Spring 2013

EMISSARY

Mathematical Sciences Research Institute

www.msri.org

Math's Versatility: From Planet to Theater

A word from Director Robert Bryant

Welcome to the Spring 2013 issue of the Emissary! Much has happened at MSRI since our last issue, and I hope that you'll enjoy the informative articles on our scientific programs as well as the news items about the accomplishments and recognitions of our current and former members.

The semester started very early for us because MSRI participated in the US launch of the Mathematics of Planet Earth 2013 project at the Joint Meetings in San Diego. This yearlong project is sponsored by a consortium of more than 100 universities and institutes around the world, and its goal is to help inform the public about the essential ways that mathematics is used to address the challenges — from climate science, to health, to sustainability — that we face in living on Earth. We are excited about this collaboration, and hope that you'll become involved by visiting the web site mpe2013.org to learn more about what MSRI and its co-sponsors are doing to promote mathematics awareness in these important areas. (I'll have more to say about some of these events below.)

Kim Harrington Photography

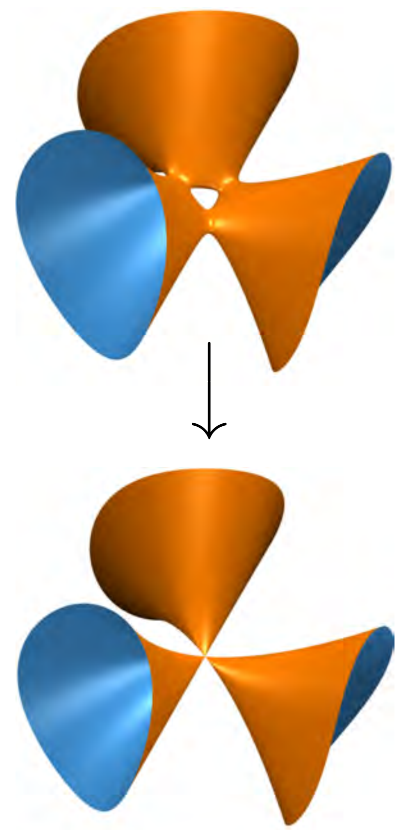
Dr. Emily Shuckburgh gave a Simons Public Lecture for MPE2013. See page 9.

This spring's programs, Commutative Algebra (which is the continuation of a yearlong program that started in the fall) and Noncommutative Algebraic Geometry and Representation Theory, got off to a great start, with a series of heavily attended workshops that highlighted the deep connections between the two programs. You'll find articles about each program and some of the scientists and postdoctoral fellows who are participating in them further along in this issue.

We have had some interesting public events this spring, starting with our annual collaboration with PlaygroundSF in February. This year, the theme was "fearful symmetry" (drawn from the famous Blake poem). One of our visiting members, Dave Benson, and I met with the playwrights on February 13 to hold a discussion with them about how mathematicians think about symmetry in music, mathematics, and physics, why it is so important, and how it can be both enlightening and misleading. After that, each of the playwrights had five days to write a 10-page play using the theme, and Dave and I sat on the jury to help select the top six (of nearly 30 submissions) that would be given staged readings on February 25 at the Berkeley Repertory Theatre.

The playwrights certainly came up with some creative and fascinating ways to make use of the theme, and we were quite pleased with many of the entries. On the night of the performance, we had one of the larger audiences that PlayGround has had (MSRI night

(continued on page 11)



A singular resolution: The D_4 singularity $x^2y - y^3 + z^2 = 0$ and its desingularization. See the article about the NAGRT program starting on page 6.

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Alissa S. Crans

Celebration of Mind

This past October, MSRI welcomed approximately 80 puzzle-lovers of all ages to the first "Celebration of Mind" event that MSRI has hosted. Celebration of Mind is a worldwide celebration of the legacy of Martin Gardner, promoted by the Gathering 4 Gardner (G4G) foundation and held annually on or near his birthdate.

The night opened in the Simons Auditorium with a viewing of an episode of David Suzuki's "The Nature of Things" that featured Martin Gardner. This was followed by two-hour session in which guests explored numerous display tables distributed throughout the building. The tables presented an array of puzzles, games, activities, and a handful of books from Gardner's vast collection. There were dozens of physical puzzles, as well as over 40 books and 15 logic puzzles. We are extremely grateful to Stan Isaacs, long-time officer of the Golden Section of the Mathematical Association of America, for generously sharing many of the puzzles and books from his personal collection.

As part of the activities, Elwyn Berlekamp, an MSRI Board of Trustees member, a leading expert in the theory of games, and a G4G officer, ran a dots and boxes "simul"; Nancy Blachman, founder of the Julia Robinson Mathematics Festivals and a G4G conference organizer, performed and explained card tricks; and Matthias Beck, Associate Professor of Mathematics at San Francisco State University and the 2012 MSRI-UP Research Director, led a Zometool construction.

Gary Antonick, author of the "Numberplay" blog at the New York Times, also attended the event and mentioned it in his October 29, 2012, column. In the column, he shared a puzzle (the 13-Link Chain Puzzle) that was suggested to him by another guest at our event!

It was a great evening filled with friendly, inquisitive people, dozens of challenging puzzles, engaging conversations, and delicious desserts that MSRI is delighted to have hosted.

Matthias Beck (San Francisco State Univ.) works with a young Martin Gardner fan, Nico Brown, on a Zometool construction.

Math Midway Events

Also during the fall, we collaborated on two evening activities with our friends at the Lawrence Hall of Science (LHS). Both events took advantage of the "Math Midway" — an interactive, "traveling carnival" style mathematics exhibition developed by the new Museum of Mathematics (MoMath) in New York — that was being hosted by LHS.

In October, Paul Zeitz, Professor of Mathematics at the University of San Francisco and Director of the San Francisco Math Circle, led a discusion for middle school students, teachers, and parents that was followed by an exploration of the exhibit. This event was also part of the annual Bay Area Science Festival.

The December event was a scavenger hunt through the exhibit organized for mathematics educators at all levels. There were numerous tasks that involved the activities displayed in the exhibit, such as locating points of symmetry on the "Monkey Mat," creating a specific tessellation using the "Miles of Tiles," and illustrating a regular hexagon in a cube in the "Ring of Fire." This event was additionally co-sponsored by the San Francisco Math Circle and Circle for Teachers, the Oakland/East Bay Math Circle, and the Bay Area Circle for Teachers.

On a related note, the directorate of MSRI has been, and continues to be, engaged in conversations with the directors of MoMath. Alissa S. Crans, Director of Educational and Outreach Activities, is serving on the organizing committee for the MOVES Conference (Mathematics of Various Entertaining Subjects) designed to highlight recreational mathematics. More information, including instructions for registering, can be found on the conference website: http://momath.org/moves-conference.

2012–13 Chancellor's Scholar — Dylan Thurston

The UC Berkeley Chancellor's Scholarship award carries a purse of $50,000 and is open to nominees from MSRI only. Chancellor's Scholars must be top researchers and must also be known for excellent teaching.

