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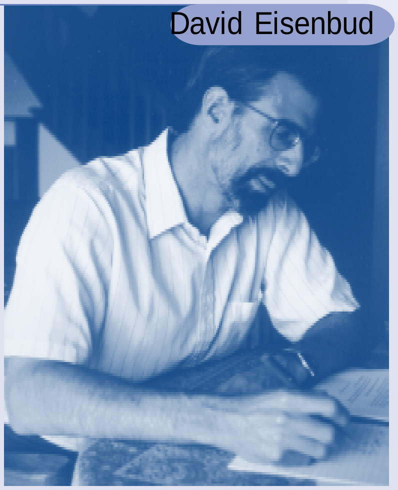
Notes from the Director

David Eisenbud



MSRI is humming with two new major programs, one on Noncommutative Algebra, and one on Galois Groups and Covering Spaces. The enthusiasm, excitement, and drive of the arriving members are palpable, and fill me each year with fresh admiration and enthusiasm for this great place. I have particular involvement in the mathematics going on: my own thesis

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Inside This Issue

Random Matrices

Joe P. Buhler, *Deputy Director, MSRI*

The ordinary rhythms of MSRI become much more intense during workshop weeks. Packed lectures alternate with breaks during which the lobby and halls buzz with discussion.

As luck would have it, I arrived at MSRI to begin learning the Deputy Director's responsibilities just before a workshop last June. Hugo Rossi and I had agreed to be jointly in residence during early June and early July in order to attempt a knowledge transfer — Hugo intended to teach me all about being Deputy Director. The first of those periods coincided with the workshop "Random Matrices and their Applications: Quantum Chaos, GUE Conjecture for Zeros of Zeta Functions, Combinatorics, and All That." Held June 7-11, it was the final workshop in the Random Matrices program at MSRI.

I had been expecting to sneak out of some of the "knowledge transfer" responsibilities to hear a few of the talks. To my surprise, Hugo had similar plans. Despite our different backgrounds (in number theory and analysis), the preliminary list of talks fascinated both of us. This workshop was unusually diverse, even by MSRI standards. The attendees included analysts, physicists, number theorists, probabilists, combinatorialists. It was delightful to be able to attend some of the talks; even though they impinged a little on the time that Hugo and I had to share, they also provided a vivid reminder of why I'd decided to take on the demanding task of contributing to the smooth functioning of MSRI over the next couple of years.

What follows is a brief description of aspects of the Random Matrix workshop, aimed at readers who are new to this circle of ideas. It is impossible to capture the breadth of this area in any concise way since there are many possible perspectives and interpretations: the pace of current research is so rapid, and is heading in so many different directions, that none of the existing surveys or major articles capture more than a fraction of what is going on. The interested reader could look at [TW2] [KS2] [GGW], [AD], [BD], and [Ok], as well as the references contained therein, to get a sense of the extent of the field.

Alternatively, you can actually view the lectures in the MSRI workshop yourself! Indeed, both of the workshops for last spring's Random Matrix program can be found on the MSRI web site (www.msri.org) in streaming video format. A glance at the list of the talks should convince the reader of the breadth of topics covered. (To view the lectures, you need the RealPlayer video software.)

What is a random matrix? In order to define this concept, we need a probability distribution on some set of matrices. If one has in mind a finite set of matrices, there is an obvious way to generate a random element: choose each matrix with equal probability. More generally, if the set of matrices is a compact group (e.g., the group of n by n complex unitary matrices) there is again an obvious notion of a random matrix obtained by using the normalized Haar measure on the group as the probability

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Mathematics in Arcadia: A conversation

Tom Stoppard

Tom Stoppard is famous for his plays, and more recently for the screenplay of the movie *Shakespeare in Love.* Some of his plays make references to mathematics: most prominent among these, *Arcadia* includes a heroine who is a mathematical prodigy (in the first scene, for example, we find that her tutor has set her the task of finding a proof of Fermat's theorem to keep her occupied). This conjunction of mathematics and theater led Robert Osserman and David Hoffman to conceive a very special MSRI event.

On February 19, 1999, Stoppard and Osserman engaged in a public conversation about Stoppard's use of mathematical ideas in *Arcadia*. The conversation was intertwined with three scenes from the play, performed by members of UC Berkeley's Center for Theater Arts. This event was videotaped by MSRI, and information about the videotape can be found on the MSRI web site.

In the short essay below, Osserman reflects on Stoppard's background, the play *Arcadia*, and the mathematics therein.

Tom Stoppard and Robert Osserman in conversation

Reflections on Stoppard and Mathematics

Robert Osserman

A column by Jon Carroll in the San Francisco Chronicle starts as follows:

> "On July 3, 1937, in the town of Zlin, in the southeast corner of what is now called the Czech Republic, a boy named Thomas was born into the family of Eugene Straussler, a Jewish physician employed by a shoe company.
>
> The early life of Thomas Straussler was not easy. When he was 2, just ahead of the Nazi invasion of his country, he fled with his family to Singapore. In 1942, he moved with his mother and brother to India, while Eugene stayed behind to face the Japanese occupation.
>
> Eugene died in a Japanese prison camp; Thomas' mother married a British major who soon took his family to England. The boy Thomas took his stepfather's surname, which was Stoppard. He grew up to be the most brilliant playwright never to have fulfilled his promise, only because his promise has always been so extraordinary.
>
> It is a fact universally acknowledged that he has now written his masterpiece, and that it is called 'Arcadia'."

Jon Carroll goes on to note that a central character in the play is a young genius named Thomasina, who is 13 when the play begins and almost 17 when the play ends. What he fails to note is that the particular form of her genius is mathematical, and that she is probably the most exuberant, engaging, and appealing mathematical character ever to appear on stage. Even more surprising is the number of mathematical topics that are explored and explained in the course of the play, from Fermat's Last Theorem to iterated algorithms. The play itself has been a great success, both critical and popular, since the often abstruse topics from mathematics and mathematical physics are embedded in a dialog that is sparkling with wit, and finally very moving.

Arcadia is not Stoppard's first foray into mathematics and science. His earlier play *Hapgood* not only uses quantum mechanics as an underlying metaphor, it discusses such topics as what makes a science paper beautiful, the size and structure of an atom, and it gives an accurate description of the Heisenberg uncertainty principle. All that in the context of a spy story that may be unique in that the key to unraveling one part of the mystery is a mathematical theorem. The reader of the play (although not the audience watching it) is clued in early by the stage directions for the opening scene:

> "Ridley now goes on a perambulation. The essence of the situation is that Ridley moves around and through, in view and out of view, demonstrating that the place as a whole is variously circumnavigable in a way which will later recall, if not replicate, the problem of the bridges of *Königsberg*."

Indeed, later in the play, one of the characters describes the problem, notes that Euler proved it was impossible to traverse the path that Ridley seemed to, and deduces that what appeared to be a single character must have been two separate ones (twins, as it turns out.)

What may be unique about Stoppard among major writers and playwrights (perhaps even more so for minor ones) is the way he really "gets" mathematics. He is not only able to make brilliant use of themes from mathematics and science such as chaos and heat loss at a metaphorical level, but he has a feeling for what makes those subjects interesting and exciting to mathematicians and scientists. Given all the distorted depictions of mathematics and mathematicians in the popular culture, we are very fortunate to have someone of Stoppard's caliber to present a more accurate and sympathetic view.

Topology and time series processing

Paul W. Gross

Paul Gross

For the past 14 months or so, I have been a postdoc at MSRI supported by Hewlett-Packard. MSRI/HP postdocs spend a summer working with a research group at Hewlett-Packard Labs in Palo Alto, CA, and during the academic year they spend about 4 days per month continuing with the same group. MSRI and HP established the joint post-doc program in order to expose interested mathematicians to industrial problems requiring mathematics. For me, this experience has provided the opportunity to learn about some mathematics unfamiliar to me and to make new connections to my research outside of HP. The following is an overview of some aspects of topology in time-series (or signal) processing which are being investigated and used at HP.

It is no secret that many of the basic building blocks of engineering systems are nonlinear systems of various types. The things an engineer typically manipulates, transistors, magnetic materials, fluids, pendulum systems, and production line parameters, have nonlinear regions of operation. In many cases it is precisely the nonlinearity which makes them useful. This stands in contrast to the fact that technology has come a very long way based on the simple idea of design by linearizing about some operating point and constraining the operating region. Indeed, the classical techniques of linear systems are well-known and loved by engineers everywhere.

