Fall 2012

EMISSARY

Mathematical Sciences Research Institute

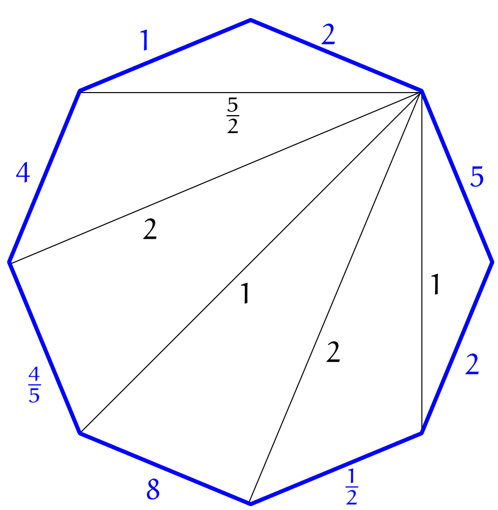
www.msri.org

Lively Work, Lively Celebrations

A word from Director Robert Bryant

Welcome to the Fall 2012 edition of the Emissary! Much has happened at MSRI since our last issue, and I hope that you'll enjoy the informative articles on our scientific programs as well as the news items about the accomplishments and recognitions of our current and former members and the introductions of our new staff members.

This fall's programs, Commutative Algebra (which is a year-long program, continuing in the spring) and Cluster Algebras, got off to an exciting start with a combined two-and-a-half-week opening workshop that proved extremely popular. More than 180 participants registered for it, and the Simons Auditorium was filled to capacity. The blackboard discussions taking place all over the Institute, both inside and outside, were even livelier than usual, and the excitement of the participants was contagious. Now that the opening workshops are over, the numbers have thinned out, but the lively interaction in the seminars and the informal discussions in the hall are keeping up the activity level. You'll find articles about each program and some of the scientists and postdoctoral fellows who are participating in them further along in this issue.



Starting from elementary school and working up to the cutting edge of current research: See the article about the Cluster Algebras program starting on page 8.

David Eisenbud

The Alan Turing celebration: (from left to right) Richard Karp, Director of the Simons Institute for the Theory of Computing; Robert Bryant; and Andrew Hodges, acclaimed biographer of Turing and speaker at the event.

On September 4, we officially welcomed our new sibling, the Simons Institute for the Theory of Computing, to Berkeley by jointly sponsoring with them a celebration of the 100th anniversary of the birth of Alan Turing, one of the pioneers of computer science and artificial intelligence. The event, Alan Turing: A Centenary Celebration, which we held in the auditorium of the Berkeley City College in downtown Berkeley, was an enormous success. It was filled to overflowing, with more than one hundred people standing, for a presentation by Andrew Hodges, the author of the acclaimed Turing biography, *Alan Turing: The Enigma*. The presentation was followed by a panel discussion moderated by Richard Karp, the director of the Simons Institute for the Theory of Computing, that featured Hodges together with luminaries of the fields in which Turing worked: Martin Davis, Peter Norvig, Luca Trevisan, Don Knuth, and Dana Scott.

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Standing room only at the Alan Turing celebration

A Word from the Director

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Several MSRI people have been in the news this summer and fall, and it is a pleasure to take this opportunity to congratulate them:

First, Endre Szemerédi, one of MSRI's research professors in our Fall 2008 program, Ergodic Theory and Additive Combinatorics, was awarded the 2012 Abel Prize by the Norwegian Academy of Sciences and Letters "for his fundamental contributions to discrete mathematics and theoretical computer science, and in recognition of the profound and lasting impact of these contributions on additive number theory and ergodic theory."

Next, Professor Ricardo Cortez, a professor at Tulane and one of the directors of MSRI-UP, our celebrated mathematics summer research program for undergraduates that emphasizes participation by underrepresented minority students, as well as past chair of MSRI's Human Resources Advisory Committee, was awarded the 2012 Blackwell–Tapia prize "in recognition of his outstanding contributions to the fields of fluid dynamics and mathematical modeling, and for serving as a role model for diversity in the mathematical sciences."

Erlend Aas / The Abel Prize

Endre Szemerédi

Ricardo Cortez

Finally, the eight members of the US team who participated in the 2012 China Girls Math Olympiad, who were jointly sponsored by the Mathematical Association of America and MSRI, turned in an outstanding performance in the August competition in Guangzhou, PRC, winning three gold, four silver, and one bronze medal! For more information about this amazing group of talented young mathematicians, see the story below in this Emissary.

This summer and fall, we are pleased to welcome two new staff members: Alissa Crans, our new Director of Educational and Outreach Activities, and Mick Santos, our new workshop coordinator. (You can find more about each of them below in this Emissary.) Both Alissa and Mick are as excited to be joining the team at MSRI as we are to be welcoming them. When you next come to visit MSRI, please stop by the administration offices and meet them.

Another change for this issue is that Silvio Levy, who served for so many years as MSRI's librarian and editor of the Emissary, and who continued to edit the Emissary on a freelance basis for several years after his retirement, has taken on other responsibilities that will no longer leave him time to serve as our editor. In his place we welcome Tracy Hicks, who lives here in the Bay Area and has had extensive experience editing newsletters and science articles.

Looking forward to 2013, I'm excited about the January launch of the program Mathematics of Planet Earth 2013, which will be a yearlong international collaboration of mathematics institutes around the world to draw attention to the many ways in which mathematics is used to help our world address issues of the environment, climate, sustainability, and health. For more information about this exciting program (and MSRI's role in it) please visit mpe2013.org. One of the upcoming events we are particularly excited about is the public lecture we have planned at the San Francisco Palace of Fine Arts on March 4 by Dr. Emily Shuckburgh, who is renowned for her study of the oceans and their effect on the dynamics of the Earth's weather. We hope to see you there for this exciting event!

Lastly, I want to take this opportunity to express a special, personal 'thank you' to MSRI's donors and sponsors! Each fall, we include a list of the wonderful supporters whose generosity has made possible all of the excellent programs and outreach that MSRI provides to the community and to our society, and those folks (you included!) have our most sincere thanks.

Boij–Söderberg Theory:
Syzygies and Cohomology

David Eisenbud and Frank-Olaf Schreyer

One of the great motivating sources of ideas, examples, problems and applications for commutative algebra is algebraic geometry, the study of geometric forms (called algebraic varieties) defined by polynomial equations. In fact, ideas and examples from algebraic geometry are important in almost every branch of mathematics, from number theory to partial differential equations and mathematical physics. Commutative algebra comes into the study via the set of functions — say, with values in the complex numbers — defined on (perhaps some open set) of the variety; since you can add and subtract and multiply such functions, they form a commutative ring. With the work of Zariski, Serre, Grothendieck and many others, commutative algebra became a more and more central part of algebraic geometry, and geometric problems became correspondingly central in commutative algebra.