The 2012–13 Chancellor's Scholar, Dylan Thurston of Columbia University, is a mathematician of remarkable breadth and originality, with outstanding achievements in low-dimensional topology, cluster algebra theory, and computational geometry. He is also a great communicator, an engaging and enthusiastic teacher, and a dedicated research supervisor, the qualities that made his selection as the Chancellor Professor particularly fitting.

The graduate course he taught at Berkeley last fall, "The Geometry and Algebra of Curves on Surfaces," started from the fundamentals of hyperbolic geometry, led to Penner's theory of decorated Teichmüller spaces, and continued to the recently developed connections between combinatorial topology, hyperbolic geometry, and cluster algebras. It was attended by a diverse audience of graduate students, postdocs, and faculty, including a number of MSRI members.

— Sergey Fomin

Matrix Factorizations

Daniel Murfet

This March, there was a focus period on matrix factorizations in the Commutative Algebra program at MSRI. The study of matrix factorizations was initiated by David Eisenbud in 1980 and has grown into a vibrant field of research with connections to many areas of mathematics and mathematical physics. The fundamental observation is that for a polynomial $W \in \mathbb{C}[x_1, \ldots, x_n]$ the equation

$$D^2 = W$$

may have no solution in polynomials, but it may acquire solutions when we enlarge our sphere of consideration to include *matrices*. For example $x_1^2 + x_2^2$ has no square root, but nonetheless

$$\begin{pmatrix} 0 & x_1 - ix_2 \\ x_1 + ix_2 & 0 \end{pmatrix}^2 = (x_1^2 + x_2^2) \cdot I.$$

A *matrix factorization* of W is one of these matrix square roots, that is, a block matrix with polynomial entries

$$D = \begin{pmatrix} 0 & F \\ G & 0 \end{pmatrix}$$

satisfying $D^2 = W \cdot I$. Does a matrix factorization always exist? The example $F = I, G = W \cdot I$ shows that this is the case, but this matrix factorization is *contractible*: there is a matrix of polynomials H with $I = DH + HD$.

The more meaningful answer is that non-contractible matrix factorizations exist precisely when the hypersurface $W = 0$ in $\mathbb{C}^n$ is singular. Moreover, the structure of the space of possible matrix factorizations encodes homological and geometric information about the singularity; this explains the role of matrix factorizations in singularity theory and commutative algebra.

Beyond these subjects, the last decade has seen matrix factorizations appear in surprising places including string theory and low-dimensional topology, and my purpose in this article is to briefly explain these two connections.

The Dirac Equation

To motivate the relation between matrix factorizations and string theory, we begin in an earlier era of mathematical physics with a famous construction of Dirac.

In the late 1920s physicists were seeking a description of the electron compatible with both relativity and quantum mechanics. These two theories were in tension: according to nonrelativistic quantum dynamics, the state of the electron is a complex-valued solution $\psi(x, t)$ of the Schrödinger equation. In this equation time plays a special role, whereas according to relativity a Lorentz transformation relating the measurements of two observers may mix the time coordinate of one with the spatial coordinates of the other, and so the correct wave equation for the electron should be covariant with respect to such transformations.

The first attempt was the Klein–Gordon equation

$$\left[-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} - \frac{m^2c^2}{\hbar^2} \right] \psi = 0$$

with time combined into the Lorentz invariant operator

$$\Box = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}.$$

As a description of the electron this was a failure, but Dirac hit upon the idea of looking for a first-order operator D giving a square root of $\Box$. From the factorization

$$\left(\Box - \frac{m^2c^2}{\hbar^2} \right) = \left(D + \frac{mc}{\hbar} \right)\left(D - \frac{mc}{\hbar} \right), \qquad (\dagger)$$

one would obtain solutions of the Klein–Gordon equation from solutions of the first-order differential equation $(D - \frac{mc}{\hbar})\psi = 0$. To find this square root D, he began with an ansatz involving unknown quantities γ_μ:

$$\left[\sum_{\mu=1}^{4} \gamma_\mu \frac{\partial}{\partial x_\mu} \right]^2 = \Box = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} + \frac{\partial^2}{\partial x_4^2},$$

where we set $x_4 = ict$. In order for the left hand side to match the right, these γ_μ must satisfy the equations

$$\gamma_\mu^2 = 1, \qquad \gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 0, \quad (\mu \neq \nu).$$

Using 4×4 matrices of complex numbers satisfying these conditions Dirac wrote down his famous equation

$$\left[\sum_{\mu=1}^{4} \gamma_\mu \frac{\partial}{\partial x_\mu} - \frac{mc}{\hbar} \right] \vec{\psi} = 0,$$

which is Lorentz invariant, and is in fact the correct wave equation for the relativistic electron. In this equation $\vec{\psi}$ stands for a tuple of four functions.

A quick look at (†) shows that each of the functions in this tuple is a solution of the Klein–Gordon equation, but what is the significance of $\vec{\psi}$ being a tuple rather than a single function? Resolving this mystery led directly to the theoretical prediction of the existence of antimatter!

Let us now return to the world of polynomials, and reinterpret Dirac's construction as a matrix factorization of a quadratic form

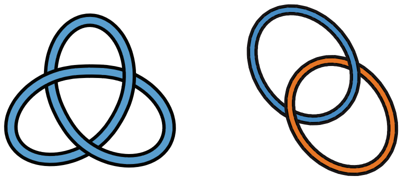
$$\left[\sum_{\mu=1}^{4} \gamma_\mu x_\mu \right]^2 = (x_1^2 + x_2^2 + x_3^2 + x_4^2) \cdot I.$$

This construction was generalized by Buchweitz, Eisenbud and Herzog, who in 1987 classified matrix factorizations of nondegenerate quadratic forms.

Although it differs in many respects from this example, the modern connection between matrix factorizations and string theory is similar in spirit. A Landau–Ginzburg model is a topological field theory defined on a Riemann surface, whose Lagrangian depends on a polynomial W. In 2002 Kapustin and Li realized that in order to extend the theory to Riemann surfaces with boundary in a way preserving supersymmetry, one has to choose a non-trivial square root D of W. The fundamental quantities of the theory are then expressed in terms of residues, traces and the derivatives $\partial_\mu D$, in a way analogous to the role of the γ_μ in the computation of correlation functions in the quantum theory of the electron.

From Singularities to Knots

Another application of matrix factorizations is to the study of knots. A knot is a smoothly embedded circle in $\mathbb{R}^3$; a link is a disjoint union of non-intersecting knots. The trefoil knot and Hopf link are as shown below:



The basic problem in knot theory is to distinguish knots by computing topological invariants, for example the Jones polynomial. In 2005 Khovanov and Rozansky introduced a new invariant $\mathcal{H}_N(L)$ of oriented links L (here $N > 1$ is an integer) using two particular matrix factorizations D_1, D_2 of the isolated singularity

$$W = x_1^{N+1} + x_2^{N+1} - y_1^{N+1} - y_2^{N+1}.$$

This invariant is a finite-dimensional graded vector space

$$\mathcal{H}_N(L) = \bigoplus_{i,j\in\mathbb{Z}} \mathcal{H}_N(L)^{ij},$$

which is *homological*, in the sense that cobordisms between links determine morphisms of vector spaces. These invariants are being actively investigated by topologists, with $\mathcal{H}_2$ (also known as Khovanov homology) being the best understood; it is known for example that $\mathcal{H}_2$ distinguishes knots more finely than the Jones polynomial.