With new materials, shrinking manufacturing processes, and demanding applications, it is often impossible or undesirable to ignore nonlinear behavior. The engineer must consider nonlinear effects, and in order to design, must have adequate models and simulations for the design process. Again a problem arises in that many times the physics of the device or process is poorly understood so that good physical models do not exist or become so large that they are not particularly useful for computational purposes.

An alternative to building functional models derived from physical considerations is to construct “behavioral models” based on experimental measurements. Certain aspects of this approach which have been brewing in the nonlinear science literature for several years are making their way into industry as means of generating device models and diagnostic procedures. The idea is to start with the general assumption that some (nonlinear) system of interest is described by a differentiable dynamical system and develop techniques for building models describing the behavior of the system based on time-series measurements taken in the laboratory. In short, the approach is to take measurements from the device of interest under a range of operating parameters and use the computer to build a state-space dynamical model based on the data.

More precisely, the physical system is regarded as a continuous flow. If the system is sampled at regular intervals τ of time, a series of measurements v can be regarded as a discrete dynamical system. In this context, the discrete system is $v : X \emptyset X$ on a topological space X which is also the space of the underlying continuous system. The objective is to develop theorems, techniques, and algorithms to construct a geometric object with properties of X so that geometric, topological, or statistical information of the state-space of the underlying continuous system can be recovered. Following are three *aspects of topological time-series processing* where the objective is to determine topological properties of X.

Time-delay Embedding

A fundamental idea in state-space reconstruction, often attributed to D. Ruelle, is that of time-delay embedding. A theorem of Whitney says that any *d*-dimensional manifold maps to R^{2d} without self-intersections. The map is called an embedding. Embeddings make their way into time-series as follows. One represents the state of the sampled system at (discrete) time *t* by a *delay coordinate vector* of the time-series measurements,

$$v = (v_t, v_{t-r}, \ldots, v_{t-(m-r)r}).$$

A theorem by F. Takens, based on the Whitney embedding theorem, says that when $\boldsymbol{X}$ is a compact manifold of dimension d, if $m > 2d$ then $\boldsymbol{v}$ is an embedding. Local differential structure is preserved, so the state space reconstructed via ***v*** preserves quantities such as predictability of trajectories. In practice, algorithms based on delay-embedding work quite well, though one must note that the embedding theorem is based on an assumption which is generally untrue, namely that the measurements are noise-free. The Takens theorem has been generalized in various directions, notably for the case when *M* is replaced by a fractal set and embedding theorems also exist for forced, or input/output systems. Forced systems are of particular interest in the engineering context, and practical algorithms for estimating embedding underlie most methods for reconstructing or extracting geometric properties such as Lyapunov exponents.

Symbolic Dynamics

For time-series which come from nonlinear systems, the problems of measurement error and noise are complicated by the fact that dynamical behavior can vary widely under small changes or drift in system parameters. Instead of trying to describe the diversity of behavior, one may try to describe some minimal global dynamics and properties which are stable under moderate perturbation. Methods for doing this give a fairly "coarse-grained" picture of the behavior which trades information about the smooth structure of the system for an algebraic approach that may be computationally tractable. Again topology arises, primarily through work of C. Conley and K. Mischaikow.

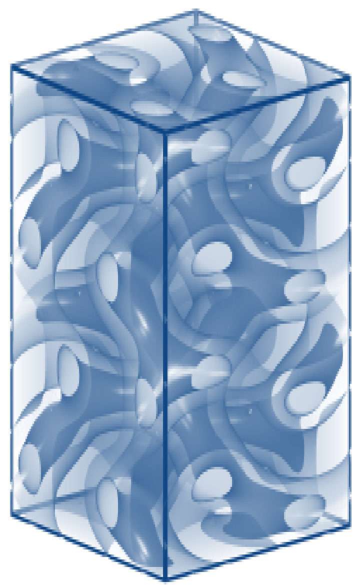
The key to this approach is through Conley index theory which studies the algebraic topology of "isolating neighborhoods" of invariant sets of the dynamical system. For time-series data, the idea is to construct a cellular decomposition (satisfying certain properties related to measurement error) of the delay-embedded data, assign symbols to cells, and study the dynamics of the new symbolic system, in particular identifying isolating neighborhoods and computer the associated Conley index. The symbol dynamics coupled with the index give a partial picture of fixed points and periodic orbits in the invariant sets. The dynamics are described on the scale of the cellular partition, but some algebraic information from the index can be lifted back to the underlying smooth system.

Beyond the Conley index itself, there are interesting problems in efficient computation of the index. The problems lie in relating the algebraic topology to the most natural data structures for the computer, and how the topology can lead to algorithms. For applications, it is promising that the combination of engineering design criteria with Conley theory will be a useful tool for modeling systems which use symbolic dynamics of a chaotic signal as the basis for a communication system. Algorithms and software for computing the Conley index from time-series data have been the focus of my efforts at Hewlett-Packard.

Braid Analysis

Another topological method useful for studying chaotic time-series comes through knot and braid theory. Periodic orbits or closely recurrent trajectories give rise to knots, and collections of intertwining orbits give rise to braids. By straightforward algorithms which look for close recurrence in time-series embedded in low dimensions, it is possible to extract periodic orbits and study their braid types. In certain one-dimensional maps, the existence of periodic orbits can be inferred from a single parameter (the kneading invariant) of the map. In higher-dimensional maps, the topological properties of the braid can be used to infer the existence of other periodic orbits so that numerical extraction of partial orbit "spectra" turns into a powerful way to study the system.

It isn't altogether clear whether the methods mentioned here are useful as general methods for engineering systems. However, they have been used with some success in the engineering context for detecting bifurcations, leading to new interpretations of phenomena which have traditionally been little understood. These methods have also been useful for developing fast methods for computing models of some types of devices.



The structure illustrated is called the "core-shelled double gyroid." It consists of five separate solid regions, separated by four divding surfaces. The surfaces are level surfaces of the function:

$$v(x,y,z) = \sin 2x \cos y \sin z + \sin 2y \cos z \sin x + \sin 2z \cos x \sin y - 0.15 (\cos 2x \cos 2y + \cos 2y \cos 2z + \cos 2x)$$

In certain copolymers (compound polymers), consisting of chains of distinct polymers joined together, the component materials tend to self-segregate. But since they are joined together, they can never move too far from the other materials in the copolymer. Under the right conditions, stable states are formed in which each of the component materials occupies a separate region of space, defined by the dividing surfaces.

Interfaces close to the level surfaces above have been found experimentally in annealed copolymers consisting of five polymers joined end-to-end in the configuration A-B-C-B-A, where A, B, and C denote polymers of three different materials. Related structures of current research interest in material science are considered in the paper" Ordered Bicontinuous Nanoporous and Nanorelief Ceramics form Self-Assembling Polymer Precursors," by V.Z-H Chan, James Hoffman et al, which has been accepted for publication in the journal Science. You can find more at www.msri.org/publicaitons/sgp/SGP/. This site was recently featured by the Canadian Mathematical Society (www.camel.math.ca) as "cool site of the week."

Joe P. Buhler

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measure. If the group isn't compact (e.g., *n* by *n* complex Hermitian matrices), then we have to specify a suitable probability measure. One aspect of ``universality'' that is referred to in the random matrix literature is that the asymptotic behavior of, for example, the eigenvalues is robust, in the sense that a number of reasonable probability models give the same behavior.

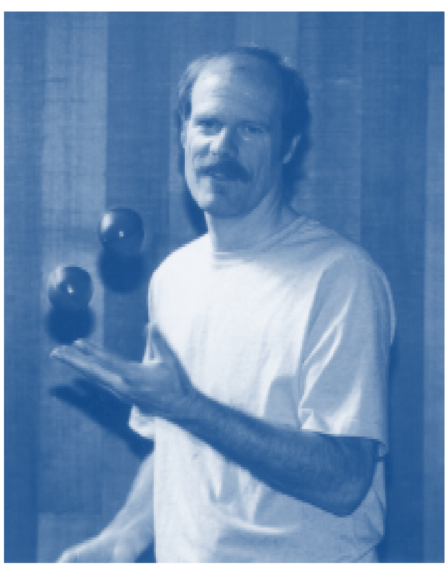
The commonly cited origins of the ''random matrix'' idea are in Wigner's work on the scattering of neutrons off heavy nuclei (though one can find various precursors in statistics). The observed energy levels in the scattering of many-body quantum system are the eigenvalues of the Schrödinger operator. The lowest eigenvalues can be easily computed, but the higher ones cannot be computed in practice in any reasonable way. A naive first guess might be that the eigenvalues should behave as a Poisson process, but Wigner had the marvelous idea that the large eigenvalues of the Schrödinger operator should be distributed in a way very close to the distribution of the eigenvalues of a random large unitary matrix.