However, the roots of the interaction between commutative algebra and algebraic geometry go back much further. A cornerstone of the development was laid by David Hilbert. In a series of papers around 1890 he proved four of the results that make the links between the subjects really useful: His “Nullstellensatz” is the foundation of the interaction; his “Basis Theorem” is the fundamental finiteness result in the subject; his “Syzygy Theorem” is the basis for many of the computations we can make in the subject today and is one of the starting points for the use of homological algebra; and finally, using the Syzygy Theorem he established the existence and some of the properties of the “Hilbert Polynomial”, which is the first source of discrete invariants of varieties.

Of course plenty of questions remain open — even questions about these basic invariants. But a recent step forward, the development of a subject now called Boij–Söderberg Theory, has significantly advanced our understanding. In the rest of this article we will describe the background and beginnings of this theory.

Background of the Theory

The most fundamental numerical invariant Hilbert deduced from his syzygy theorem is the Hilbert polynomial. It gives a way of extracting discrete geometric invariants from a projective algebraic variety. It is defined as follows: If $X \subset \mathbb{P}^n$ is a variety, say over the complex numbers $\mathbb{C}$, then the *homogeneous coordinate ring* S_X *of* X is the factor ring

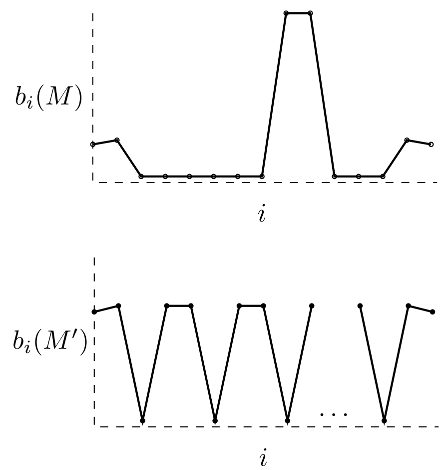
$$S_X := \mathbb{C}[x_0, \dots, x_n]/I_X,$$

where I_X is the ideal of all homogeneous polynomials vanishing on X. (Similar considerations hold over any base field, and nearly everything we will discuss in this exposition can be made independent of characteristic.) Hilbert showed that the dimension of the d-th graded component of S_X is equal, for large integers d, to the value $P_X(d)$ of a polynomial P_X. The polynomial P_X is now universally called the Hilbert polynomial of X.

The Hilbert polynomial P_X captures the most important invariants of X. Its degree is the dimension of X. If, for example, X is a Riemann surface (dimension $= 1$) embedded in some way in projective space, then $P_X(d) = ad + b$ is a polynomial of degree 1. The constant coefficient b tells us the genus g of X — the only topological invariant of X as an abstract manifold (more precisely, $b = 1 - g$) — and the linear coefficient a is the number of points in which a general hyperplane of $\mathbb{P}^n$ will meet X — the only other topological invariant of the embedding $X \subset \mathbb{P}^n$.

Hilbert's theorem is actually more general. It applies to any graded module, such as the module of twisted global sections of an arbitrary sheaf on $\mathbb{P}^n$, of which the homogeneous coordinate ring is a special case.

There are two useful and well-known ways to give invariants that refine the Hilbert polynomial: the *graded Betti table* and the *cohomology table*. Until recently, the exact connection between them was unclear, but the development of Boij–Söderberg theory in the last six years has made it precise: they are, in a natural sense that we will describe, dual to one another!



For a module M over a regular local ring with residue field K, the graph of the function $i \mapsto \beta_i(M) := \dim \mathrm{Tor}_i(M, K)$ can have almost any shape. The figure is from Berkesh–Erman–Kummini–Sam (2011) .

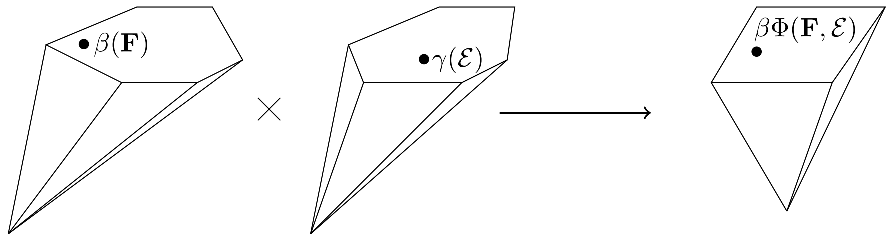
What is a Betti Table? First the definitions. We begin with a finitely generated graded module M over the polynomial ring

$$S := \mathbb{C}[x_0, \dots, x_n].$$

We will compare it to a free module by choosing a minimal set of generators for it, and noting their degrees; the free module “most” like M is the one with generators of those degrees. By way of notation, we suppose that there are $\beta_{0,j}$ generators of degree j (so that $\beta_{0,j} = 0$ for almost all j) and write the free module as

$$F_0 = \oplus_j S(-j)^{\beta_{0,j}}.$$

There is a natural surjection $F_0 \to M$ sending generators of F_0 to generators of M and preserving degrees. If M was not actually free, then this surjection has a kernel, called the *first syzygy module of* M.



The duality between Betti tables and cohomology tables involves three cones. Namely, given the Betti table $\beta(\mathbf{F})$ of a complex of S-modules, and the cohomology table $\gamma(\mathcal{E})$ of a complex of coherent sheaves on $\mathbb{P}^n$, the categorified pairing produces the Betti table $\beta(\Phi(\mathbf{F},\mathcal{E}))$ of a complex of $K[t]$-modules. (Figure from a preprint of Eisenbud–Erman (2012).)

We may analyze M — or, better, *resolve* M more and more finely by repeating this process over and over, to obtain a *minimal free resolution of* M. Hilbert proved that the process terminates after at most $n+1$ steps, because the syzygy module of the syzygy module of the ..., repeated $n+1$ times, will be a free module. Thus we obtain an exact sequence

$$0 \to F_m \to \cdots \to F_1 \to F_0 \to M \to 0$$

of graded S-modules, where the maps all preserve degrees and the F_i are all free modules. We may write $F_i = \oplus S(-j)^{\beta_{i,j}}$ for some collection of numbers $\beta_{i,j}$, almost all equal to 0. These $\beta_{i,j}$ depend only on M and not on the choices of generators that we made, and the collection of numbers $\{\beta_{i,j}\}$ is called the *Betti table* of M.

The Betti table really does refine the Hilbert polynomial as an invariant; indeed, one may compute the dimension of the degree d part of M as the alternating sum of the degree d parts of the modules F_i. Using the formula

$$\dim_{\mathbb{C}} S(-j)_d = \binom{n+d-j}{n}$$

we get

$$\dim_{\mathbb{C}} M_d = \sum (-1)^i \beta_{i,j} \binom{n+d-j}{n}.$$

Since $\binom{n+d-j}{n}$ is equal to the polynomial function

$$P_{S(-j)}(d) = \frac{(n+d-j)(n-1+d-j)\cdots(1+d-j)}{n\cdots 1}$$

for $d \geq -n+j$, we see that $\dim_{\mathbb{C}} M_d$ agrees with a polynomial function for all d such that

$$d \geq \max_{j \mid \beta_{i,j} \neq 0} -n+j.$$

In fact, this was Hilbert's original proof of the existence of the Hilbert polynomial!