The matrix factorizations D_1, D_2 are of the form

$$\begin{pmatrix} 0 & 0 & b_1 & b_2 \\ 0 & 0 & -a_2 & a_1 \\ a_1 & -b_2 & 0 & 0 \\ a_2 & b_1 & 0 & 0 \end{pmatrix} \qquad (\ddagger)$$

for some polynomials a_i, b_i in $\mathbb{C}[x_1, x_2, y_1, y_2]$ satisfying

$$W = a_1 b_1 + a_2 b_2.$$

For instance, there are clearly polynomials b_1, b_2 such that

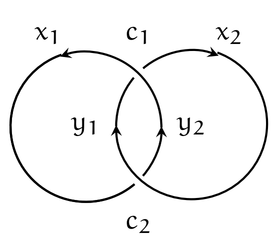
$$W = (x_1 - y_1)b_1 + (x_2 - y_2)b_2$$

and, although it is less obvious, there are b_1', b_2' with

$$W = (x_1 + x_2 - y_1 - y_2)b_1' + (x_1x_2 - y_1y_2)b_2'.$$

These two expressions, when used to fill in the matrix ($\ddagger$), define the matrix factorizations D_1, D_2. As an aside for the experts: while the b_i', b_i' are not *unique*, different choices determine the same matrix factorizations.

Let us now sketch Khovanov and Rozansky's construction using the Hopf link L_{Hopf} as a running example. We begin with a projection of the oriented link onto the plane, with the "edges" between crossings labelled by variables

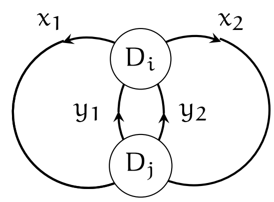


According to the orientation of the strands at the crossing c_1 the x-variables are *outgoing* and the y-variables are *incoming*. To each crossing c we associate the polynomial W_c which is the sum of the $(N+1)$st powers of the two outgoing variables minus the same power of the two incoming variables. In the case of the Hopf link we have simply

$$W_{c_1} = W, \quad W_{c_2} = -W.$$

For each crossing c we can take the matrix D_i and in each entry replace x_1, x_2 by the outgoing variables at c, and y_1, y_2 by the incoming variables at c. This substitution defines two matrix factorizations $D_i(c)$ of W_c using the four variables incident at c.

In the next step, we label the crossings with D_1 or D_2 in all possible ways,



and to such a labelling we associate the operator

$$D_{ij} = D_i(c_1) \otimes 1 + 1 \otimes D_j(c_2)$$

on $V^{\otimes 2}$, where $V = \mathbb{C}[x_1, x_2, y_1, y_2]^{\oplus 4}$. For a general link one defines in the same way an operator on $V^{\otimes m}$, where m is the number of crossings, and V is the free module of rank four over the polynomial ring in the edge variables.

These operators all square to zero, since at the crossing c we have placed a matrix factorization of W_c and the sum of these polynomials over all crossings is zero. Returning to the Hopf link, the relevant calculation is

$$D_{ij}^2 = D_i(c_1)^2 \otimes 1 + 1 \otimes D_j(c_2)^2 = W_{c_1} + W_{c_2} = 0.$$

The pair $(V^{\otimes 2}, D_{ij})$ is thus a complex, and on the direct sum of the cohomologies of these complexes over all possible labellings, which in our case is

$$C = \bigoplus_{1 \leqslant i,j \leqslant 2} H^*(V^{\otimes 2}, D_{ij}),$$

Khovanov and Rozansky define another differential ∂ that encodes for each crossing c whether it is an over or under-crossing. Finally, their invariant is the vector space

$$\mathcal{H}_N(L_{\mathrm{Hopf}}) := H^*(C, \partial).$$

Remarkably, this turns out to be a finite-dimensional vector space independent of the planar projection chosen to represent the link. For example: the $N = 3$ invariant of the Hopf link, expressed as a Poincaré polynomial, is

$$\sum_{i,j} \dim_{\mathbb{C}} \mathcal{H}_3(L_{\mathrm{Hopf}})^{ij} t^i q^j = 1 + q^2 + q^4 + q^4 t^{-2} + 2q^6 t^{-2} + 2q^8 t^{-2} + q^{10} t^{-2}.$$

The t grading is homological with ∂ having degree one, while the q grading is polynomial with x_i in degree two.

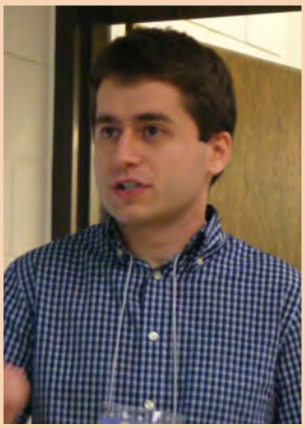
The subject of knot homology has also been studied by mathematical physicists. In 1989 Witten showed that the Jones polynomial of a knot K is related to the expectation value of a natural observable in a topological field theory called Chern–Simons theory. These ideas had a broad impact on topology and representation theory, for example in the theory of quantum groups and modular tensor categories. Homological knot invariants also naturally arise in topological field theory as spaces of states associated to $K \times \mathbb{R}_+$, and the implications of this perspective for mathematics are just beginning to become clear.

As a result of this link with physics, we were visited by Nils Carqueville and Ana Ros Camacho (both mathematical physicists) as part of the March focus period on matrix factorizations.

Focus on the Scientists: Claudiu Raicu and Michael Artin

Claudiu Raicu is an instructor at Princeton University and is currently visiting MSRI as the initial Huneke Postdoctoral Fellow. (See the article on page 10 for more about the Huneke Fellowship).

Claudiu works on problems that are a mixture of algebraic geometry, commutative algebra, representation theory and their computational aspects. He has worked on defining equations of secant varieties, syzygies of Segre–Veronese varieties, finite equivalence relations in algebraic geometry, as well as Boij–Söderberg theory on complete intersections.



Claudiu Raicu

Already, Claudiu has solved several conjectures posed by other mathematicians. For example, he proved a well-known conjecture of Garcia–Stillman–Sturmfels on the defining equations of secant varieties of Segre products for an arbitrary number of factors. His solution was based on a subtle use of the representation theory of the symmetric group. This was quite surprising to some experts, as it seemed to translate the question to a more difficult one. Remarkably, the approach succeeded.

He also established the Geramita conjecture on the ideals of minors of catalecticant matrices. In another direction, Claudiu found a counterexample to a conjecture of János Kollár stating that a finite equivalence relation is effective.