The accuracy of this stochastic model of the eigenvalues is striking, and the success of this idea has led to its proliferation in many parts of physics during the last forty years. Indeed, the rubric ''distributed like the eigenvalues of random matrices'' is now seen to model many instances of stochastic, or pseudo-stochastic, behavior. For instance, in Wigner's context the random behavior arises from the complicated many-body quantum system, but it is now known that this behavior is exhibited by chaotic quantum systems even if there are only a few degrees of freedom.

More recently, the random matrix heuristic has come up in various areas of mathematics, and one has the sense that this is an emerging paradigm of great importance. The handful of different limiting distributions that arise play a role similar to the normal distribution in the law of large numbers, and seem to give new descriptions of asymptotic behavior of many different phenomena. Even as people try to construct proofs, new instances are being discovered empirically. The talks in the workshop gave evidence both of new discoveries and of new proofs.

A number of examples arise in quantum systems that have a sufficiently chaotic Hamiltonian. A conjecture attributed to Bohigas, Giannini and Schmidt predicts that that these systems will have the same asymptotic behavior as random matrix models, where the precise model depends on the symmetry of the system. An example of such a system is a ''quantum billiard ball'' bouncing around suitable regions in the plane.

To illustrate the breadth of this heuristic, it is useful to examine two recently famous examples of these ideas that arise ''in nature.'' One of these connections concerns the Riemann zeta function, and the other has to do with the length of the maximal increasing subsequence in a random partition. In the case of the zeta function, much remains to be proved, but exciting recent work of Rudnick and Sarnak, and Katz



Joe Buhler manipulating objects (photo by Brad Yazzalino)

Random Matrices and Their Applications: Quantum Chaos, GUE Conjecture for Zeros of Zeta Functions, Combinatorics, and All That, June 7-11,1999

- **Oriol Bohigas** On the Distribution Of the Total Energy of a System of Non-interacting Fermions
- **Gernot Akemann** Universality of Unitary Ensembles in the Microscopic Scaling Limit
- **Shinsuke Nishigaki** Deformed Random Matrix Ensembles and Anderson Hamiltonian
- **Stephane Nonnenmacher** Algebraic Study of the Pair Correlation Function for Quantum Maps
- **Martin Zirnbauer** Dual Pairs in Random Matrix Theory
- **Pavel Bleher** Double Scaling Limit for Orthogonal Polynomials
- **Shinobu Hikami** Random Matrix with Deterministic Source
- **Alexei Khorunzhy** On Universality of the Smoothes Eigenvalue Density of Large Random Matrices
- **Steve Zelditch** Universality and Scaling of Correlations of Zeros and Complex Manifolds
- **André Voros** Exact Quantization Method for General 1D Polynomial Potentials
- **John Hannay** The Chaotic Analytic Function
- **Patricio Leboeuf** Random Polynomials in Quantum Chaos: a Review
- **Eugene Bogomolny** Spectral Statistics
- **Paul Zinn-Justin** Matrix Models, Links and Knots
- **Mark Srednicki** Correlations in Chaotic Eigenfunctions in Space and Time
- **Alexander Soshnikov** Universality at the Edge of the Spectrum in Wigner Random Matrices
- **Peter Sarnak** Random Matrix Theory and Zeroes of Zeta Functions - a Survey
- **Andrew Odlyzko** The 10^{22}-nd Zero of the Riemann Zeta Function
- **Michael Rubinstein** Zeros of L-functions: Computations and the Behavior of Low Zeros
- **Jon Keating** RMT and V(1/2+*it*)
- **Steven Gonek** The Second Moment of the Reciprocal of the Riemann Zeta Function and it's Derivative
- **Alex Gamburd** Spectraa of Elements in Group Rings and Random Matrix Theory
- **Ali Ozluk** Zeros of Quadratic L-functions Close to the Real Axis
- **Atle Selberg** Linear Combinations of L-functions and Zeros on the Critical Line
- **Dennis Hejhal** Linear Combinations of L-functions and Zeros off the Critical Line
- **Jeff Lagarias** Complements to Li's Criterion for the Riemann Hypothesis
- **Audrey Terras** Artin L-functions of Graph Coverings
- **Kannan Soundararajan** Beyond Pair Correlation
- **Svetlana Katok** Spanning Sets for Automorphic Forms and Dynamics of the Frame Flow on Complex Hyperbolic Spaces
- **Zeev Rudnick** The Quantized Cat Map and Quantum Ergodicity
- **Daniel Goldston** The Logarithmic Derivative of the Riemann Zeta-Function and Pair Correlation of Zeros
- **David Farmer** Mean Values of L-functions and Symmetry
- **Philippe DiFrancesco** Coloring Random Triangulations
- **Kurt Johansson** Discrete Orthogonal Polynomial Ensembles
- **Eric Rains** Algebraic Aspects of Increasing Sequences
- **Jinho Baik** The Asymptotics of Monotone Subsequences of Involutions
- **Persi Diaconis** From Random Permutations to Random Matrix Theory (and Back)
- **Craig Tracy** On the Distributions of the Lengths of the Longest Monotone Subsequences in Random Words
- **Alexei Borodin** Z-Measures on the Partitions, Robinson-Schensted-Knuth Correspondence
- **Andrei Okounkov** Representation Theory and Measures on Partitions

and Sarnak, gives strong evidence of the validity of the heuristic. In the case of random permutations, the basic idea has recently been proved, and the workshop provided an occasion for further progress in this direction.

The application of the random matrix heuristic to zeta functions has many aspects; one of the most famous is the connection between the spacings of the zeros of the Riemann zeta function and the separation of eigenvalues of random unitary matrices. Let A be a random n by n unitary matrix, chosen uniformly according to the Haar measure on the compact group of all unitary matrices. If the eigenvalues are e^{2piq_j} where

$$0 \leq q_1 \leq q_2 \leq \cdots \leq 1$$

then we can consider the spacings $q_{j+1} - q_j$ between these *eigenvalues. Since there are n* such spacings it is natural to normalize these values by considering the random variable

$$X_j := n(q_{j+1} - q_j)$$

whose expected value is 1. One finds that the distribution function of these random variables is given by the "Gaudin distribution" which was investigated by Gaudin and Mehta in the early 60's. This distribution is shown by the continuous curve in the accompanying Figure; the function has been expressed as a Fredholm determinant, though no truly simple expression for the distribution is known.

Now we consider spacings between zeros of the zeta function. Let $V(s)$ be the Riemann zeta function, so that $V(s)$ is analytic for complex s • 1, and is given by the sum and product

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p \text{ prime}} \left(1 - \frac{1}{p^s}\right)^{-1}$$

on the half-plane Re(s) > 1. This function is of enormous importance in number theory. The celebrated Riemann Hypothesis asserts that the non-real zeros of the zeta function have real part equal to 1/2 so that they have the form

$$s = \frac{1}{2} + i\gamma$$

for real g Since $V(\bar{s}) = \overline{V(s)}$, it suffices to consider zeros with positive imaginary part. It is known that the number of zeros whose imaginary part is at most T is approximately $T\log(T)/2p$. Thus the zeros become more dense as the imaginary part increases. All known zeros do indeed satisfy the Riemann Hypothesis, and in order to analyze their spacing it is useful to order the zeros $1/2+ig_j$ by their imaginary parts

$$0 \leq g_1 \leq g_2 \leq \cdots,$$

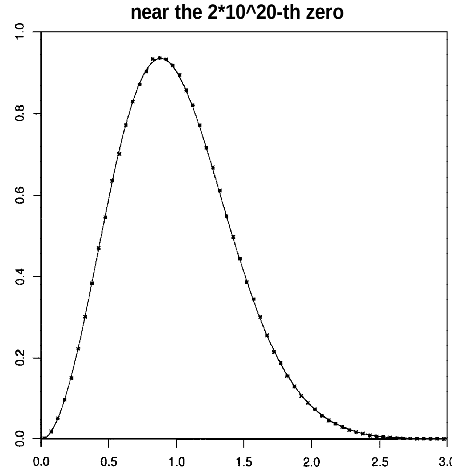
and to normalize them to have average spacing 1 by defining

$$\hat{\gamma}_j = \frac{\gamma_j \log \gamma_j}{2\pi}.$$

Finally, we consider the spacings between consecutive normalized zeroes:

$$X_j = \hat{g}_{j+1} - \hat{g}_j$$

Although the X_j are deterministic, it is useful to think of them as random variables and to plot their distribution. Their expected values are 1, and it seems plausible that they might behave as identically distributed random variables. Empirically, one finds that these variables are distributed according to the Gaudin distribution! Indeed, the figure shows the fit between the Gaudin distribution and a histogram of nearest neighbor spacings among the (approximately) one million zeros near the $2 \cdot 10^{20}$ zero. The dots in the figure are the relative frequencies of bins of width .05. The fit is remarkable.