What is a Cohomology Table? Finitely generated graded S-modules are almost the same thing as coherent sheaves on $\mathbb{P}^n_{\mathbb{C}}$: by a famous theorem of Serre, the category of coherent sheaves is equivalent to the category of finitely generated graded modules modulo the subcategory consisting of those of finite length. Explicitly, one may go from a coherent sheaf $\mathcal{F}$ to a module by taking the *module of twisted global sections*

$$\mathcal{F} \mapsto \Gamma_*(\mathcal{F}) := \bigoplus_d H^0(\mathcal{F}(d)).$$

The value at any integer d of the Hilbert polynomial of the module $\Gamma_*(\mathcal{F})$ is equal to the *Euler characteristic*

$$\chi(\mathcal{F}(d)) := \sum_{i=0}^{n} (-1)^i \dim_{\mathbb{C}} H^i(\mathbb{P}^n, \mathcal{F}(d)).$$

We define the *cohomology table of* $\mathcal{F}$ to be the collection of numbers

$$\gamma_{i,j} := \dim_{\mathbb{C}} H^i(\mathbb{P}^n, \mathcal{F}(j)),$$

and we see at once that the cohomology table of $\mathcal{F}$ determines the Hilbert polynomial, as well.

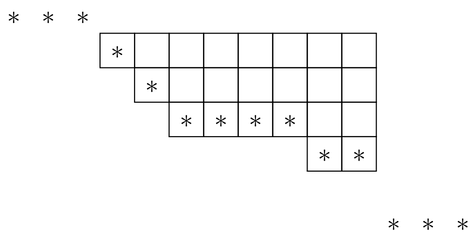
Boij–Söderberg Theory

Which tables of numbers occur as Betti tables or as Cohomology tables? A complete answer would solve many outstanding problems in algebraic geometry and commutative algebra, and seems very distant, although much interesting work has been done on the question.

In 2006 Mats Boij and Jonas Söderberg, pursuing a proof of the "multiplicity conjecture" of Herzog, Huneke and Srinivasan, decided to take seriously the fact that the Betti table of the direct sum of two modules is the (component wise) sum of the two Betti tables of the modules. Thus the collection of all Betti tables forms a semigroup embedded in the infinite lattice $\oplus_{-\infty}^{\infty}\mathbb{Z}^{n+2}$. Restricting themselves, for simplicity, to the case of Betti tables of modules of finite length, they realized that if a module M has a *pure* resolution in the sense that, for each i, the value of $\beta_{i,j}$ is nonzero for at most one j, then the Betti table of M will generate an extremal ray in the positive rational cone generated by the set of all Betti tables. They proposed the conjecture that this cone is closed, and that these are all the extremal rays. This statement does not tell us all the possible Betti tables — but it does tell us all the possibilities *up to a rational multiple*, a very big step forward. The conjecture seemed distant; for example, at the time it was made, few examples of pure resolutions were known.

The conjecture was proved by the two of us in 2008, after some preliminary work by Boij–Söderberg and Eisenbud–Fløystad–Weyman. The key turned out to be the realization that if $\mathcal{F}$ is a coherent sheaf on $\mathbb{P}^n$, then from the cohomology table of $\mathcal{F}$ one can immediately construct linear functionals on $\oplus_{-\infty}^{\infty}\mathbb{Z}^{n+2}$ that are non-negative on every Betti table; and that moreover the functionals constructed in this way precisely define the rational cone of Betti tables. Moreover, the relationship is dual — Betti tables define functionals that are collectively positive precisely on cohomology tables. Thus it sufficed to produce enough modules with pure — and thus extremal — resolutions, and enough coherent sheaves with similarly extremal cohomology tables (called by us "supernatural sheaves").

Details of all this, together with a more complete history, can be found in our paper [2009]. Here we content ourselves with a pretty picture, shown to us by Jerzy Weyman, that solves the problem of constructing supernatural sheaves in characteristic 0. Let $\lambda = (\lambda_{n-1}, \dots, \lambda_0)$ be a partition of $|\lambda|$ into n parts, and let Q be the tautological rank n quotient bundle on $\mathbb{P}^n$. One formulation of Bott's Theorem says that the positions of the non-zero cohomology groups of the Schur functor $S_\lambda Q$ in the cohomology table can be read from the Ferrers diagram, as in the diagram below, where we put $h^j(\mathcal{F}(k-j))$ in the (k,j)-th position. Here we show the case $\lambda = (8, 7, 6, 2, 0, 0)$; the bundle $S_\lambda Q$ on $\mathbb{P}^6_{\mathbb{C}}$ has nonzero cohomology groups only where the diagram has stars:



Our paper was more than enough to prove the multiplicity conjecture, and it has had a number of other applications. For example, Berkesch, Erman, Kummini and Sam used our constructions to answer the question of what "shapes" the graphs of the functions $i \mapsto \beta_i(M)$ could take when M is a module over a regular local ring with residue field K and $\beta_i(M)$ denotes the dimension of the K-vector space $\mathrm{Tor}_i(M, K)$. The answer, surprisingly, is: practically anything! — as indicated in the first figure in this article.

Among the many questions left open by our paper was a philosophical mystery. Each cohomology table contributed *many* linear functionals on Betti tables, and conversely each Betti table contributed many linear functionals positive on cohomology tables, so the two cones were *not* dual in the ordinary sense. What was the nature of this many-many duality? Eisenbud and Erman have answered this question, and at the same time opened up many possible further applications, by reinterpreting the duality as a pairing involving not two but three cones, as the figure at the top of the previous page, mediated by a pairing

$$D^b(S) \times D^b(\mathbb{P}^n) \xrightarrow{\Phi} D^b(\mathbb{C}[t]),$$

from the product of the derived categories of graded S-modules and of coherent sheaves on $\mathbb{P}^n$ to that of torsion $\mathbb{C}[t]$-modules.

Many other questions in this area remain open, and because of all the exciting developments, this field is one focus of the current program at MSRI.

Focus on the Scientist: Christine Berkesch

Ezra Miller

Christine Berkesch

Christine Berkesch grew up in Indiana and attended Butler University for her undergraduate studies. She then joined the Mathematics department at Purdue University, where she completed her Ph.D. in 2010. She received an International Postdoctoral Fellowship from the NSF to work with Mikael Passare at Stockholm University during the following academic year. In the first half of 2011, Christine also participated in the program on "Algebraic geometry with a view towards applications" at the Institut Mittag-Leffler. She is now on leave from her Assistant Research Professor position at Duke University.

Her research focuses on homological questions at the interface of algebra, geometry, and combinatorics. She has worked with hypergeometric systems on toric varieties as well as with monomial ideals, multiplier ideals, and free resolutions.