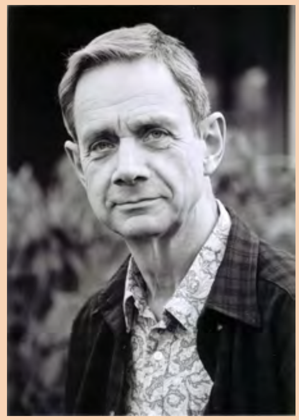
Claudiu grew up in Romania and received his undergraduate degree at the University of Bucharest. In 2007 he started graduate school at UC Berkeley, where he completed his degree with David Eisenbud in 2011.

In addition to his research, Claudiu is involved in several outreach activities. In Romania he was an instructor for the Math Olympiad. In Berkeley, he has been an instructor in the Berkeley Math circle, will give a lecture at the Marin Math Circle, and will help with this year's Bay Area Math Olympiad.

— Jerzy Weyman

Michael Artin is one of the organizers of the program on Noncommutative Algebraic Geometry and Representation Theory, currently running at MSRI.

Michael's role in the creation of modern algebraic geometry is well known and needs no introduction. What might be less known is that his first article devoted to noncommutative algebras, "On Azumaya algebras and finite dimensional representations of rings," was published in 1969, predating a number of his greatest contributions to algebraic geometry.



Michael Artin

In the article, Michael characterizes Azumaya algebras by their representations, obtaining an analogue of a similar result for C^*-algebras. His theorem is essentially a statement about the solutions of matrix equations, and in this sense it can be thought of as a first step into noncommutative algebraic geometry. Besides being a cornerstone in the theory of PI-rings, the theorem led to important questions in invariant theory which gave rise to much further research.

In subsequent years Michael started a long term collaboration with Bill Schelter. In their search for noncommutative analogues of polynomial rings, the two discovered a class of graded algebras intimately connected to the theory of elliptic curves. It turned out that such algebras could be advantageously studied by thinking of them as coordinate rings of noncommutative projective planes. In other words: by doing noncommutative algebraic geometry!

Despite considerable progress in recent years, the classification of general noncommutative surfaces remains wide open. Michael conjectures that up to birational equivalence, the list of noncommutative surfaces is actually quite small. Some of the surprising consequences of this provocative conjecture appear at first sight easy to disprove, but so far no one has succeeded in doing so. Instead, there is a large body of examples supporting the conjecture.

Michael is also the recipient of the 2002 Steele Prize for Lifetime Achievement and the 2013 Wolf Prize. He received Honorary Doctoral degrees from the universities of Antwerp and Hamburg.

— Michel Van den Bergh

Noncommutative resolutions of singularities

Graham J. Leuschke

This spring's semester program in Noncommutative Algebraic Geometry and Representation Theory (NAGRT) brings together researchers in many different aspects of noncommutative algebra, such as deformation theory and Poisson structures, enveloping algebras, noncommutative projective algebraic geometry, D-modules, Calabi–Yau conditions, symplectic reflection algebras and the many guises of Hecke algebras. I've been fortunate to participate in both the NAGRT program and the year-long program in Commutative Algebra, which have brought together these two areas in a couple of workshops so far, with more to come. The stimulation and cross-pollination between the programs have been a delight to witness and exhilarating to participate in.

The NAGRT program actually kicked off last summer, with a two-week Summer Graduate Workshop (http://tinyurl.com/NAGRT-SGW) featuring lecture series by rising stars in the area on four of the main themes of the program: noncommutative projective geometry, deformation theory, symplectic reflection algebras, and noncommutative resolutions of singularities. I would like to use this space to sketch a very informal introduction to the last of those topics; readers whose interest is piqued would be wise to pick up Michael Wemyss' notes from the workshop next. This topic has quickly generated a large body of research, starting from its introduction in string theory (!) around the turn of the millennium and picking up steam with Michel Van den Bergh's introduction of "noncommutative crepant resolutions" in 2004. Applications have been found, for example, in the context of the minimal model program of Mori and Reid; in cluster theory (the topic of an MSRI program last fall); in the study of Orlov's singularity category; and in the representation theory of maximal Cohen–Macaulay modules.

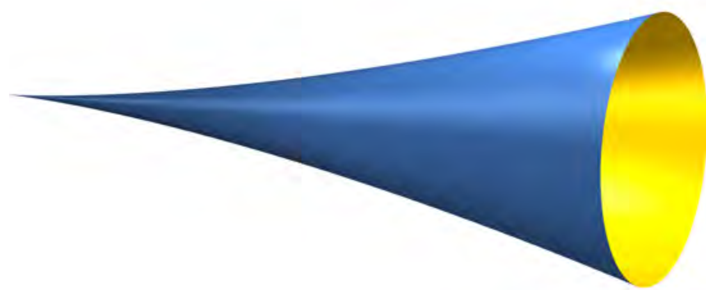
Commutative Resolutions of Singularities

First let us understand the commutative situation. A *resolution of singularities* of an affine algebraic variety (the locus of solutions of a system of polynomial equations) is a parametrization by the points of a non-singular variety — that is, a manifold — via a map which is a bijection almost everywhere. In other words, the smooth space looks essentially the same as the original one, except that any pinches, creases, self-intersections or other kinds of *singularities* are smoothed out. This operation is basic in algebraic geometry: it allows the reduction of many calculations and constructions to the case of a smooth variety.

Applications are ubiquitous. For example, on a smooth variety one has access to differential forms and to strong vanishing theorems for cohomology, both of which one can try to push back down to the singular space. Resolutions of singularities are known to exist for all varieties over $\mathbb{C}$ by the work of H. Hironaka in the 1960s, for which he was awarded the Fields Medal in 1970.

The Devil of Algebra

From an algebraic point of view, however, this situation leaves something to be desired. In general, the classical correspondence between algebraic geometry and commutative algebra gives a lovely dictionary to translate information back and forth: a finitely generated commutative algebra over the complex numbers, say $R = \mathbb{C}[x_1, \ldots, x_n]/I$, where I is an ideal generated by some polynomials $f_1, \ldots, f_r$, is realized geometrically by the common vanishing locus $\operatorname{Spec} R := V(f_1, \ldots, f_r)$ of the f's. Conversely, an affine variety $X \subset \mathbb{A}^n_{\mathbb{C}}$ corresponds to the quotient $\mathcal{O}_X = \mathbb{C}[x_1, \ldots, x_n]/I(X)$ of the polynomial ring by the ideal $I(X)$ of polynomials vanishing on X. The correspondence is very nearly one-to-one by Hilbert's Nullstellensatz; if one is willing to restrict to integral domains (or to incant the magic word "scheme" from time to time), it becomes so. In that case the points of X correspond bijectively to the maximal ideals, or equivalently the simple modules, of $\mathcal{O}_X$. In some sense (which can be made precise) affine varieties over $\mathbb{C}$ *are the same thing as* commutative integral domains finitely generated over $\mathbb{C}$. The dictionary extends to all sorts of properties on both sides; for example, $X = \operatorname{Spec} R$ is non-singular if and only if $R = \mathcal{O}_X$ is what's called a regular ring, meaning that it has finite global homological dimension (essentially this means that it is just a polynomial ring).



The A_2 singularity $x^3 + y^2 + z^2 = 0$, also called "trumpet."