The extensive computations represented by these figure were done by Andrew Odlyzko [Od], who was motivated by earlier work by Montgomery on pair correlations of zeros of the zeta function. Rudnick and Sarnak [RS] have subsequently been able to prove several consequences of the random matrix predictions about the correlations. In ground-breaking work, Katz and Sarnak [KS1] have proved, more or less, the analogues of these conjectures for the spacings of zeros of zeta functions of curves over finite fields. This provides evidence for the validity of the heuristic for the Riemann zeta function, and support for the long-standing notion that perhaps the zeros have a spectral interpretation that might lead to the Riemann Hypothesis in a natural way.

Now I'll turn to even more recent news. It has been shown that a natural property of random permutations can be modeled by looking at the largest eigenvalue of random Hermitian matrices! Specifically, a suitably scaled variant of the "longest increasing subsequence of a random permutation of n objects" random variable has the same asymptotic distribution as a suitably scaled variant of "the largest eigenvalue of a random Hermitian matrix."

One way to make the notion of a random Hermitian matrix precise is to let the diagonal entries be standard (real) Gaussian random variables, and let the entries above the diagonal be standard complex Gaussian random variables (the entries below the diagonal are of course determined by the Hermitian property. As alluded to above, the results here will hold under many other reasonable definitions.) Let λ_n denote the largest of the (real) eigenvalues of such a random matrix. It is known that the mean of the random variable λ_n is $2n^{1/2}$, and further properties have been extensively studied. The distribution function for λ_n can be expressed in various ways, e.g., as a Fredholm determinant, but recently the following explicit expression for its limiting distribution was obtained by Tracy and Widom ([TW1])

$$\lim_{n\to\infty} \text{Prob}\left(\frac{\lambda_n - 2n^{1/2}}{2n^{-1/6}} \le x\right) = \exp\left(-\int_x^\infty (t-x)q(t)^2 dt\right)$$

where $q(x)$ is the unique solution of the Painlevė II equation

$$q''(x) = xq(x) + 2q(x)^3$$

that is asymptotic to the Airy function Ai(x) as x goes to infinity. Thus if we suitably scale the largest eigenvalue we find a well-defined limiting distribution, often nowadays called the Tracy-Widom distribution.

Now let L(n) be the length of the longest increasing subsequence of a random permutation π, where π is chosen uniformly at random from among the n! permutations of n objects. It is known that the mean of L_n is $2n^{1/2}$, and that in order to get a limiting distribution as n goes to infinity one has to suitably scale the difference between L_n and its mean. More precisely, the theorem of Baik, Deift, and Johannson [BD] asserts that

$$\lim_{n\to\infty} \text{Prob}\left(\frac{L_n - 2n^{1/2}}{2n^{1/6}} \le x\right) = \exp\left(-\int_x^\infty (t-x)q(t)^2 dt\right)$$

where $q(x)$ is as above, so that the asymptotic distribution of the appropriately scaled form of L_n is the same as the asymptotic distribution of the scaled form of λ_n. The sharp-eyed reader will note that the scaling factor for L_n is different from the one for λ_n, and in general the appropriate scaling factors remain somewhat mysterious.

This surprising connection has led to a burst of related results ([AD], [Ok]), some of which were described at the workshop, and some of which were even discovered during the workshop. Recent work of Okounkov [Ok] gives an unexpected and exciting approach to explaining the connection between permutations and random matrices via geometry, namely by showing that each can be interpreted in terms of triangulations of Riemann surfaces and the monodromy of branched covers of surfaces.

Reference

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[TW2] C. A. Tracy and H. Widom, Universality of the Distribution Functions of Random Matrix Theory, submitted to the proceedings of *Séminaire de Mathématiques Supérieures: Integrable Systems: From Classical to Quantum*

[TW1] Tracy and H.Widom, Level-spacing distributions and the Airy kernel, *Commun*. *Math. Phys.* **159** (1994) 151-174.

Lunch on the MSRI patio on a warm fall day

Special Events

Galileo — A Dialog on Science, Mathematics, History, and Drama

Friday, October 15, 1999, 4-5:30 p.m.
Berkeley Repertory Theatre

This fall the Berkeley Repertory Theatre is performing *The Life of Galileo,* David Hare's new English version of Brecht's play. In association with the Theatre, MSRI is hosting an unusual public dialog between George V. Coyne, S.J., Robert Osserman, and Michael Winters.

George Coyne is the Director of the Vatican Observatory, and served on a commission appointed by Pope John Paul II to study the Galileo affair, and was in charge of publishing the findings of the commission.

Robert Osserman is MSRI's Special Projects Director. He is a Professor Emeritus at Stanford University, and is the author of *Poetry of the Universe: a Mathematical Exploration of the Cosmos*.

Michael Winters is an actor who has appeared in a number of stage, screen, and television productions (including a role in *Ally McBeal*). Currently, he has the lead role in the *The Life of Galileo.*

The Future of Mathematical Communication

MSRI workshop, December 1-5, 1999

Organizing Committee: François Bergeron, Jonathan Borwein (co-chair), Joe Buhler (co-chair), Bradd Hart, Martin Groetschel, Peter Michor, Andrew Odlyzko.

This meeting is intended to provide a snapshot, at the end of the millennium, of the present state of mathematical communication, and a provocative look at the future from a variety of perspectives. The meeting is a followup to a meeting with the same title held at MSRI five years ago.

In addition to the workshop itself, there will be several associated events: a streaming video training session, a one-day symposium on electronic publishing in the sciences, and a public session of talks and discussion under the auspices of the IMU's Committee on Electronic Information and Communication.

The symposium will be held in the Anderson Auditorium on the UC Berkeley campus, and the speakers will be Phil Agre, Will Hearst, Pam Samuelson, and Hal Varian.

The conference is being sponsored by MSRI, the Pacific Institute for the Mathematical Sciences, the International Mathematical Union, Centre de Recherches Mathematiques, and the Fields Institute, with additional support from the American Mathematical Society, the Canadian Mathematical Society, Cambridge University Press, Springer Verlag, Wolfram Research, Inc., and Waterloo Maple, Inc.

Modularity of Elliptic Curves and Beyond

MSRI workshop, December 6-10

Organizing Committee: Brian Conrad, Jean-Marc Fontaine, Barry Mazur, Ken Ribet (chair), Richard Taylor

A famous conjecture asserts that every elliptic curve over the rational numbers is "modular." A proof of this conjecture for a broad class of elliptic curves was a key idea behind the proof of Fermat's Last Theorem.

Christophe Breuil, Brian Conrad, Fred Diamond, and Richard Taylor are preparing a paper that contains a complete proof of the modularity conjecture for all elliptic curves. The argument follows the basic direction of Andrew Wiles and Richard Taylor's earlier work in conjunction with Fermat's Last Theorem.

MSRI will host a "Hot Topics" workshop to celebrate this development, and discuss current work and future directions in elliptic curves, modular forms, and Galois representations. Details and further information can be found on our web site.

LECTURES AVAILABLE IN STREAMING VIDEO

Ever wish you could beat your kids at dots-and-boxes? We now have a lecture by **Elwyn Berlekamp** in our archive that will show you how — and show you the connection of this children's game with deep ideas in combinatorial game theory.