Christine's first major contribution used homological and combinatorial techniques to derive a formula for the rank — that is, the dimension of the solution space — of an arbitrary A-hypergeometric system. Christine discovered the surprising fact that for systems on a fixed toric variety, the locus of continuous parameters giving rise to systems whose ranks are larger than expected does not contain enough combinatorial data to compute the ranks themselves. Her discovery (with Forsgård and Passare) of families of integral solutions that are holomorphic as functions of the parameters holds promise for the goal of constructing sheaves of solutions.

In another direction, Christine has explored applications and generalizations of Boij–Söderberg theory. In collaboration with Erman, Kummini, and Sam, she categorified the poset structure on extremal rays of the cone of Betti tables and also constructed families of minimal free resolutions from higher tensors, providing a unifying view on a wide variety of complexes. Christine's work in Boij–Söderberg theory also aims to generalize the theory to multigraded and (ungraded) local settings, as well as to singular settings such as hypersurface rings.

In addition to her research, Christine is actively involved in mentoring, with particular emphasis on increasing the presence of women in mathematics, through such programs as EDGE, the Noetherian Ring, and the new MATH MENT♀RING program that she is developing with colleagues at Duke.

Two New Staff Members Join MSRI

Alissa S. Crans

Alissa S. Crans, our new Director of Educational and Outreach Activities, earned her Ph.D. in mathematics from the University of California at Riverside in 2004 under the guidance of John Baez. She is currently an Associate Professor of mathematics at Loyola Marymount University and previously was a VIGRE Ross Assistant Professor at The Ohio State University and a visiting Dickson Instructor at the University of Chicago.

Alissa's research interests lie in the field of higher-dimensional algebra, and her recent work, funded by an NSA Young Investigator Grant, involves categorifying algebraic structures called quandles with the goal of defining new knot and knotted surface invariants. She is also interested in the connections between mathematics and music; the 2009 paper she co-authored in this area received the 2011 Merten M. Hasse Prize for expository writing from the Mathematical Association of America.

Alissa has extensive experience mentoring and supporting women mathematicians through her involvement in the Summer Mathematics Program (SMP) at Carleton College and teaching in the Enhancing Diversity in Graduate Education (EDGE) program and the Summer Program for Women in Mathematics (SPWM) at George Washington University. She is extremely active in helping students increase their appreciation and enthusiasm for mathematics through co-organizing the Pacific Coast Undergraduate Mathematics Conference, now in its 8th year.

She received the 2011 Henry L. Alder Award for distinguished teaching by a beginning college/university mathematics faculty member. In addition, Alissa has been the invited speaker at several meetings, both academic- and outreach-oriented.

Mick Santos

Mick Santos, our new Workshop Assistant, is continuing on with MSRI after being a Program Intern. Mick recently finished his studies in UC Berkeley in the field of Integrative Biology. He is pursuing a career in possibly Public Health, Computer Science, or Education. He enjoys learning and improving the administrative tasks that are done behind the scenes. Most importantly, it is especially rewarding for him when he has the chance to meet and welcome the workshop attendees to MSRI. By primarily assisting in the coordination of MSRI's workshops, Mick continues to foster his passion for education and serving the community.

CME Group–MSRI Prize in Innovative Quantitative Applications

Michael Marsland, Yale photographer

Robert Shiller

The seventh annual CME Group–MSRI Prize was awarded to Robert Shiller on October 12 at a luncheon in Chicago. Dr. Shiller is the Arthur M. Okun Professor of Economics, Department of Economics and Cowles Foundation for Research in Economics, Yale University, and Professor of Finance and Fellow at the International Center for Finance, Yale School of Management.

The annual prize recognizes originality and innovation in the use of mathematical, statistical or computational methods for the study of the behavior of markets, and more broadly of economics.

Prior to the lunch and award presentation, a moderated discussion on "The U.S. Housing and Financial Crisis: Aftermath and Afterthoughts" was held with Congressman Barney Frank (D-Massachusetts, 4th District) and Keith Hennessey, former Director of the U.S. National Economic Council.

You can read more about the CME Group–MSRI Prize and also find a link to video of the Frank/Hennessey discussion at http://www.msri.org/web/msri/public-events/show/-/event/Em9750.

Girls Math Olympiad in China

U.S. team members all score medals

The 2012 U.S. CGMO team, from left right: Alicia Weng, Danielle Wang, Jingyi Zhao, Gabriella Studt, Laura Pierson, Cynthia Day, Victoria Xia, and Courtney Guo.

" This was probably the best summer I've ever had. . . even better than I'd expected. China was just amazing and awesome. "

For the sixth consecutive year, MSRI and the Mathematical Association of America (MAA) sent a team of eight talented girls to China to compete in the China Girls Mathematical Olympiad (CGMO) in August. Every member of the team scored a medal, winning a total of three gold medals, four silver medals, and one bronze medal at the highly competitive, girls-only math Olympiad, which was held in Guangzhou in southern China's Guangdong Province.

Gold medals were awarded to Victoria Xia, 16, from Vienna, Virginia — she received a perfect score and won her second consecutive gold medal — who is a junior at the Thomas Jefferson High School for Science and Technology; Danielle Wang, 15, from Campbell, California, a sophomore at Westmont High School, who also won a gold medal last year; and Alicia Weng, 16, from West Hills, California, who is a junior at North Hollywood High School. Silver medals were won by Cynthia Day, 17, from San Jose, California, who recently graduated from Lynbrook High School and is a freshman at Stanford University this fall, and was also a medalist at the 2009 and 2010 CGMO; Courtney Guo, 16, a U.S. citizen who is a junior at the International School of Beijing in China; Laura Pierson, 12, from Oakland, California, who is the youngest student to ever compete on the U.S. team, and is a seventh grader at Hillcrest Middle School; and Jingyi Zhao, 16, from Culver, Indiana, who is a junior at Culver Academies. A bronze medal was awarded to Gabriella Studt, 16, from Silver Spring, Maryland, who is a junior year at Montgomery Blair High School.

The girls sat the rigorous exam on two days: each day they were presented with four proof-based math problems and had four hours in which to solve them. The competition was founded in 2002 as a regional competition for teams of female students from China and other eastern Asian countries. It later expanded to invite teams from more countries, and this year drew 204 girls from ten countries. MSRI sent the first team from the United States in 2007.

The team's mathematical skills were honed earlier in the summer when they spent three weeks at the MAA Mathematical Olympiad Summer Program at the University of Nebraska at Lincoln. To read the travelogue that the girls started writing in the summer and picked up, again, in China, go to http://www.msri.org/cgmo/2012; the quote above is from Gabby Studt's entry during her flight home.

The team's students were selected from the top ranks of female finalists in the 2012 USA Mathematical Olympiad. Head coach, Zuming Feng, is a math teacher on the faculty of Phillips Exeter Academy, the leader of the USA International Mathematical Olympiad team, and the director of the Mathematical Olympiad Summer Program since 2003. Former U.S. team members and gold medalists, Jennifer Iglesias, who entered Carnegie Mellon University's Ph.D. program in mathematics this fall, and Sherry Gong, who began a Ph.D. program in mathematics at MIT, returned again as assistant coaches to help coach the team in China and during the team's training in June at the Summer Program.

