Inverse Problems

Inverse Problems are problems where causes for a desired or an observed effect are to be determined. They lie at the heart of scientific inquiry and technological development. Applications include a number of medical as well as other imaging techniques, location of oil and mineral deposits in the earth's substructure, creation of astrophysical images from telescope data, finding cracks and interfaces within materials, shape optimization, model identification in growth processes and, more recently, modelling in the life sciences.

The workshop will consist of four 4 minicourses of 2 hours each that will give an introduction to several of the topics discussed in the Introductory Workshop the following week as well as topics that will be discussed during the Fall semester. A brief description of each minicourse follows.

- **An Introduction to Microlocal Analysis**

  Lecturer: Tanya Christiansen (U. of Missouri, Columbia)

  Microlocal analysis is useful in understanding solutions of differential equations. Pseudodifferential operators arise, for example, in inverting elliptic differential equations. We introduce pseudodifferential operators and their mapping properties. We will see that the notion of the "wave front set" of a function is very helpful in describing its singularities.

  References

  [1] J. Brüning and V. Guillemin (editors), Mathematics past and present: Fourier integral operators.

  [2] L. Hörmander, The Analysis of linear partial differential operators III.

  [3] M. A. Shubin, Pseudodifferential operators and spectral theory.

  [4] F. Treves, Introduction to pseudodifferential operators and Fourier integral operators.

- **An Introduction To Seismic Imaging**

  Lecturer: Alison Malcolm (MIT)

  This course will give a broad overview of seismic imaging techniques, highlighting their underlying relationships to imaging in other fields (e.g. radar and ultrasound). We will begin with the Generalized Radon Transform, progress to one-way methods using a microlocal splitting of the wave equation into up- and down-going waves, and finish with a discussion of so-called reverse-time migration in which the full wave equation is run backwards in time to form an image. We will discuss the approximations underlying each method and their relative importance and will briefly discuss extensions beyond single-scattering.

  References

  [1] N. Bleistein, J.K. Cohen and J.W Stockwell, Mathematics of multidimensional seismic imaging, migration and inversion.

  [2] A. Grigis and J. Sjöstrand, Microlocal analysis for differential operators: an introduction.

- **An Introduction to Asymptotic Expansions for Small Inhomogeneities in EIT and Related Problems**

  Lecturer: Sharil Moskow (Drexel U.)

  We discuss the basic tools and derivation of series expansions for potential data in the presence of small volume inhomogeneities which are different from a smooth background conductivity.

We explain what properties can be recovered from the series terms and give a few ideas about how these expansions can be used to do inversion.

References

[1] H. Ammari and H. Kang, Reconstruction of Small Inhomogeneities from Boundary Measurements Lecture Notes in Mathematics **1846**, Springer Verlag, Berlin 2004. H. Ammari and H. Kang, reconstruction of small inhomogeneities from boundary measurements, Springer Verlag.

[3] D.J. Cedio-Fengya, S. Moskow, and M.S. Vogelius, Identification of conductivity imperfections of small diameter by boundary measurements. Continuous dependence and computational reconstruction. *Inverse Problems* **14** (1998), pp. 553–595.

[4] G. Folland, Introduction to Partial Differential Equations, Princeton Univ. Press.

[5] A. Friedman and M. Vogelius Identification of small inhomogeneities of extreme conductivity by boundary measurements: a theorem on continuous dependence, *Arch. Rational Mech. Anal.* **105** (1989), pp. 299–326.

[6] R. Kress, Linear integral equations, Springer Verlag.

- **Coherent Imaging in Random Media**

  Lecturer: Chrysoula Tsogka (U. of Crete)

We consider the problem of arrayimaging in cluttered media, in regimes with significant multiple scatteringof the waves by the inhomogeneities.In such scatteringregimes, the recorded traces at the array have long and noisy codasand classic imaging methods give unstable results.We will discuss coherent statistically stable imaging methodologies for imaging in such regimes.

References

[1] N. Bleistein, J.K. Cohen and J.W Stockwell, Mathematics of multidimensional seismic imaging, migration and inversion, Springer, 2001.

[2] Baggeroer, AB and Kuperman, WA and Mikhalevsky, PN, "An overview of matched field methods in ocean acoustics", IEEE Journal of Oceanic Engineering,vol. 18, no. 4, pp. 401-424, 1993. Random Layering Effects in Imaging, SIAM Multiscale Model. Simul., 8:3, pp. 751-781 (2010)

[3] L. Borcea, F. Gonzalez del Cueto, G. Papanicolaou and C. Tsogka, Filtering Deterministic Layer Eects in Imaging, SIAM Multiscale Model. Simul., 7:3, pp. 1267-1301 (2008)

[4] L. Borcea, G. Papanicolaou and C. Tsogka, Adaptive interferometric imaging in clutter and optimal illumination, Inverse Problems, Vol. 22, pp. 1405-1436, 2006.

[5] L. Borcea, G. Papanicolaou and C. Tsogka, Interferometric array imaging in clutter, Inverse Problems, Vol 21, pp. 1419-1460, 2005.

[6] J.P. Fouque, J. Garnier, G. Papanicolaou, and K. Solna, Wave propagation and time reversal in randomly layered media, Springer Verlag, 2007.