

Exercises for Lecture 1.
E. Kerman

1. Let $x: S^1 \rightarrow M$ be a nondegenerate 1-periodic orbit of a Hamiltonian vector field X_H on (M, ω) . Show that x is isolated away from all other 1-periodic orbits.
2. Find a function $f: \mathbb{R}^{2n} \rightarrow \mathbb{R}$ such that the origin is a nondegenerate critical point of f but not (the image of) a nondegenerate periodic orbit.
3. Let Λ be the space of smooth maps from S^1 to M which are contractible, i.e., homotopic to constant maps. For $x \in \Lambda$, the tangent space $T_x\Lambda$ is the space of smooth sections ξ of $(TM)|_{x(S^1)}$.

(a) Consider the action functional $\mathcal{A}_H: \Lambda \rightarrow \mathbb{R}$ defined by

$$\mathcal{A}_H(x) = \int_{S^1} H(t, x(t)) dt - \int_{D^2} \bar{x}^* \omega,$$

where (again) \bar{x} is a smooth map from the unit disc $D^2 \subset \mathbb{C}$ to M such that $\bar{x}(e^{2\pi i t}) = x(t)$. Compute $(d\mathcal{A}_H)_x(\xi)$.

(b) Let J_t is an S^1 -family of almost complex structures. J_t determines a metric on Λ as follows:

$$\langle \xi, \eta \rangle = \int_0^1 \omega(\xi(t), J_t \eta(t)), dt.$$

Show that the negative gradient trajectories of \mathcal{A}_H with respect to the metric determined by J_t are Floer trajectories.