Cluster Algebras

Hugh Thomas

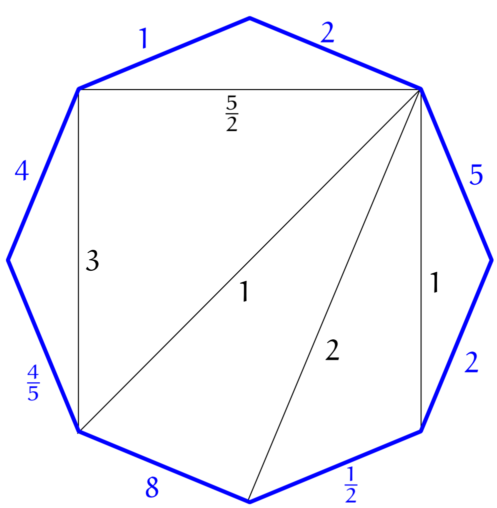
One of the two programs at MSRI this fall is focused around cluster algebras. Cluster algebras were discovered some ten years ago by Sergey Fomin and Andrei Zelevinsky. Links to a surprising number of areas of mathematics have subsequently appeared. This means that two people who work on cluster algebras could potentially have very little background in common, so the program has been a welcome opportunity for people knowledgeable about different aspects of the topic to share their experience.

This article is a very brief and informal introduction to cluster algebras. For many resources on cluster algebras, including links to the original papers of Fomin and Zelevinsky, and to surveys treating cluster algebras from a variety of perspectives, the reader is encouraged to consult the Cluster Algebras Portal, which is located at http://www.math.lsa.umich.edu/~fomin/cluster.html.

Building a Cluster Algebra

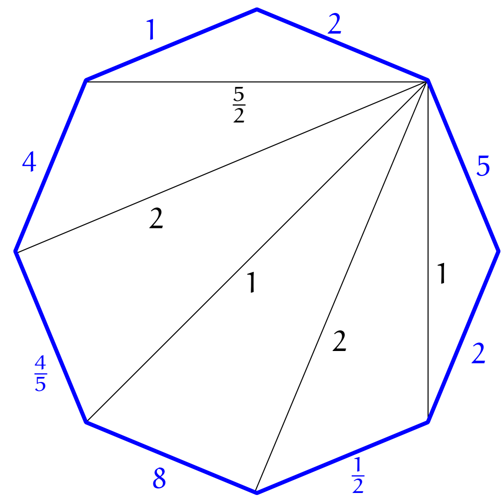
To introduce cluster algebras, I am going to begin with an elementary example. In this case, I mean "elementary" as in "elementary school." Sixth grade, I would say, since I'm going to need fractions.

Take a polygon, and divide it into triangles by lines which do not cross in the interior. Assign a rational number to each of the diagonals in the triangulation, and also to each of the edges of the polygon. The result should look something like this:



Now, erase one of the diagonals of the triangulation. This leaves an open quadrilateral. Draw in the other diagonal. Assign a number to the new edge based on the following rule: if the sides of the quadrilateral are numbered a, b, c, d in cyclic order, and the old diagonal was numbered e, then the new diagonal's label is given by $(ac + bd)/e$.

For example, if we had erased the edge with the label 3 in the old diagram, the new label would be $(\frac{4}{5} \cdot \frac{5}{2} + 4 \cdot 1)/3 = 2$, and the new diagram would be:



By repeatedly applying this rule, you can successively replace one diagonal after another. The surprise is that the label which is assigned to a diagonal does not depend on the sequence of diagonal flips used to arrive at it, and if you return to a diagonal which you have seen before, it will receive the same label each time.

Two Surprises

If we check in with our elementary school students a few years later, after they have learned about rational functions, then they will be able to carry out a somewhat souped-up version of the previous exercise, which starts by labeling the diagonals of the chosen triangulation, and the boundary edges, by a collection of distinct indeterminates. The same procedure as before can be carried out with this starting labeling. It will still be true that each diagonal receives a well-defined label, independent of the sequence of flips which produces it.

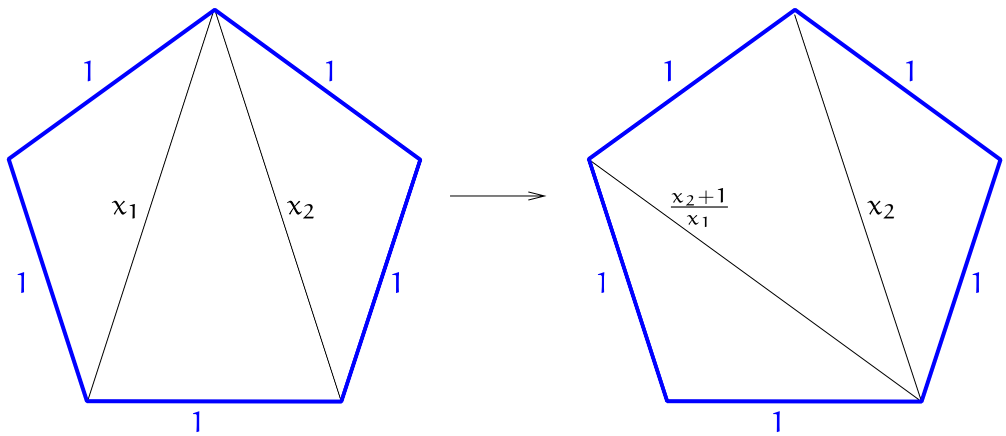
At this point, there are two further surprises, which were not yet visible in elementary school. As we carry out this procedure of swapping diagonals and calculating the corresponding new labels, we add, multiply, and divide. As a result, even though we start with a single indeterminate on each edge, we might reasonably expect to see complicated rational functions appearing as labels. But in fact, this doesn't happen: we only ever see Laurent polynomials (that is to say, rational functions whose denominator is a monomial). This is an instance of the "Laurent Phenomenon," about which I will have more to say later.

Let me illustrate this very concretely. I start with a pentagon with a fixed triangulation, and, for simplicity, let me assign the boundary edges the label 1. We label the diagonals of the triangulation x_1 and x_2, as in the left-hand diagram on the top of the next page.