Here comes the fly in the ointment, at least for a naive commutative algebraist. Resolutions of singularities of affine varieties are hardly ever affine varieties again. Instead they are *glued together* out of affine varieties, in the same way that a complex manifold is glued together out of copies of $\mathbb{C}^n$: there are charts, and transition functions, so that locally one can pretend one is working with an affine variety, even though globally it's something a bit different. However, this gluing process breaks the dictionary with commutative algebra. There is a global algebraic object associated to a resolution of singularities of an affine variety $X = \operatorname{Spec} R$, called the Rees ring (or *blowup algebra*) $R[It]$, but now the connection between the geometry and the algebra becomes more subtle. Smoothness of the geometry corresponds to regularity of certain rings of fractions of $R[It]$. It's not impossible to make the translation, but it's annoying.

Too Few Commutative Rings

A stubborn commutative algebraist might hope that all this is unnecessary, that it is possible to resolve the singularities of an affine variety (= ring) with another affine variety. To discuss this possibility, we'd better look at the actual definition. A morphism of varieties $\pi\colon Y \longrightarrow X$ is a resolution of singularities if

(i) Y and X are *birational*, meaning that π is surjective and the fields of rational functions $K(Y)$ and $K(X)$ coincide;

(ii) π is *proper*, in the sense that the inverse image of a compact set is compact; and

(iii) Y is non-singular.

Passing to algebra reverses the arrows, so given, say, an integral domain R, we hope to find a ring homomorphism $\varphi\colon R \longrightarrow S$ satisfying appropriate versions of (i)–(iii). Birationality is easy: we can require the induced map on quotient fields $Q(R) \longrightarrow Q(S)$ to be an isomorphism. It is a standard fact from algebraic geometry that a proper map between affine varieties is necessarily a finite map, so we assume $R \longrightarrow S$ is finite, that is, S is a finitely generated R-module via φ. Finally, S should be a regular ring. In other words, perhaps a resolution of singularities of R is an intermediate algebra $R \subseteq S \subseteq Q(R)$ which is a finite R-module and is a regular ring.

Unfortunately, this hope is soon dashed. The ring $\mathbb{C}[x,y,z]/(x^3 + y^2 + z^2)$, corresponding to the "trumpet" singularity (shown in the figure on the previous page), has no regular finite extensions in its quotient field. One really needs the additional flexibility of gluing together affine varieties to resolve this singularity.

Allow a Little Noncommutativity

The idea of a non-commutative desingularization, now, is a natural one: find a replacement for resolution of singularities using possibly *non-commutative* rings. We are therefore interested in defining what it should mean for a ring homomorphism $R \longrightarrow \Lambda$, where R is commutative and Λ is not necessarily so, to be called a non-commutative resolution of singularities of R.

Peeking back at properties (i)–(iii) in the definition of a resolution of singularities with the goal of translating them to a non-commutative scenario, we hit an immediate stumbling block. Insisting that a map of rings $R \longrightarrow \Lambda$ as above induces an isomorphism on quotient fields is clearly too strong: it forces Λ to be commutative!

We extricate ourselves from this awkward situation by adopting the representation-theoretic point of view. One of the guiding principles in algebraic geometry and commutative algebra in the last 50 years, since Grothendieck, has been that properties of a variety X (respectively, a ring R) are encoded in the category $\mathsf{Qch}\, X$ of quasi-coherent sheaves that X supports (respectively, the category $\mathsf{Mod}\, R$ of modules over R). We lose no information in this transition: an affine variety X or commutative finitely generated $\mathbb{C}$-algebra R can easily be reconstructed from $\mathsf{Qch}\, X$ or $\mathsf{Mod}\, R$.

On the other side, it can happen that two non-commutative rings Λ and Γ support indistinguishable categories of modules; in this case we say Λ and Γ are *Morita equivalent*. The canonical example is that a field K and the matrix ring $\mathrm{Mat}_{n\times n}(K)$ are Morita equivalent. It's common practice in non-commutative algebra not to distinguish between Morita-equivalent rings. In particular, we can ask only that (i) hold up to Morita equivalence, that is,

(i′) $R \longrightarrow \Lambda$ is *birational*, meaning that the "fraction field" $K \otimes_R \Lambda$ of Λ is a matrix ring over the fraction field K of R.

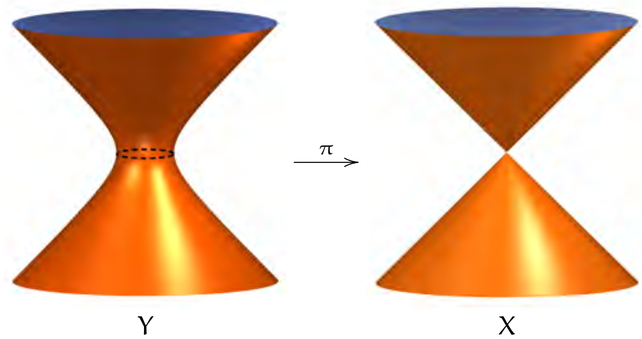
This leads to a provisional definition: A *non-commutative desingularization* of a ring R is an R-algebra $R \longrightarrow \Lambda$ (with R mapping into the center of Λ) which is finitely generated as an R-module, birational in the sense of (i′), and has finite global dimension.

Unfortunately, finite global dimension is a relatively weak property for non-commutative rings, and so there are various strengthenings of this definition. For example, Dao–Iyama–Takahashi–Vial define a *non-commutative resolution* of R to be an R-algebra of finite global dimension which is isomorphic to the ring of endomorphisms $\mathrm{End}_R(M)$ of some finitely generated faithful R-module M. Even stronger, Van den Bergh defined a *non-commutative crepant resolution* to be an endomorphism ring $\Lambda \cong \mathrm{End}_R(M)$ having finite global dimension, such that in addition M is reflexive and Λ is maximal Cohen–Macaulay as an R-module. These "NCCRs" have attracted a great deal of interest.

There are also weaker definitions. Kuznetsov and Lunts recently announced the existence, in all situations where a resolution of singularities is known to exist, of a *categorical resolution of singularities*, which is defined entirely at the level of derived categories of sheaves, without reference to rings at all.

Drawing Pictures of Noncommutativity

Leaving aside the pros and cons of the various definitions, let us consider an example where there is consensus on a good answer. Put $R = \mathbb{C}[x,y,z]/(xy - z^2)$. Then R is the coordinate ring of the quadric cone X in the figure below.



The A_1 singularity $X\colon (z^2 = xy)$ and its resolution Y.