Also newly available is the 1999 von Neumann Symposium that introduced our two programs this semester: **Noncommutative Algebra** and **Galois Groups and Covering Spaces**. Many of the lectures are accessible to an audience with just a basic graduate algebra course. By the time this newsletter comes out, lectures from our workshops on **Galois Theory** and **Mathematics of Imaging** will be available too.

Postings from last spring include a lecture by **Donald Knuth** on **The Birth of the Giant Component of a Random Graph** and the complete lectures from our two Summer Graduate Programs, **Lie Groups, Moving Frames, and Exterior Differential Systems** with **Robert Bryant** and **Jeanne Clelland**; and **Nonlinear Dynamics of Low-Dimensional Continua** with **L. Mahadevan** and **Annette Hosoi**.

In all, over 400 lectures are now available on demand from MSRI's streaming video archive. Come to our website and click on video to browse the collection yourself! Instructions for getting the (free) web tool necessary are there too.

Programs and Events

Semester and Year-long Programs

Fall 1999-Spring 2000	**Noncommutative Algebra Program**
Fall 1999	**Galois Groups and Fundamental Groups**
Spring 2000	**Topics in Numerical and Applied Mathematics Program**
Fall 2000-Spring 2001	**Operator Algebras**
Fall 2000	**Algorithmic Number Theory**
Spring 2001	**Spectral Invariants**
Fall 2001	**Integral Geometry**
Fall 2001	**Inverse Problems**
Spring 2002	**Infinite Dimensional Lie Algebras and Mathematical Physics**

1999-2000 Events

August 16-27	**Von Neumann Symposium in Noncommutative Algebra and Galois Groups and Fundamental Groups**
October 4-8	**Workshop on Constructive Galois Theory**
October 11-15	**Workshop on Galois Actions and Geometry**
October 15	**Galileo Dialog** (see Special Events)
October 25-29	**Workshop on Hopf Algebras**
November 1-5	**Workshop on Mathematics of Imaging**
December 1-5	**Workshop on The Future of Mathematical Communication 1999** (see Special Events)
December 6-10	**Hot Topics Workshop: The Modularity of Elliptic Curves, and Beyond** (see Special Events)
January 5-7	**Workshop on Computational Algebraic Analysis**
January 10-14	**Workshop on Combinatorial Algebra**
February 7-11	**Workshop on the Mathematics of Quantum Computation**
February 14-25	**Workshop on the Interactions between Algebraic Geometry and Noncommutative Algebra**
March 6-17	**Workshop on Homogenization and Effective Media Theories**
March 20-31	**Superconvergence in Finite Element Methods**
March 27-31	**Workshop on Quantum Groups, in Morelia, Mexico**
April, 3-14	**A posteriori Error Estimation and Adaptive Approaches in the Finite Element Method**
April 17-28	**Elastic Shells: Modeling, Analysis and Numerics**
May 8-12	**Workshop on Geometric and Topological Aspects of Group Theory**
June 12-23	**Summer Graduate Program I, Mathematical Challenges in Molecular Biology** (with PMMB)
June 26-28	**The Mathematics of Genomics and Proteomics Workshop** (with PMMB)
July 10-21	**Summer Graduate Program II, Combinatorial Game Theory**
July 24-28	**Workshop on Combinatorial Game Theory**

Susan Montgomery, Co-Organizer of the Noncommutative Algebra Program

New members of MSRI's Governance Committees

This year the Board of Trustees passed a resolution to enlarge the board by electing six trustees each year instead of four. The six newly elected trustees are **Robert Bryant** (*Duke University*), **Richard Brualdi** (*University of Wisconsin, Madison*), **William Randolph Hearst III** (*Kleiner, Perkins, Caufield and Byers*), **Cathleen Synge Morawetz** (*Courant Institute*), **James Simons** (*Renaissance Technologies*), and **Carol Wood** (*Wesleyan University*). Joshua Leslie's term as secretary of the board having expired, the Board elected board member **Robert Oliver** as its new secretary. Oliver is Professor Emeritus at UC Berkeley in Industrial Engineering and Operations Research, and Chairman of the Board of Fair, Isaac, Inc.

On the recommendation of the Human Resources Advisory Committee (HRAC), the board also elected **William Hawkins** (*University of the District of Columbia*) and **William A. Massey** (*Bell Labs*) to the HRAC. **Robert Megginson**'s term as chair expired, and **William Yslas Velez**, Professor of Mathematics at the University of Arizona, was elected to replace him.

Finally the board elected **Sun-Yung Alice Chang** (*Princeton University*) and **Benedict Gross** (*Harvard University*) to the Scientific Advisory Committee. It also appointed **Elias M. Stein** of Princeton University and **Margaret Wright** of Bell Labs as cochairs of this committee.

Journalist

in Residence Program

It is often said that mathematics is too abstract; too remote from "real life" to be of interest to the general public. It is a commonplace that mathematicians are too far "up in the clouds" to be able to explain their science to the layman. Yet, mathematics underlies almost all of real life as we know it today (computers, satellites, finance), and successful attempts to explain mathematics are popular with the general public (i.e., K. C. Cole's *The Universe and the Teacup*, R. Osserman's *The Poetry of the Universe*). Mathematics CAN be presented intelligibly to the public and there is genuine interest when that happens.

Supported by grants from the Gabriella and Paul Rosenbaum and William Randolph Hearst Foundations, MSRI has initiated a Jounalist-in-Residence Program to help make contacts between mathematical scientists and journalists who can spread more and better information to the public. In this issue we include two articles produced by recent Journalists-in-Residence, Ivars Peterson and Brian Hayes, on a different topic: their impressions of MSRI.

MSRI's Journalist-in-Residence program is supported by a board of distinguished advisors, chaired by William R. Hearst III, and already has a distinguished alumni as well.

Journalist-in-Residence Program Board

David Eisenbud	*MSRI*
Joe P. Buhler	*MSRI*
David Hoffman	*MSRI*
Robert Osserman	*MSRI*
Orville Schell	*Dean, Department of Journalism, UC Berkeley*
Timothy Ferris	*Professor Emeritus, Department of Journalism, UC Berkeley*
William R. Hearst III	*Partner, KPCR*
John Wilkes	*Head, Science Communication Program, UC Santa Cruz*

Journalist-in-Residence Program

Spring 1998	
K. C. Cole	*Science Writer for the L.A. Times*
Fall 1998	
Allyn Jackson	*Senior Writer and Deputy Editor for the Notices of the AMS*
Spring 1999	
Brian Hayes	*Computing Science Writer for American Scientist*
Summer 1999	
Ivars Peterson	*Mathematics/Computer Writer and Online Editor at Science News*
Fall 1999	
Beverly Wachtel	*Producer/Writer for Earth and Sky Radio Series*
Spring 2000	
Robert Coontz	*Free-lance Science Writer*
Fall 2000	
James Holt	*Book Columnist, Wall Street Journal*

Serving as MSRI's Journalist-in-Residence during the summer of 1999 proved a refreshing break from my weekly routine of reporting and editing for *Science News*, a newsmagazine based in Washington, D.C.

After nearly 18 years of writing articles and books about mathematics, I was somewhat prepared for the highly interactive marketplace of mathematical ideas that I found at MSRI. What I didn't anticipate were the little surprises and distinct pleasures of MSRI's physical setting.

One typically foggy morning soon after I had arrived at MSRI, I happened to glance out of my office window. Peering through the waves of mist swirling around the building, I caught a glimpse of goats busily chomping on thick clumps of tall yellow grass scattered across the steep hillside down below. It turned out that the goats served as natural grass eliminators, able to go where no lawn mower dare venture, as part of an effort to reduce the risk of fire sweeping through the Berkeley hills.

There's even more to see in the brilliant sunshine of a Berkeley summer afternoon. Housed in a three-story structure clad in weathered wood, MSRI occupies a spectacular site – longitude 122 degrees, 14 minutes, 23 seconds West, latitude 37 degrees, 52 minutes, 49 seconds North, and elevation 1,260 feet above sea level. From that height, you get an unobstructed view of San Francisco Bay and the Golden Gate Bridge glinting in the distance.

In such a setting, it isn't difficult to imagine MSRI as an important center of the mathematical world. Indeed, the Institute's programs, conferences, and workshops attract mathematicians from all over the globe. Some stay just for a few days; others spend a semester or more at the center, often focusing on a hot topic in mathematical research.