Now, I may choose a diagonal to replace. Let's say I choose to replace the diagonal labelled by x_1. In the diagram on the right, the resulting new edge gets the label

$$x_3 = \frac{x_2 + 1}{x_1}.$$

It is not interesting to flip the diagonal labelled by x_3, since we get back to our original picture, with the original labels, but we can



flip the diagonal labelled by x_2, and obtain a new diagonal labelled by

$$x_4 = \frac{x_1 + x_2 + 1}{x_1 x_2}.$$

The first of the two surprises I mentioned kicks in at the next step. When we calculate the label for the next diagonal, we get

$$x_5 = \frac{x_4 + 1}{x_3} = \frac{x_1 x_2 + x_1 + x_2 + 1}{x_2(x_2 + 1)} = \frac{x_1 + 1}{x_2}.$$

We see that the numerator of x_3 has wound up in the denominator of x_5, so we ran the risk of leaving the Laurent world — but the numerator of x_5 also had a factor of $(x_2 + 1)$, so the binomial in the denominator cancels, and the Laurent property still holds.

What about the next step? The elementary school behavior already tells us the answer — we have calculated a label for each of the five diagonals of the pentagon, so at the next step, we will be recalculating the label for one of our initial diagonals. And, indeed, if we carry out the calculation, we find that

$$x_6 = \frac{x_5 + 1}{x_4} = \frac{(x_1 + x_2 + 1)x_1}{x_1 + x_2 + 1} = x_1.$$

The second surprise that is now visible is that the numerators of the labels on the diagonals always have non-negative coefficients. This might not seem like much of a surprise, because so far, I haven't even mentioned subtraction. So how could a negative sign have crept in? Well, it turns out that negative signs are sneaky things, and can appear where you don't expect them. For example, $(x^3 + y^3)/(x + y) = x^2 - xy + y^2$. So, we are actually entitled to be surprised that, when we reduce to lowest terms (canceling the non-monomial factors in the denominator), then what is left in the numerator still has no negative signs.

General Cluster Algebras — the Same Thing, Only Without the Triangles

What I have discussed so far is as much as I could reasonably explain to tenth graders. If they were to complete high school, go to university, and continue to graduate school, they might encounter cluster algebras again, in a variety of settings: total positivity in algebraic groups, representation theory of algebras, mathematical physics, low-dimensional geometry, or others. I am not going to attempt to summarize these different perspectives here. Instead, I am going to attempt to explain in a little more detail what a cluster algebra is.

In the setting that I have been discussing, associated to triangulations of a polygon, the cluster algebra is simply the algebra over $\mathbb{Z}$ generated by all the labels that appear on all the edges and diagonals. This setting can be vastly generalized. First of all, we can replace the polygon by (essentially) an arbitrary orientable surface with a finite collection of marked points on the boundary. Triangulations are still recognizable, though the diagonals are now isotopy classes of curves between marked points rather than straight lines. With significantly more effort (and some modification to the notion of triangulation), marked points in the interior of the surface can also be allowed. As before, the cluster algebra is the algebra generated by the labels on the arcs.

It turns out that the cluster algebra concept can be generalized much further. (In fact, in what is now a time-honored tradition in cluster algebra papers, I am revising history, as this more general description came first chronologically.) In order to explain the more general version of cluster algebras, I would like to recall an apocryphal story about Einstein, when he was asked to explain how radios work. He said "First, let me explain the telegraph. Essentially, it is a very long cat. You tug on its tail in Pittsburgh, and its head meows in Cleveland. *(Pause.)* Radio is the same thing, only without the cat."

Cluster algebras in general, then, are the same thing as what I have already explained, only without the triangulations. Slightly more precisely, it is possible to encode a triangulation in an integer matrix, so that the operation of flipping a diagonal acts in a well-defined way on the matrix. The rule for assigning a label to the new edge, also, can be interpreted purely in terms of the matrix. At this point, the triangulation is no longer necessary, and we can start with any (suitable) matrix, including ones which could not possibly arise from triangulations.

The rational functions that are produced by iterating this procedure in all possible ways are called "cluster variables." The Laurent Phenomenon still persists in this generality: the cluster variables are Laurent polynomials. However, the non-negativity of the coefficients appearing in the numerators of the cluster variables, for all its seeming obviousness, remains a conjecture, one which dates back to the origin of cluster algebras, and which continues to motivate new work.

As always, the cluster algebra is the $\mathbb{Z}$-algebra generated by the cluster variables. Even if you accept that the cluster variables are interesting, it might not be clear that the right thing to do with these Laurent polynomials is to use them to generate an algebra. I will give a couple of reasons why it might be interesting to do so.

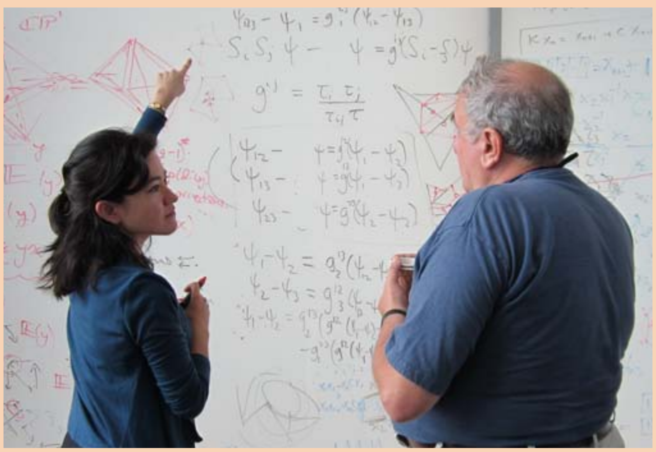
Beyond Cluster Variables

One reason to be interested in the cluster algebra, rather than just the cluster variables, is that some of the algebras which appear in this way are algebras which we already consider to be interesting, such as homogeneous co-ordinate rings of homogeneous varieties (and also quantized versions of these, appearing as quantum cluster algebras, which I do not have space to explain here). The cluster algebra perspective provides a new tool for studying these familiar rings; in particular, for trying to construct nice linear bases.

Another reason to be interested in the cluster algebra, rather than the set of cluster variables, is the following. As I have mentioned, the cluster variables have the property that they are Laurent polynomials with respect to the variables with which we began. But much more is true. We group the cluster variables into overlapping sets called "clusters." In the surface case, the clusters correspond to sets of edges which together form a triangulation, while in general, a cluster is simply the set of cluster variables that occur together at some stage of the iterative construction. Now it is important to observe that there is nothing special about the initial cluster and the initial cluster variables. We can equally well express a cluster variable as a rational function in the cluster variables appearing in *any* cluster, and the expressions we obtain will *all* be Laurent polynomials. It follows immediately that this property extends to all elements of the cluster algebra: any element can be expressed as a Laurent polynomial in the variables of any cluster. In fact, under good circumstances, more is true: this is a necessary and sufficient characterization of the elements of the cluster algebra, which provides a further justification for the significance of the cluster algebra itself.

Focus on the Scientist: Lauren Williams

Bernd Sturmfels



Lauren Williams with Andrei Zelevinsky, who is also involved in the MSRI Cluster Algebras program

Lauren Williams works in combinatorics, focusing on its enumerative, algebraic, and geometric aspects, and its connections to other branches of mathematics including discrete probability, representation theory, and mathematical physics. In this highly competitive field, she is widely regarded as a young star. During the current fall semester, Lauren is one of the five organizers for the special program on Cluster Algebras.