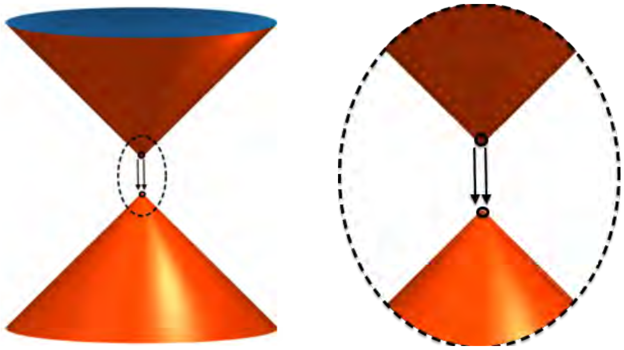
The usual resolution of singularities of the cone is one of the first anyone sees: it is obtained by *blowing up* the vertex, replacing it by a copy of the projective line $\mathbb{P}^1$. The result Y is topologically a cylinder, and the map $\pi\colon Y \longrightarrow X$ from the cylinder to the cone is just the contraction of the $\mathbb{P}^1$ to a point. Set $\Lambda = \mathrm{End}_R(R \oplus I)$, where $I = (x, z)$ is the ideal of R generated by x and z. Then Λ is a noncommutative ring, which can also be written as a subring of the 2×2 matrices over R

$$\Lambda = \begin{pmatrix} R & I \\ I^{-1} & R \end{pmatrix},$$

since $\mathrm{End}_R(I) = R$. Relatively standard computations then show that there is an equivalence of derived categories

$$\mathcal{D}^b(\mathsf{Qch}\, Y) \xrightarrow{\simeq} \mathcal{D}^b(\mathsf{Mod}\, \Lambda).$$

Following the philosophy that we should identify an algebro-geometric object with the modules or sheaves it supports, we are thus led to the conclusion that Λ *is* a resolution of singularities of the cone X.



Schematic of the non-commutative desingularization of the cone.

Here is a slightly more concrete approach to the same example, based on work of Crawley-Boevey (and earlier work by Kronheimer, Cassens–Slodowy, Reiten–Van den Bergh, and Ito–Nakamura). The cone X and its resolution Y can both be realized as moduli spaces of certain semisimple representations of preprojective algebras of type A_1. Explicitly, the points of X are in bijection with equivalence classes of diagrams

$$\begin{tikzcd} \mathbb{C} \arrow[r, bend left=40, leftarrow, "a'"] \arrow[r, shift left, "a"] \arrow[r, shift right, leftarrow, "b"] \arrow[r, bend right=40, "b'"'] & \mathbb{C} \end{tikzcd}, \qquad (\dagger)$$

where a, a', b, b' are complex numbers such that $aa' = bb'$. Two such diagrams are equivalent if they are the same up to scaling by non-zero complex numbers in each copy of $\mathbb{C}$. The resolution Y, on the other hand, is realized by representations (†) such that a and b' are not both zero. In particular, the preimage of the vertex is given by such representations which are not simple, which, given that $a \neq 0 \neq b'$, amounts to saying that $a' = 0 = b$. In other words, the "exceptional fiber" of the noncommutative resolution is parametrized by representations with dimension vector $(1, 1)$ of the *Kronecker quiver*

$$\begin{tikzcd} \mathbb{C} \arrow[r, shift left, "u"] \arrow[r, shift right, "v"'] & \mathbb{C} \end{tikzcd}$$

in which u and v are not both zero. This gives exactly a $\mathbb{P}^1$ of such representations, again up to scaling by $\mathbb{C}^*$ in each vertex. Thus the commutative resolution of singularities is manifest within the category of modules over the preprojective algebra, that is, the noncommutative ring whose elements are formal linear combinations of paths in the diagram (†) subject to the relations $aa' = b'b$ and $a'a = bb'$.

The remarks above constitute just a couple of myriad possible perspectives on this example, and of course the example can be generalized in half a dozen different ways. Ideas like these form a crossroads where the participants in this semester's program can meet on common ground and extend our understanding in new directions.

Forthcoming Workshops

June 15, 2013 to July 28, 2013: *MSRI-UP 2013*

June 17, 2013 to June 21, 2013: *Bay Area Circle for Teachers (BACT) Summer Workshop 2013*

June 24, 2013 to June 28, 2013: *Pacific Rim Mathematical Association (PRIMA) Congress 2013*

For a complete list of all upcoming workshops and programs, please see www.msri.org/scientific. A list of programs for 2014–15 is included in the call for membership applications on page 9.

Endowed Positions at MSRI

MSRI is grateful for the generous support that has created four endowments that support faculty and postdoc members of its programs each semester.

The **Viterbi Postdoctoral Fellows** are funded by an endowment from Trustee Andrew Viterbi, Erna Viterbi, and the Viterbi Family Fund of the Jewish Community Foundation. Both the **Eisenbud** and **Simons Professorships** are funded by an endowment from the Simons Foundation. The **Huneke Postdoctoral Fellows** are supported by a new endowment, announced on page 10.

Spring 2013 Special Postdocs

Commutative Algebra
Huneke Fellowship: Claudiu Raicu, Princeton University

Noncommutative Algebraic Geometry and Representation Theory
Viterbi Fellowship: Maria Chlouveraki, University of Edinburgh

2013–14 Eisenbud and Simons Professorships

Mathematical General Relativity (Fall 2013)
Piotr Chruściel, University of Vienna
Gregory Galloway, University of Miami
James Isenberg, University of Oregon
Hans Ringström, KTH Royal Institute of Technology
Vincent Moncrief, Yale University
Robert Wald, University of Chicago

Optimal Transport: Geometry and Dynamics (Fall 2013)
Cédric Villani, Institut Henri Poincaré & Université de Lyon
Wilfrid Gangbo, Georgia Institute of Technology
Robert McCann, University of Toronto
Ludovic Rifford, Université de Nice – Sophia Antipolis

Algebraic Topology (Spring 2014)
Charles Rezk, University of Illinois at Urbana-Champaign
Doug Ravenel, University of Rochester
Bill Dwyer, University of Notre Dame
Michael Mandell, Indiana University at Bloomington

Model Theory, Arithmetic Geometry, and Number Theory (Spring 2014)
François Loeser, Université Pierre et Marie Curie
Ehud Hrushovski, Hebrew University
Minhyong Kim, University of Oxford
Sergei Starchenko, University of Notre Dame

Clay Senior Scholars

The Clay Mathematics Institute Senior Scholar awards support established mathematicians who will play a leading role in a topical program at an institute or university away from their home institution. MSRI will host five Clay Senior Scholars in 2013–14:

Igor Rodnianski, Massachusetts Institute of Technology
Vincent Moncrief, Yale University
Cédric Villani, Institut Henri Poincaré and Univ. de Lyon
Lars Hesselholt, Nagoya University
Ehud Hrushovski, Hebrew University

Mathematics Weighs in on Climate Disruption

MPE2013 Simons Public Lecture at the Palace of Fine Arts

Anne Brooks Pfister

Dr. Emily Shuckburgh, a mathematician and climate scientist with the British Antarctic Survey, delivered the second Simons Public Lecture on "Climate Disruption: What Math and Science Have to Say" on March 4, 2013, at the Palace of Fine Arts in San Francisco, to an audience in the nearly packed 960-seat venue.

The talk was one in a series of nine public lectures, funded by the Simons Foundation, scheduled at nine international locations — ranging from Melbourne to Berlin — which are part of the Mathematics of Planet Earth (MPE2013) activities being held this year worldwide. It was co-hosted by MSRI and the American Institute of Mathematics.