"For the semester that we're running a program, we are usually the strongest center in the world in the field of that program," remarks David Eisenbud, MSRI's director. Last spring's program was devoted to random matrices and their applications. In August, attention shifted to noncommutative algebra and Galois groups.

In many ways, MSRI programs represent an effort to overcome the fragmentation of mathematical research into private conversations and highly specialized endeavors accessible only to a handful of experts. The idea is to gather a diverse group of mathematicians representing, when possible, different approaches to and interests in a given topic. In some cases, physicists and other scientists join in the discussions and presentations. The resulting interactions turn the programs into exciting learning experiences for everyone involved, as they did in the recently completed session on random matrices, which had links to both number theory and quantum mechanics.

It's not unusual to hear a variety of accents and languages when MSRI members and visitors compare notes and trade tips. Each office has a blackboard (and plenty of chalk). Additional blackboards are strategically located in the atrium, along corridors, and even outdoors at the patio, ever ready to bear the scribblings that inevitably accompany an impromptu seminar.

Even in an age of instant communication via e-mail, telephone, fax, and the Internet, nothing beats face-to-face encounters – at

Reflections

Ivars Peterson *Science News (ip@sciserv.org)*

a blackboard or over a table-to work things out. Afternoon teatime, in particular, draws people out of their offices and away from their solitary pursuits.

Remarkably often during my three-month sojourn at MSRI, I witnessed a mathematician standing at the ubiquitous blackboard, coffee cup or cookie in one hand and stick of chalk in the other, answering a question or patiently explaining some new mathematical wrinkle to interested bystanders. A book could be written about mathematical advances that came about because of chance encounters at afternoon tea!

The skylights, floor-to-ceiling windows, white walls, gray carpeting, and potted bamboo plants create a subdued environment pleasantly conducive to mathematical thought and interchange. "It's a nice place to work," Eisenbud insists, gently understating the pleasure he takes in being at the Institute.

A noncirculating library with an extensive array of journals, a quirky collection of old and new books, and an assortment of mathematical games and puzzles provide handy reference material and entice the mind. Members can also glance at the morning *New York Times* or the latest issues of *Nature, Science, Science News, The Chronicle of Higher Education, Scientific American, American Scientist, Communications of the ACM,* and *The Nation.*

Some MSRI efforts reflect the enormous need for programs that bridge the gap between mathematics and the other sciences and foster interdisciplinary approaches. The coming year will feature workshops devoted to mathematics and imaging, the future of mathematical communication, quantum computation, and computational biology.

This summer saw the addition of a two-week program for graduate students interested specifically in the application of mathematics. The subject of the course was low-dimensional continuum mechanics, and students had a chance to try out their understanding of both mathematical model and physical theory in simulation projects ranging from microfluidic mixing to turbulent convection and pattern formation in liquids. Next year's course will focus on mathematical issues in molecular biology.

Even during a lengthy hike (led by intrepid outdoorsman David Eisenbud) into nearby Tilden Park for a lunchtime picnic and barbecue, the students taking this summer's course continued to puzzle over their projects, in between comparing experiences at different universities, exchanging gossip, telling travel tales, and pondering job prospects. Those conversations, too, represent an affirmation of the collaborative nature of contemporary mathematical research.

Mathematical outreach can extend to all sorts of audiences. In July, a class of high school math students visited MSRI to learn a little about what mathematicians do and what makes them tick. From Hugo Rossi, outgoing Deputy Director, they obtained a glimpse of both the immense appeal and the inevitable frustrations of mathematical research at the frontiers of thought. They even got a brief lesson in the curious arithmetic of the ancient Egyptians.

The students also saw an impressively dramatic demonstration of juggling with balls and clubs, performed by Joe Buhler, incoming Deputy Director. They got a feel for the combinatorics of juggling-how numbers can be used to represent different juggling patterns. To their delight, the students discovered that they could tap into some of the mathematical talent on display to glean hints on how to handle homework problems involving slope (rise over run) and linear equations.

Nonetheless, the gap between what the general public gleans of mathematical research and what mathematicians actually do and think remains enormous. A year ago, MSRI organized a three-day conference on "Mathematics and the Media," which brought together a diverse group of mathematicians and science journalists. Meeting discussions vividly illustrated the immense difficulties involved in translating interesting mathematical research into terms understandable and meaningful to reporters and the general public. In the end, mathematicians gained a better appreciation of the obstacles and deadlines that journalists face in writing their articles, and journalists obtained glimpses of some cutting-edge mathematics.

I have long argued that mathematicians ought to make a greater effort to communicate their ideas and research effectively not only to the general public but also to their own mathematical colleagues and to scientists and engineers. I have attended far too many mathematics lectures where even mathematicians quickly lose the talk's thread and begin to nod off, and I have glanced at too many research papers that fail to explain why a given topic is worth pursuing or to put the material in a broader context.

In a 1991 essay, I wrote: "Research worth publishing should also be worth communicating. There is room in the mathematical literature for at least a small concession to a nonmathematical audience that may actually find the work of interest. And if mathematics is more than just a private game... then mathematicians must take some responsibility for communicating their ideas in ways that convey the meaning of their work to broader audiences."

MSRI offers more for the mind than mathematics. Special art exhibitions add provocative color and form to the white walls and the space within the high-ceilinged atrium. The current display features the collage-style work of Berkeley artist Mari Marks Fleming – visually rhythmic speculations on "time, nature, and the space between."

A more permanent fixture is an artwork by sculptor and mathematician Helaman Ferguson. Called *The Eight-Fold Way,* the sculpture sits in the middle of the patio, framed by a backdrop of hills, pines, and eucalyptus trees. Carved out of a block of white, marble, the roughly tetrahedral form rests on a black serpentine column. Covered with mysteriously indented curves and sinuous ridges, the sculpture invites comment and touch.

My final image is of a late August afternoon concert in the atrium – of sunlight streaming in through the skylights to illuminate the members of the Peregrine Trio and of the sublime music of Mozart, Beethoven, and Haydn sailing throughout the building. It seemed a fitting finale to a stimulating summer spent immersed in a world devoted to the pursuit of mathematics.

Aftermath

Brian Hayes

As every visitor to MSRI immediately discovers, the INSTITUTE occupies the brow of a steep hill, more than a thousand feet above the Berkeley campus. On my first day as MSRI's Journalist in Residence last January, I set out to climb the hill by bicycle, taking what seemed a direct and obvious route. I couldn't make the grade. Halfway up, the pedals refused to turn. It was a mortifying failure, but I kept trying, and eventually I reached the summit by a longer, more roundabout approach. The experience neatly prefigured the mathematical challenges of my residency, which also presented a steep learning curve and yielded only to an indirect attack. (And could it be significant that the successful bike route began on Euclid Avenue?)

Mountain climbing is a common metaphor for what science writers are supposed to do. As Journalist-in-Residence, my main role – as I understood it was to climb the hill, learn as much as I could of the mathematics being done up there, and then explain it to the public waiting below. But an observer from outside can serve another, more reflexive, purpose as well. Here in the pages of *The Emissary* I address a few words to the mathematical community itself, reporting back on what the world of research mathematics looks like to a visitor from another world. (I would rate myself an interested, engaged and sympathetic outsider – but an outsider all the same.)

During my six months' residency, I attended roughly a hundred talks in the MSRI lecture room. I learned quite a lot of mathematics this way, but I learned other things as well. As I took my notes on the talks, I got into the habit of reserving the left margin of each page for observations on the practice and presentation of mathematics, and on social interactions in the lecture room. The paragraphs that follow are based on my marginalia, organized under four main headings.

Chalk. Dead diatoms are becoming rare elsewhere in the academic world, but mathematicians are still expected to be masters of blackboard technique. Although the MSRI lecture room is equipped with projectors for transparencies and videos and computer displays, most speakers relied primarily or exclusively on chalk and slate. By the end of the term I had acquired a new respect for the particular virtues and versatilities of this ancient writing medium.