Lauren's research interests span a wide range of areas, from pure enumeration to topological combinatorics to integrable systems to statistical mechanics. Her main results on cluster algebras concern an important class of them that can be associated with oriented surfaces with boundary and marked points, a topic prominently featured in an ongoing graduate course at UC Berkeley by the Chancellor Professor Dylan Thurston. In joint work with Gregg Musiker and Ralf Schiffler, Lauren proved that the longstanding Laurent positivity conjecture of Fomin and Zelevinsky holds for this class of cluster algebras.

In the field of integrable systems, Lauren wrote a series of four remarkable papers with Yuji Kodama which explore deep connections between soliton solutions to the Kadomtsev-Petviashvili equations and the combinatorics of totally nonnegative cells in Grassmannians.

Lauren made a major contribution to discrete probability by providing, in joint work with Sylvie Corteel, an explicit combinatorial formula for the stationary distribution of the asymmetric exclusion process (ASEP). Their work leads to new insights into the hierarchy of classical orthogonal polynomials.

All of Lauren's projects are highly innovative. She is willing to take the risks associated with getting into an entirely new area. For instance, not many combinatorialists would dare to attack problems concerning integrable systems and differential equations for water waves. She shares her vision and enthusiasm with numerous graduate students and postdocs, who regard her as an exceptional mentor.

Lauren grew up in Southern California and went to Harvard University for her undergraduate studies. After receiving a Ph.D. from MIT in 2005, she was a postdoctoral fellow at Berkeley and Harvard. In Fall 2009, after spending the last postdoctoral semester at MSRI during the program on Tropical Geometry, she moved down the hill to join UC Berkeley as an Assistant Professor.

Puzzles Column

Joe P. Buhler and Elwyn Berlekamp

1. Show that every isosceles right triangle can be tiled by smaller isosceles right triangles of distinct sizes.

2. You are given n coins, of three kinds — gold, silver, and bronze — and your job is to identify a gold coin. You are guaranteed that the gold coins are a (strict) majority of the coins. Unfortunately, the coins are all gold-plated and are indistinguishable without sophisticated equipment.

Your only tool is a machine that accepts two coins and tells you whether or not they are made of the same material. Your algorithm to identify a gold coin should describe which pairs of coins should be compared; the choices may depend dynamically on the results of earlier comparisons. Such a procedure is said to be of depth "k" if the maximum number of times that any given coin participates in a comparison is k, for all possible initial compositions of the coins.

Find a gold coin algorithm of minimal depth. The depth should be a constant, independent of n.

Comment: We do not know the precise origin of this problem. It was brought to our attention by Stan Wagon, who pointed us to a version on the "Math Factor" website at the University of Arkansas.

3. N distinct points are given in the plane. Show that it is possible to make arbitrarily small adjustments to their positions so that they can be connected by straight line segments to form a simply connected polygon.

4. A coin has probability p of coming up heads, and probability q of coming up tails. It is known that $(p, q) = (55\%, 45\%)$, i.e., that the coin is biased either 55/45 towards heads or 55/45 towards tails. An economics professor partitions his class of $2n$ students into two teams of n boys and n girls, and asks them to determine whether the coin is biased towards heads or tails using the following procedure. The teams line up, and for $k = 1$ to $k = n$, the k-th boy and k-th girl each flip the coin, privately. Each of them then announces "heads" or "tails." Their announcements are concurrent. The announcements are heard by all. Each coin flip is known only to the person making the flip.

The announcements are governed by the following invariable rule: the announcer computes and announces the more likely bias, heads or tails, using all available information, including the one coin flip visible to him or her, and all prior announcements. Each announcer must seek to maximize the chance of a correct announcement by exploiting everything known to the announcer at the time of the announcement.

After all n couples have made their announcements, what is the probability that all $2n$ announcements were correct? What is the probability that all $2n$ announcements were false?

Comment: This problem illustrates an "information cascade" as described on Wikipedia.

5. Motivated by current events (see the solution on the web for a full discussion), we extend the previous information cascade problem by adding "shills." As in Problem 4, there is a biased coin. The size of its bias, but not its direction, is known. In other words, there are numbers u and $v = 1 - u$ such that the probability of heads is either u or v. Subject to the constraints we will specify, each player in this game seeks to determine the direction of this bias.

In addition to the biased coin, there are many cards, each of which has one of the following labels: "heads," "tails," and "report." Two decks of cards are created: one contains only "report" cards, and the other contains $r = 75\%$ "report" cards, and $s = 1 - r = 25\%$ cards that we will call "shill" cards. These shill cards are either all heads or all tails. The choice of whether the shills are "heads" or "tails" is made once and for all by an initial flip of a (fair!) coin. Both decks are shuffled. One of these decks is randomly assigned (by another flip of a fair coin) to the all-boys team and the other to the all-girls team. No one but the referee knows which team's deck contains the shill cards, nor whether the shills are heads or tails.

The game now proceeds as follows. As in Problem 4, the boys and girls line up and the successive couples flip the coin, privately. Each then examines the top card from his or her team's deck. After privately perusing the card, the player returns the card to its deck and reshuffles it. He and she then announce "heads" or "tails" according to the following rules:

a. Both members of any couple announce concurrently.

b. A student with a shill card must ignore all prior information, including the current coin flip and all prior announcements, and must announce the bias specified on the shill card.

c. As before, a student with a "report" card announces the more likely bias of the coin based on all available information, including prior announcements and the single coin flip which he or she has observed.

d. Each then passes its team's deck along to the next member of the same team, and the next couple then behaves according to these same rules.

e. All announcements are public, and all coin flips and cards are private.

Here are some questions to be answered after all $2n$ announcements have been made:

1. What is the probability that all announcements were correct?

2. What is the probability that all announcements were false?

3. What is the probability that there was a complete deadlock: all boys announced one bias and all girls announced the other?

Hints: It may be easier to first consider only the limiting case of arbitrarily large n. It may also be simpler to first consider the special case $s = 0$ (Problem 4) and a new special case $s = 1$.

New Viterbi Scholar

Fan Qin is the latest recipient of a Viterbi Postdoctoral Fellowship at MSRI. Fan did his undergraduate studies at Tsinghua University in Beijing with Jie Xiao. From October 2008 to May 2012, he did his thesis with Bernhard Keller at the University Paris Diderot. He has obtained important results on Berenstein–Fomin–Zelevinsky's quantum cluster algebras. In particular, in joint work with Yoshiyuki Kimura, he has proved Fomin–Zelevinsky's positivity conjecture for all acyclic quantum cluster algebras, generalizing earlier results by Nakajima.

The Viterbi postdoctoral fellowships are funded by a generous endowment to MSRI from Andrew Viterbi, who is currently a trustee of MSRI. In 1985, Dr. Viterbi helped found Qualcomm, where he was vice chairman of the board from 1996 until his retirement in 2000. Before founding Qualcomm, he co-founded Linkabit Corporation, a digital communications company. Dr. Viterbi is a member of the National Academy of Engineering and the National Academy of Sciences and was a member of President Clinton's Information Technology Advisory Committee.

