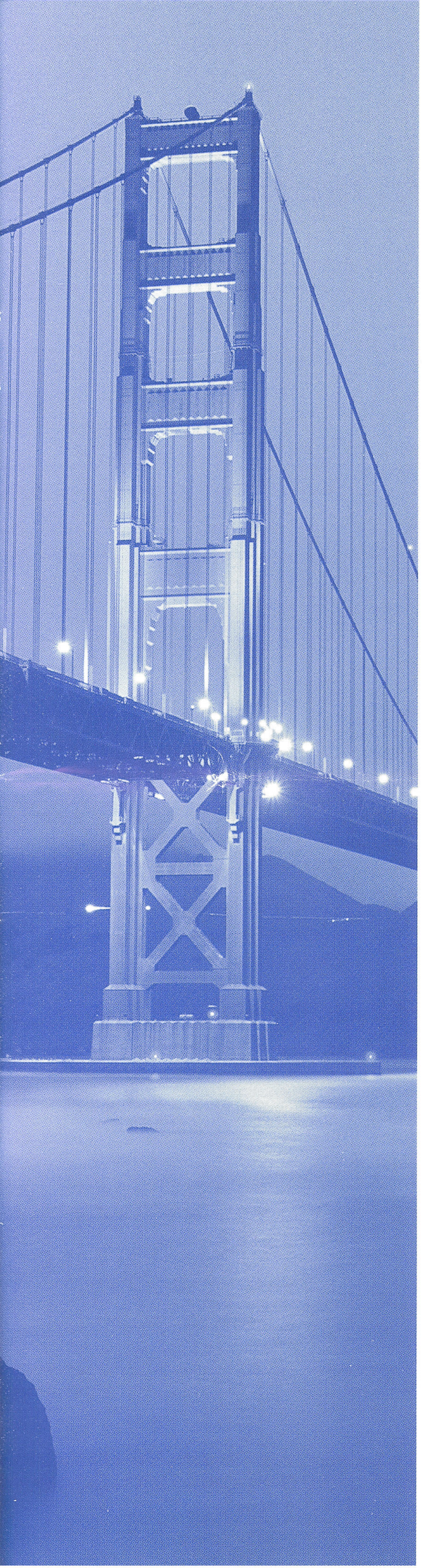
Hussein's craziness combined! So, although it is not yet completely implemented, it is very promising. Also H. Park's talk was very interesting and clear: how symbolic computation can be used in real life. Well, that is the main application nowadays – entertainment; everything else is subordinate to entertainment: the internet, watching TV. Park's contributions to this are very gratifying. Another thing which was very interesting although a little bit dry was Hendrik Lenstra's talk. But he has his dry sense of humor, and it was a very nice talk mathematically. James Shank also gave a very stimulating talk about SAGBI bases – I never knew about SAGBI, but now I know a little bit. Also, he mentioned my good friend, Gert Almkvist, so I have to write him an email that he got mentioned.

"Vickie Powers and Bruce Reznick - a double feature - were also very entertaining and very stimulating. It was very nice to meet Vickie Powers because I really enjoyed her article in the *Monthly* about two years ago about how Bruce won a bottle of champagne from Eberhard Becker for solving a long-standing problem on sums of squares and positive polynomials. I could tell from her article that she is a great speaker and I was not wrong. And of course I like Bruce Reznick's inscription on his tombstone – I hope not too soon! "Real sums of squares do not wimp out with generic theorems!" And William Sit's was a very nice talk: many things in addition to the beautiful method of how symbolic computation can help immensely with numerical computations – and being so modest he didn't make explicit enough how important this is, so some people in the audience expressed some skepticism – nevertheless, it is very interesting, I think. And also something I was very amused by - he introduced a new unit of complexity - cups of coffee: how many cups of coffee it takes to obtain a result.