It's no mystery why a mathematical "talk" is almost always a visual as well as a verbal presentation. The two-dimensional notation of mathematics is hard to squeeze into one-dimensional speech. But any graphic device would satisfy this need; why is the blackboard favored over transparencies or Powerpoint? An efficiency expert would find the practice outrageously wasteful. Everyone must sit and wait for the speaker to write out equations that could have been prepared in advacne, probably more neatly and with fewer errors. But waiting for the equation to unfold is just the point: It turns out that mathematics is far easier to grasp when you can see it in the process of being written, rather than having it presented as static text. The speaker, by adjustments in timing and emphasis, directs attention to the more important parts of an expression. Indeed, an equation is seldom written in strict left-to-right order. Instead the more important terms come first, followed by less-critical constants and coefficients, with details such as ranges of integration filled in last. The sequence carries meaning; it reveals a hierarchical structure in the equation, which tends to get flattened out in a published paper.

Virtually all of the speakers during my months at MSRI performed with grace and confidence at the blackboard. Perhaps it is naive of me even to mention this, as if I were noting in astonishment that professional pianists are all comfortable sitting at the keyboard, or that chess masters know how to move the pieces. Nevertheless, blackboard mannerisms made a strong impression. Learning to wield the chalk with authority, and to deftly erase an errant symbol with the heel of the same hand, seems to be an essential step in the education (or acculturation) of a mathematician. Many go on to master the higher blackboard management, expertly juggling the six sliding panels at the front of the room, and occasionally even managing the obscure technique of using all nine boards.

Group Dynamics. Is it acceptable to talk through someone else's talk? That depends on who's talking.

Under the social contract of the lecture room, interrupting the speaker is not necessarily rude. For example, calling out a typographical correction – "I think you mean minus beta, no?" – is generally welcomed as a friendly intervention. (And it's proof that someone in the audience is awake and paying attention.) Interrupting with a question – a request for clarification – "I don't understand how you derived that partition function" – is somewhat more assertive but seldom appears hostile. Some speakers explicitly invite this kind of dialogue.

On the other hand, interrupting to dispute the speaker's results, or to argue for your own interpretation of them, is pretty clearly an act of aggression – or so it appears to a bystander like me. The issue here is not one of manners. Every group and institution evolves a style of discourse that serves its own purposes, and there's no sense in being judgmental about it. (The British Parliament and the U.S. Congress have very different rules of debate, but they are both successful bodies.) Hijacking someone else's talk may well be the best way to sweep aside needless verbiage and focus on points in contention.

But as I sat in the lecture room meekly observing these sometimes-heated exchanges, one aspect of the interactions began to disturb me. I could not help noting that a speaker's likelihood of being challenged in the middle of a talk depends to some extent on the speaker's seniority, or perhaps on some similar measure of stature within the community. A distinguished professor seldom has to fend off a hostile interruption, but a young postdoc is more likely to face at least one challenge from the floor. And being interrupted by abelligerent question is not the end of it. Someone else may interrupt the speaker again to give the answer!

Boundaries and Rivalries. A whole genre of wan academic humor thrives on the friction between scientific disciplines. "A mathematician, a physicist and an engineer walk into a bar...," the joke begins. As a professional observer of mathematics, physics and engineering, I have certainly been aware of differences in style and philosophy. But my few months of total immersion at MSRI, in a group that had strong representation from both physics and mathematics, altered my view of the issue. The problem of working together across disciplinary boundaries is no joke.

Physics and mathematics have the closest possible family ties, with a number of major founding figures (Newton, Lagrange, Laplace) claimed by both tribes. It's therefore a little unsettling to realize that the two disciplines have deep disagreements not only about formalities such as notation and terminology but also about fundamentals, including what constitutes a valid proof. Problems considered settled by (some) physicists are still classified as open questions by (some) mathematicians. With that deep a dispute, communication is sometimes reduced to shouting across the barricades.

Of course mathematics itself has its own internal divisions into specialties and subspecialties, whose intricacies can baffle an outsider. ("A differential geometer, an algebraic geometer and a symplectic geometer walk into a bar....") Apparently the divisions can even baffle an insider. At many of the talks I attended, I was not the only member of the audience struggling to follow the argument. Maintaining open lines of communication appears to be a challenge even within mathematics.

Bridging such inter- and intradisciplinary chasms is one of the principal goals of MSRI (and of various other institutions). Creating a true meeting of the minds looks to be a harder problem than I ever imagined. Bringing people together in the same room is often not enough. But bridges can be built. It does happen. I saw it happen repeatedly in the lecture hall at MSRI. Although the dialogue sometimes failed, it was more often successful.

Intensity. If we can't always count on everyone playing peacefully in the sandbox, the reason is not hard to find. Mathematical gatherings get intense because people doing mathematics care passionately about it. The emotional tension is a gauge of intellectual commitment. You don't tell a football player who just lost the World Cup final, "It's only a game," and you don't tell a mathematician hot on the trail of a new result, "It's only mathematics."

Personal ambition is surely a factor here, as it is in any other discipline, but there is more. Lewis Thomas, in his essay *Natural Science* (in "Lives of a Cell", Viking, 1974) described the phenomenon eloquently:

"Scientists at work have the look of creatures following genetic instructions; they seem to be under the influence of a deeply placed human instinct. They are, despite their efforts at dignity, rather like young animals engaged in savage play. When they are near to an answer their hair stands on end, they sweat, they are awash in their own adrenaline. To grab the answer, and grab it first, is for them a more powerful drive than feeding or breeding or protecting themselves against the elements."

The nature of the mathematical enterprise may raise the stakes even higher than they are elsewhere in the world of science and scholarship. In other fields, an idea that proves fruitful for a time but eventually has to be discarded is counted a partial success. In mathematics, a proof that turns out to have a serious flaw is nothing but an embarrassment. Even though Hilbert's dream of a complete and consistent formal system has been set aside, the collected literature of mathematics is treated as a sacred text to be guarded against corruption and dilution. Shoddy or trivial work is positively offensive; the emotion it evokes is something akin to disgust. By the same token, a result that provides illumination is greeted with a powerful sense of joy.

Strong feelings appear to be intrinsic to the practice of mathematics. They are not to be blunted or softened. Anyone who imagines that this is a purely cerebral, intellectual and emotionlessenterprise has missed the point entirely.

In June I came back down the hill – the descent was a challenge to the nerves rather than the muscles, and went by entirely too fast – and now I find myself in the aftermath of my residency at MSRI. "Aftermath" is one of those words that seem to have lost their roots. In newspapers today it usually refers to the events following a tornado or an earthquake or some such natural disaster. Originally, though, an aftermath was a second mowing – a bonus crop that a lucky farmer might squeeze into the growing season after the first harvest.

My aftermath, happily, falls into the bonus category. The first harvest was learning the mathematics itself, reaping all I could in the time allotted. The bonus was getting a clearer view of how mathematics is done.

MSRI, second and third floors

David Eisenbud

(continued from page 1)

was about non-commutative algebra, and though I haven't worked in that area much over the last 30 years I'm delighted to get a look back into that subject. A lot has changed since I was a student, but some of the preoccupations of that period are still current.

The big news for the Institute is that the NSF "recompetition" for institute funding is over, and MSRI will be refunded. Having spent a lot of effort on the recompetition proposal and its aftermath over the last two years, I was mightily relieved to be finished with it. I felt strongly, on the basis of the lively programs going on at MSRI, that this institute *should* be refunded, so the outcome was rather a confirmation than a surprise... but a relief all the same.

The Institute For Mathematics And Its Applications in Minnesota (IMA) was also refunded, and a new Institute for Pure and Applied Mathematics (IPAM) was started at UCLA. I wish the IMA and our new sibling much success and excitement! IPAM's plans focus strongly on interdisciplinary pursuits.

Among the good effects of the recompetition at MSRI has been a very significant increase in the resources UC Berkeley will give to MSRI. We have worked to attract funding from new private and industrial sources as well, and this effort is already bearing fruit. I'm grateful, on behalf of MSRI, to all the friends and colleagues who played a role in the process: Hugo Rossi, with whom I worked very closely on every detail; the office staff, who worked long and hard on the document with us; Cal Moore who helped us greatly with matters of University support; supporters of the Institute who read drafts of the proposal and donated their ideas; the many organizations and individuals who rallied moral and material support for the Institute; and of course the US mathematical science community, whose broad support and enthusiasm for the Institute must finally have been the most important factor in our winning of the recompetition. May MSRI deserve all this, by continuing to serve mathematics well for many many years to come!