Fan Qin

Forthcoming Programs & Workshops

Programs

January 14, 2013 to May 24, 2013: *Noncommutative Algebraic Geometry and Representation Theory*, organized by Mike Artin, Viktor Ginzburg, Catharina Stroppel, Toby Stafford (chair), Michel Van den Bergh, and Efim Zelmanov

August 19, 2013 to December 20, 2013: *Mathematical General Relativity*, organized by Yvonne Choquet-Bruhat, Piotr T. Chruściel, Greg Galloway, Gerhard Huisken, James Isenberg (chair), Sergiu Klainerman, Igor Rodniansky, and Richard Schoen

August 19, 2013 to December 20, 2013: *Optimal Transport: Geometry and Dynamics*, organized by Luigi Ambrosio, Yann Brenier, Panagiota Daskolopoulos, Lawrence C. Evans, Alessio Figalli, Wilfrid Gangbo, Robert J. McCann (chair), Felix Otto, and Neil S. Trudinger

January 20, 2014 to May 23, 2014: *Model Theory, Arithmetic Geometry and Number Theory*, organized by Ehud Hrushovski, François Loeser, David Marker, Thomas Scanlon, Sergei Starchenko, and Carol Wood (chair)

January 20, 2014 to May 23, 2014: *Algebraic Topology*, organized by Vigleik Angeltveit, Andrew J. Blumberg, Gunnar Carlsson, Teena Gerhardt, Michael A. Hill (chair), and Jacob Lurie

August 11, 2014 to December 12, 2014: *New Geometric Methods in Number Theory and Automorphic Forms*, organized by Pierre Colmez, Wee Teck Gan (chair), Michael Harris, Elena Mantovan, Ariane Mézard, and Akshay Venkatesh

August 18, 2014 to December 19, 2014: *Geometric Representation Theory*, organized by David Ben-Zvi (chair), Thomas Haines, Florian Herzig, Kevin McGerty, David Nadler, Ngô Bảo Châu, Catharina Stroppel, and Eva Viehmann

January 12, 2015 to May 22, 2015: *Dynamics on Moduli Spaces of Geometric Structures*, organized by Richard D. Canary, William Goldman, Francois Labourie, Howard Masur (chair), and Anna Wienhard

Workshops

October 29, 2012 to November 2, 2012: *Cluster Algebras in Combinatorics, Algebra, and Geometry*, organized by Claire Amiot, Sergey Fomin, Bernard Leclerc, and Andrei Zelevinsky (chair)

December 3, 2012 to December 7, 2012: *Combinatorial Commutative Algebra and Applications*, organized by Winfried Bruns, Alicia Dickenstein, Takayuki Hibi, Allen Knutson (chair), and Bernd Sturmfels

January 24, 2013 to January 25, 2013: *Connections for Women: Noncommutative Algebraic Geometry and Representation Theory*, organized by Georgia Benkart, Ellen Kirkman (chair), and Susan Sierra

January 28, 2013 to February 1, 2013: *Introductory Workshop: Noncommutative Algebraic Geometry and Representation Theory*, organized by Michael Artin, Michel Van den Bergh (chair), and Toby Stafford

February 11, 2013 to February 17, 2013: *Representation Theory, Homological Algebra, and Free Resolutions*, organized by Luchezar Avramov, David Eisenbud, and Irena Peeva (chair)

March 16, 2013 to March 17, 2013: *AWM Research Symposium 2013*, organized by Hélène Barcelo, Estelle Basor, Georgia Benkart, Ruth Charney, Frank Farris, and Jill Pipher

April 8, 2013 to April 12, 2013: *Interactions between Noncommutative Algebra, Representation Theory, and Algebraic Geometry*, organized by Victor Ginzburg, Iain Gordon, Markus Reineke, Catharina Stroppel (chair), and James Zhang

May 6, 2013 to May 10, 2013: *The Commutative Algebra of Singularities in Birational Geometry: Multiplier Ideals, Jets, Valuations, and Positive Characteristic Methods*, organized by Craig Huneke (chair), Yujiro Kawamata, Mircea Mustaţă, Karen Smith, and Keiichi Watanabe

For more information about any of these programs and workshops, see www.msri.org/scientific.

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Plato $1,922 to $5,040

Plato, in The Laws, suggested that a suitable number of citizens for the ideal city would be that number which contained the most numerous and most consecutive subdivisions. He decided on 5,040, a number with 59 divisors (apart from itself). For purposes of war and peace 5,040 citizens can be divided by any number from 1 to 10.

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Noether $1,730 to $1,921

1921 is the year of publication of Noether's "Idealtheorie in Ringbereichen" — a landmark paper ushering in the beginning of the field of Abstract Algebra, which became a dominant theme of 20th Century mathematics and flourishes into the 21st.

Anonymous

Ramanujan $730 to $1,729

1,729 is the number of Hardy's taxicab which, Ramanujan reflected from his sick bed, is the smallest number expressible as the sum of two cubes in two different ways.

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Cubic $258 to $729

729 is a number important to Plato, and a cube that is the sum of three cubes.

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Fermat $90 to $257

257 is a Fermat number, and is equal to 2 to the 2 to the 3 plus 1.

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(continued on next page)

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Fibonacci $34 to $89

Both the upper and lower end of this range are Fibonacci Numbers. 34 is also the magic constant of a 4 by 4 magic square.

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Letters of intent and proposals for semester or year long programs at the Mathematical Sciences Research Institute (MSRI) are considered in the fall and winter each year, and should be submitted preferably by **October 1** or **December 30**. Organizers are advised that a lead time of several years is required, and are encouraged to submit a letter of intent prior to preparing a pre-proposal. For complete details see http://tinyurl.com/msri-progprop.

Hot Topics Workshops

Each year MSRI runs a week-long workshop on some area of intense mathematical activity chosen the previous fall. Proposals for such workshops should be submitted by **October 31** or **December 30**. See http://tinyurl.com/msri-htw.

Summer Graduate Schools

Every summer MSRI organizes four 2-week long summer graduate workshops, most of which are held at MSRI. To be considered for the summer of year n, proposals should be submitted by **October 31** or **December 30** of year $n-2$. See http://tinyurl.com/msri-sgw.

MSRI
Mathematical Sciences Research Institute

17 Gauss Way, Berkeley CA 94720-5070
510.642.0143 • FAX 510.642.8609 • www.msri.org

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Come to the Institutes'

Open House / MPE 2013 U.S. Launch

at the January 2013 Joint Mathematics Meetings in San Diego!

Wednesday
January 9, 2013
5:30-8:00 pm

San Diego Marriott
Ballroom B
Lobby Level

MSRI Staff Roster

Phone area code 510. Add @*msri.org* to email addresses.

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Alissa S. Crans, Director of Educational and Outreach Activities, 642-4745, *acrans*

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