During the evening, Dr. Shuckburgh explained how mathematics is used in essential ways not only to help us model the Earth's climate, but also to make sense of the enormous amount of scientific data being collected in the effort to understand what is happening to our climate and what the consequences of those changes might be.

As the leader of the Open Oceans research group at the British Antarctic Survey, Dr. Shuckburgh focuses her work on the role of the polar oceans in the global climate system. Her research uses theoretical approaches and observational studies in the Arctic and Antarctica, as well as numerical modeling.

Dr. Shuckburgh reviewed the mathematical models and climate data that that she employs in her work. From the climate data, she highlighted the relatively recent increase seen in carbon dioxide concentrations and other greenhouse gases, as well as the striking reduction in Arctic sea ice that has melted over the last three decades: an area equivalent to the size of the Eastern region of the United States (a little more than 3 million square km). Since 1992, the total polar ice sheet melt has contributed to a sea level rise of 0.6 mm per year.

She also explained how the changing climate system has severe implications for the future — on coastal properties and infrastructure, food production, risk of extreme droughts and floods, and the disruption of ecosystems throughout the world — and she presented various global scenarios based on projected increases in the Earth's surface temperature.

Kim Harrington Photography

At the San Francisco 'leg' of the Simons Public Lecture series for MPE2013: (from left to right) Brian Conrey, Estelle Basor, Hélène Barcelo, Emily Shuckburgh, Robert Bryant, and Nathaniel Simons.

The predictions envision dire prospects for the planet if mitigation strategies are not implemented soon to peak global emissions and keep below a two degree rise in the global temperature. Nonetheless, she emphasized that collective action could work to solve the challenging problems that we now face. According to Dr. Shuckburgh, the science is clear and the way ahead to a habitable, healthy Earth can still be in our future, but it's in our hands.

MSRI will host another MPE2013-related event this summer for graduate students (but please be advised that it is already fully enrolled!). "Introduction to the Mathematics of Seismic Imaging," organized by Professor Gunther Uhlmann, will be held at MSRI from July 19 to August 9 (see http://tinyurl.com/MPE2013SGS).

Call for Membership Applications

MSRI invites membership applications for the 2014–2015 academic year in these positions:

Research Professors	by October 1, 2013
Research Members	by December 1, 2013
Postdoctoral Fellows	by December 1, 2013

In the academic year 2014–2015, the research programs are:

New Geometric Methods in Number Theory and Automorphic Forms, Aug 11 to Dec 12, 2014
Organized by Pierre Colmez, Wee Teck Gan (chair), Michael Harris, Elena Mantovan, Ariane Mézard, and Akshay Venkatesh

Geometric Representation Theory, Aug 18 to Dec 19, 2014
Organized by David Ben-Zvi (chair), Thomas Haines, Florian Herzig, Kevin McGerty, David Nadler, Ngô Bảo Châu, Catharina Stroppel, and Eva Viehmann

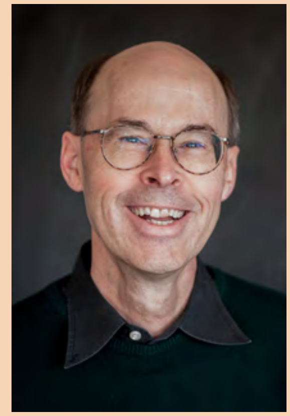
Dynamics on Moduli Spaces of Geometric Structures, Jan 12 to May 22, 2015
Organized by Richard D. Canary, William Goldman, François Labourie, Howard Masur (chair), and Anna Wienhard

Geometric and Arithmetic Aspects of Homogeneous Dynamics, Jan 19 to May 29, 2015
Organized by Dmitry Kleinbock (chair), Elon Lindenstrauss, Hee Oh, Jean-François Quint, and Alireza Salehi Golsefidy

MSRI uses **MathJobs** to process applications for its positions. Interested candidates must apply online at www.mathjobs.org after August 1, 2013. For more information about any of the programs, please see www.msri.org/scientific/programs.

Huneke Postdoctoral Fellowship Launches with New Endowment

MSRI is proud to announce the beginning of the Huneke Postdoctoral Fellowship, funded by a generous endowment from Professor Craig Huneke. Currently a trustee of MSRI, Huneke is internationally recognized for his work in commutative algebra and algebraic geometry. Among his fundamental contributions is his discovery, with Melvin Hochster, of the notion of tight closure. In a long and fruitful collaboration, Hochster and Huneke forged this very original idea into a powerful tool that has led to great advances in our understanding of algebra in characteristic p, inspiring several hundred publications over the last 20 years.


David Eisenbud

Craig Huneke

Huneke is a great teacher, expositor, and organizer. He has mentored more than twenty Ph.D. students and a long list of postdocs. He is co-author, with his former student Irena Swanson, of a book on integral closure that has become the standard reference in the subject. He was co-organizer of the 2002–03 program on Commutative Algebra at MSRI. He currently holds the title of Marvin Rosenblum Professor of Mathematics at the University of Virginia, where he is the Chair of the Mathematics Department. Before moving to Virginia in 2012, he was the Henry J. Bischoff Professor of Mathematics at the University of Kansas. Claudiu Raicu (profiled on page 5) is the the first recipient of the Huneke Fellowship.

Puzzles Column

Elwyn Berlekamp and Joe P. Buhler

1. Find all of the ways that 2013 can be written as a sum of consecutive integers (not necessarily positive).

Comment: This is due, in essence, to Nelson Blachman and was included in a video of Nelson, by George Csicsery, that was linked to in the Numberplay column in the New York Times on March 4, 2013. The column itself featured a probability puzzle due to Nelson.

2. Consider a variant of the game "Battleships" played on the real line, in which your goal is to shoot and hit a battleship. There is a single battleship of length 4 that moves at a fixed (but unknown) integral velocity in a fixed, but unknown, direction (left or right), starting at time $t = 0$ at an unknown location. At each nonnegative integral time t you are allowed to shoot at some integer on the line. You are told whether or not you hit the battleship; if you hit you win, otherwise the battleship continues moving and you get another shot in one unit of time.

Find a strategy that guarantees that you will win in finite time (!).

Comment: Rumor has it that this was used as an interview question at a hedge fund.

3. (a) There are n equally-spaced objects on a line. How many permutations are there of them in which every object ends up within one unit of its original position?

(b) You again have n objects, but now they are arranged in a circle. As before, the question is how many permutations are there in which no object moves further than one unit (in either direction)?

Comment: Part (a) was overheard at an MSRI meeting of the Academic Sponsors and Trustees, leading to part (b). Curiously, rumor has it that Will Shortz used (b) for the specific case $n = 8$ in a recent puzzle tournament.

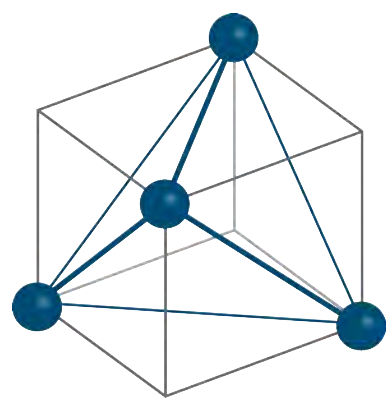
4. At some particular instant of time, N planets are located in space arbitrarily. (For our purposes, a planet is a solid opaque ball of radius 1, and planets do not overlap.) At one instant, any point on any planet is colored red if none of the other $N-1$ planets are visible from that point. Prove that the sum of the areas of all the N red regions equals exactly the surface area of one planet.