"Willy Hereman's talk fulfilled a dream of mine – all my life I wanted to know about KdV equations in dynamical systems. I already have bought five books, hoping to get around to it. But then I'd look in the introduction, you know, blah blah blah, but, well, I had other things to do so I'd never go beyond page five. But from these forty minutes of Willy Hereman's talk I think I understand what KdV and all that stuff is. So to understand something you have to program it – then you really understand something; otherwise it is really all abstract nonsense. Martin Sombra's talk was also nice, because doing Nullstellensatz was a big deal with Bernstein and Yger – a triumph to complex analysis. But now it turns out that it is a triumph allright – but not to complex analysis! That was really a red herring and everything was really the algebra embedded in it.

"All the talks here were excellent – super excellent. But only one talk was truly outstanding. Only one talk, in addition possibly to my own talk, I'll remember until my dying day and perhaps beyond. That was Dave Bayer's talk. I was so inspired by it that I'm even paying 25 bucks to buy the video for this fine talk. A modest price; I would gladly have paid much more. What was nice about Dave Bayer's talk – in addition to its technical content, which I admit I didn't quite understand - was the superstructure, the message behind it, about the joy of doing mathematics. And his debunking certain sacrosanct ideas in mathematics, for example, the obsession with minimality. I think that will help me a lot in my own research. So that was his pep talk, which was so stimulating. Also, the amount of metaphors per second exceeded any talk I ever heard before. If ever there will be an MTV – not Music, but a Math TV – there Bayer will be a superstar! That was really stimulating. One of the many metaphors in Dave Bayer's talk was: you're too busy hiking. Stop being so busy hiking – stop and enjoy the view. This was a really good thing – too many people are so busy hiking so they don't pause to enjoy the view. Even though Bayer is very good at metaphors, he doesn't heed his own metaphor at its origin: the hike, the beautiful hike David Eisenbud organized yesterday, (as usual every Wednesday). Dave Bayer was seen to hike a lot, but I didn't see him stopping to enjoy the view as often as he could have. But, mathematically, I think, he does".

From here Doron moved to the mathematical subject of his own talk, which was, along with all the remaining talks, just as he prophesized – excellent. The impressions he expressed are of course his own, and do not necessarily reflect the views of any one else at MSRI; however, they do convey a bit of the excitement with and the joy of doing mathematics which permeated the atmosphere in the building that week.



Remaining 1998-1999 Events

January 19-23, 1999:
Introductory Workshop on Random Matrix Models and Their Applications

February 19, 1999:
Mathematics in the Plays of Tom Stoppard

February 8-12, 1999:
Gröbner Bases, cosponsored by MSRI and CIMAT, Guanajuato, Mexico

February 22-26, 1999:
Workshop on Random Matrices, Statistical Mechanics, and Integrable Systems

April 12-14, 1999:
Self-Assembling Geometric Structures in Material Science: The Geometry of Interfaces in Mesoscopic Materials

June 4-5, 1999:
CAD/CDF Workshop

June 7-11, 1999
Quantum Chaos, GUE Conjecture for Zeros of Zeta Functions, Combinatorics, and All That

June 21 - July 2, 1999:
Summer Graduate Program I, L. Mahadevan, Nonlinear Dynamics with Applications to Biology and Fluids

July 12-23, 1999:
Summer Graduate Program II, R. Bryant, Geometry in Differential Equations.

July 16-18, 1999:
Conference in honor of Olga Taussky-Todd, American Women in Mathematics

1999-2000 Programs

August 1999 - May 2000
Noncommutative Algebra
Organizers: Michael Artin, Susan Montgomery, Claudio Procesi, Lance Small, Toby Stafford, Efim Zelmanov

August 1999 - December 1999
Galois Groups and Fundamental Groups
Organizers: Eva Bayer, Michael Fried, David Harbater, Yasutaka Ihara, B. Heinrich Matzat, Michel Raynaud, John Thompson

March 6 - April 28, 2000
Numerical and Applied Mathematics

Homogenization and Effective Media Theories, chair: M. Vogelius
Superconvergence in Finite Element Methods, chair: L. Wahlbin
A posteriori Error Estimation and Adaptive Approaches in the Finite Element Method, chair: R. Bank
Elastic Shells: Modeling, Analysis and Numerics, chair: D. Arnold

Aug 16-27, 1999:
Joint Introductory Workshop in Noncommutative Algebra and with Galois Groups and Fundamental Groups (AMS von Neumann Symposium)