There's been a changing of the guard at MSRI this summer: Hugo Rossi finished his two years as Deputy Director, and Joe Buhler has arrived to take on this challenging role. Hugo signed on in 1997 for just one year, but stayed on for a second. He said he "couldn't resist being here to toast our victory in the recompetition!"

Hugo Rossi and David Eisenbud toast the outcome of the recompetition.

However, with commuting between MSRI and his family in Utah, two years was all he could stay, so a year ago we began to search for a new Deputy Director. I'm delighted that we were able to recruit Joe Buhler, a Harvard-trained number theorist who has taught for years at Reed College. Joe brings a deep and playful appreciation of mathematics, long experience in the central administrative committees at Reed, tremendous sensitivity to others, and a perfectly amazing ability to juggle both tangible and intangible objects. (A picture representing the tangible case is a Page 5.)

Regular readers of this newsletter (and lots of others) already know about our broadcasts of MSRI lectures via streaming video. For example, the wonderful talks I've been listening to in the introductory workshops, expositions of matters from Gel'fand-Kirillov dimension to noncommutative algebraic geometry and etale fundamental groups, are all there for the clicking. Do come and browse — just go to www.msri.org, choose "video", and follow the instructions. We are in the process of developing international mirrors (in Brazil, England, Germany, Israel, Korea, Mexico) to make it easier to watch these programs from afar.

There are new features in this issue of the newsletter: articles by our two most recent Journalists-in-Residence, Brian Hayes (from *American Scientist*) and Ivars Peterson (from *Science News*). The Journalist-in-Residence occupies a half-time position, supported by grants from the Rosenbaum Foundation and the Hearst Foundation, and overseen by a distinguished board (William Randolph Hearst III, Timothy Ferris, John Wilkes, and Orville Schell). The ultimate goal of the program is to increase the awareness of mathematics among the public at large. All mathematicians know that most people are ignorant about what is going on in mathematics (many aren't aware that *anything* is going on!) It's hard to communicate with the public, but we hope to make a small contribution by encouraging mathematicians and writers to interact so that mathematicians can learn to talk to journalists and journalists can develop contacts with mathematicians. A number of interesting popular articles have emerged from this collaboration, and we expect more to come.

The creative energy at MSRI has allowed us to produce programs that relate mathematics to our culture. Last February we presented one that I found particularly exciting: Tom Stoppard (of *Shakespeare in Love* fame) came and held a public conversation with Robert Osserman on *Mathematics in Arcadia*. They discussed Stoppard's play *Arcadia* and his use of mathematical themes: chaos, iterative mappings, thermodynamics, Fermat's theorem — truly an amazing list, naturally and delightfully integrated. You can see the program yourself if you like, as we're distributing a videotape of the event (see our website for details).

This fall we'll follow up with a program on October 15 keyed to a local production of an English version, by David Hare, of Brecht's play 'The Life of Galileo'. One strand of the play concerns Galileo's struggles with the Catholic Church. Our contribution will be another public conversation, this time between Osserman, George Coyne, the Director of the Vatican Observatory and Michael Winters, “Galileo” in the current production. Coyne is an expert on Galileo and an active astronomer. He was co-director of a working group established in 1979 by the College of Cardinals to study the case of Galileo, a study that led to the Church's recent apology. I'm looking forward to this happening! I hope you'll be nearby and join us.

Problem Corner

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Mathematicians often like to pass around problems and puzzles. Given the large concentration of mathematicians at MSRI it's no surprise that problems of all kinds are bandied about. In an attempt to reflect this activity, we will write an occasional column for the Emissary that will highlight puzzles that we've heard recently, here at MSRI or elsewhere. We will credit sources when we know them, but the rich oral tradition alluded to above makes this difficult; indeed, we don't know the original creator of any of the problems given below. The primary criterion for inclusion is just that the problem has been discussed recently, usually at MSRI; some problems may be quite well known (e.g., the first one below), some may be easy, and some may be quite hard.

The MSRI web page (www.msri.org) will contain followup discus sions on these problems. We will post the best solutions that we receive, and will provide further information about these problems. Submissions of problems are welcome.

PROBLEM 1

In contemplating the increase in average rental rates from 1998 to 1999 in 5 Bay Area cities, Jackie Blue finds that in each city the average rent in 1999 was at least 10% larger than the average rent in 1998. Prove or disprove: The average rent in those five cities taken together in 1999 was at least 10% larger than the average rent in 1998.

COMMENT

This is a variant of a well-known conundrum that is relevant to data analysis in many situations. Jackie Blue assists with housing arrangements for long-term visitors at MSRI (and does a spectacular job of it); although this problem is fictitious, she does in fact occasionally worry about rising rental rates in the vicinity of MSRI.

PROBLEM 2

Find the 98-th digit to the right of the decimal point in the decimal expansion of (š2+1)^{500}.

COMMENT

This was one of the monthly take-home contest problems in last year's Berkeley Mathematical Circles, and is probably a variant of a problem that appeared in math contests in Eastern Europe. The monthly context problems provide practice problems for (but are often harder than) the problems in the Bay Area Mathematical Olympiad which MSRI helped found.

PROBLEM 3

Your wealthy father-in-law gives you $10,000 dollars to place a "double-or-nothing" bet on the Boston Red Sox in the World Series. In other words, your father-in-law expects to receive $20,000 if the Sox win, and $0 if they lose.

To your horror, you then discover that the only kind of bets that are allowed at your neighborhood casino are double-or-nothing wagers on individual games. (For simplicity, we assume that the casino accepts bets up until the start of each game, and that the house cut is negligible.) How much should you bet on the first game? I.e., what strategy should you follow so that your final outcome is certain to be equivalent to an overall double-or-nothing bet on the Series. (We remind readers that the World Series is a "best-of-seven" event in which two teams play a series of games and the first team to win four games is the winner.)

COMMENT

This isn't really a probabilistic question: you must achieve the same result as a single double-or-nothing bet on the series, since you cannot allow any possibility whatsoever of alienating your father-in-law. This could be viewed as a prototype of financial engineering problems in which a given risk must be partitioned in a precise way into smaller risks.

PROBLEM 4

The aforementioned father-in-law gives you $10,000 to play in the following card game: The cards of a standard deck are turned over one by one. Each card then remains face up until all are turned over. Before each card is turned over, you can bet any fraction of your current capital on a "double-or-nothing" bet on the color of the next card. How should you bet?

COMMENT

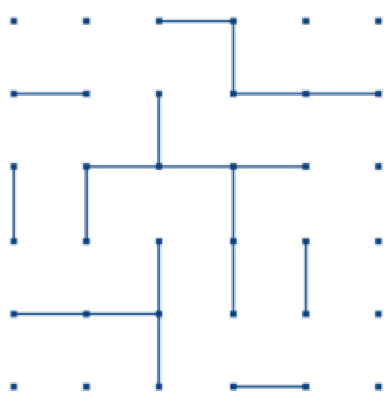
There are different ways to interpret the problem. You could interpret the question as a probabilistic problem in which the deck is shuffled and you want to maximize your expected return on the entire deal. Or you could interpret your goal to be to minimize risk, so that we should imagine an adversary who gets to choose which of the remaining cards to expose at a given turn after seeing your wager. Analogs of both of these situations arise in practice.

PROBLEM 5

Prove that any automorphism of a finite group of order n has order strictly less than n>1.

COMMENT

The shortest solution that we know is entirely elementary, but several pages long. The only published solution that we are aware of is longer. We thank Hendrik Lenstra for bringing this problem to our attention.



Move and Win

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Mathematical Sciences Institutes Reception

AMS meeting
Washington, DC,
Wednesday, January 19, 2000
5:30 p.m. - 7:30 p.m. (Before the Gibbs Lecture)

See old friends, find out what's going on, and have a bite at a reception organized by MSRI, featuring:

Centre de Recherches Mathematiques *(CRM, Montreal),*
Center for Discrete Mathematics and Theoretical Computer Science *(DIMACS, New Jersey)*,
Fields Institute *(FI, Toronto)*,
Institute for Mathematics and Its Applications *(IMA, Minneapolis)*,
Institute for Pure and Applied Mathematics *(IPAM, Los Angeles)*,
Mathematical Sciences Research Institute *(MSRI, Berkeley)*,
Pacific Institute Insititute for the Mathematical Sciences *(PIMS, Vancouver)*.