Comment: This is apparently well-known to some, though no one can cite a source; we heard the problem from Gregory Galperin. You are welcome to start with the two-dimensional analogue.

5. (a) Find six orderings of the numbers $1, 2, 3, 4$ such that any ordered 3-tuple of distinct numbers from that set can be obtained by deleting one element from one of your chosen orderings.

(b) Show that it is not possible to find six orderings of $\{1, 2, 3, 4, 5\}$ such that all ordered 3-tuples of the set can be obtained by deleting two elements of one of your chosen 5-tuples.

Comment: This problem was motivated by the January 2013 puzzle on the IBM research puzzle web site "Ponder This."



6. Let V_n denote the set of 2^n points in R^n whose coordinates are -1 or 1, i.e., the vertices of the cube in R^n that is centered at the origin and has side 2. Let $C(n)$ be the largest cardinality of a subset of V_n with the property that all pairwise distances between elements of the set are equal.

Find bounds on, or exact values of, $C(n)$ for as many values of n as you can.

Comment: This question was asked by Dan Asimov.

A Word from the Director

(continued from page 1)

at PlayGround is one of their most popular), and the audience's reception of the plays was very enthusiastic. We are waiting to learn which of the plays might make it into the "Best of PlayGround" festival at the end of the year. I think we have a couple of strong contenders.

In March, we joined forces with the American Institute of Mathematics in Palo Alto, CA, to be the local co-sponsors of the second in the series of nine lectures in the Simons Public Lecture Series. This series, sponsored by the Simons Foundation, is part of the events of MPE2013, and "our" lecture on March 4 at the Palace of Fine Arts in San Francisco was a big success.

Dr. Emily Shuckburgh from the British Antarctic Survey spoke about "Climate Disruption: What Math and Science Have to Say." It was an inspiring lecture because it addressed not only how we are studying the problem of climate disruption, but also laid out ideas for how we could begin to prepare for it. You can read a full report on the lecture on page 9. Filling a 960-seat auditorium for a lecture of this kind was a challenge for both MSRI and AIM, but by collaborating and pooling our resources and contacts, we were able to get the word out and we were gratified with a nearly full auditorium for Emily's lecture. It was recorded and will be made available (along with the other lectures in the series) by the Simons Foundation. I hope that you'll take an opportunity to watch it if you weren't able to attend.

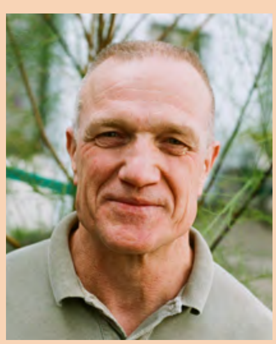
On March 18, we had the public premiere at the Roxie Theater in San Francisco of the biographical film *Taking the Long View: The Life of Shiing-shen Chern*, which was part of a small film festival featuring the mathematical films of George Csicsery. Of course, Professor Chern was MSRI's founding director, and we have been very pleased with the enthusiastic reception the film has garnered in private showings so far. We have just recently learned that *Taking the Long View* has been approved for syndication for showing on public television in the US, so we are looking forward to having more people learn about the life of this remarkable mathematician and cultural figure.

Finally, this will be the last Director's Word from me, since my term as MSRI's Director ends on July 31. I think it is appropriate for me to thank the many, many people (which includes you) who have given so generously of their valuable but limited time and their equally precious and seemingly unlimited talent to help me and MSRI in our mission to serve the communities for which it was founded. It has been a tremendous honor to be entrusted with leading such a dedicated group of people, and I find it hard to express in words just how grateful I am to all of you for this opportunity. Ultimately, it is you who have made it possible for an institution such as MSRI to prosper and give back to the community a valuable resource for furthering mathematics research, training young talent, and reaching out to delight and inform the public.

With David Eisenbud as the incoming Director and a remarkable staff, MSRI is in excellent hands, and, with your help, we can all look forward to a great future for MSRI and the society that it proudly serves.

. . . And a Word *for* the Director

The MSRI Board of Trustees honored Robert Bryant, Director of MSRI, at its March 2, 2013, annual meeting. Following the meeting, Board members celebrated the accomplishments of Dr. Bryant's term as Director (Aug 1, 2007 to Jul 31, 2013). A poem (shown at right) by trustee Ed Baker, written in tribute to Dr. Bryant's leadership, was read at the gathering and received warm and enthusiastic applause.



George Bergman

For Robert Bryant

Rock solid,
chiseled
from stone.
A granitic
island amid
the drifting tides.
Steadfast
within the unexpected
flow of turbulence.

You came
with a vision,
for mathematics.
An ascent up
Parnassus,
perched majestically
On the precipice,
the epicenter
for endless
Mathematical tremors
emanating
around the globe.

The muses were
in disarray,
instead of
Inspiration,
discovery
and new objects,
You were compelled
to enter
a management labyrinth.
To confront a tangled
web of challenges
and details.
A man of truth,
your will
was resolute.

Rock solid
steadfast,
chiseled from granite.
Your implacable
focus and determination
were just enough
To escape the
labyrinth's
mazy gloom.

You leave MSRI
in your image,
on a foundation
Of rock,
secure,
fully prepared
For mathematics'
next seismic
shifts.

— Ed Baker

Mathematical Sciences Research Institute

17 Gauss Way, Berkeley CA 94720-5070
510.642.0143 • FAX 510.642.8609 • www.msri.org

ADDRESS SERVICE REQUESTED

MSRI Staff Roster

Phone area code 510. Add @*msri.org* to email addresses.

Scientific Staff

Robert Bryant, Director, 642-8226, *director*
Hélène Barcelo, Deputy Director, 643-6040, *hbarcelo*
Alissa S. Crans, Director of Educational and Outreach Activities, 642-4745, *acrans*

Administrative Staff

Jackie Blue, Housing Advisor, 643-6468, *jblue*
Arthur Bossé, Operations Manager, 643-8321, *abosse*
Alex Gonzalez, Director of Development, 643-6056, *agonzalez*
Lisa Jacobs, Executive and Development Assistant, 642-8226, *lisaj*
Christine Marshall, Program Manager, 642-0555, *chris*
Rizalyn Mayodong, Staff Accountant, 642-9798, *rizalyn*
Megan Nguyen, Program Analyst, 643-6855, *megan*
Anne Brooks Pfister, Press Relations Officer & Board Liaison, 642-0448, *annepf*
Linda Riewe, Librarian, 643-1716, *linda*
Mick Santos, Workshop Assistant, 643-6467, *mick*
Sarah Springfield, Program Assistant, 643-6019, *officeassist3*
Stefanie Yurus, Controller, 642-9238, *syurus*