

EXERCISES FOR MSRI GRAD WORKSHOP

LENNY NG

1. LECTURE 1

- (1) For the left-hand trefoil, we have $\overline{tb} = -6$ and $\overline{sl} = -5$. Find Legendrian and transverse representatives achieving these maxima.
- (2) Prove that $\overline{tb}(K) \leq \overline{sl}(K)$ for all oriented topological knots K . (Beware: K is not necessarily the same as its orientation reverse!)
- (3) The pretzel knot $P(-3, 5, 7)$ is famously topologically slice. Use slice-Bennequin to prove that it is not smoothly slice.

2. LECTURE 2

- (1) Prove that if a knot is Legendrian simple, then it is transversely simple.
- (2) Prove that the resolution of a front is the xy projection of a Legendrian knot Legendrian isotopic to the original.
- (3) Prove that ∂ in Legendrian contact homology lowers degree by 1: for an immersed disk with boundary on the xy projection of L , positive corner at a_i , and negative corners at a_{j_1}, \dots, a_{j_k} ,

$$|a_i| \equiv |a_{j_1}| + \dots + |a_{j_k}| + 1 \pmod{2r(L)}.$$

3. LECTURE 3

- (1) Check: if (V, α) is contact then $d(e^t\alpha)$ is symplectic on $\mathbb{R} \times V$; nontrivial critical points for the action functional $\gamma \mapsto \int_\gamma \alpha$ on the free loop space of V are Reeb orbits.
- (2) Prove that the standard Legendrian right-hand trefoil (with $tb = 1$) is not destabilizable, using contact homology rather than bounds on tb .
- (3) Prove that the Chekanov 5_2 knots are Legendrian isotopic after one stabilization of each (disregard orientation if you like). Hint: choose the placement of the zigzags carefully.