presenter: Peter Winkler, Dartmouth College

title: Rough isometry between random sets of integers

problem:

Let X and Y be metric spaces; a map $\phi : X \to Y$ is said to be a *rough isometry* if there are constants $M, D, R > 0$ such that

1. for every $u, v \in X$, $\frac{1}{M} d_X(u, v) - D \leq d_Y(\phi(u), \phi(v)) \leq M d_X(u, v) + D$ and

2. for every $y \in Y$, there is an $x \in X$ such that $d_Y(\phi(x), y) \leq R$.

Let $\mathbb{Z}_p$ be the result of Bernoulli percolation on the integers $\mathbb{Z}$, that is, each k is put in $\mathbb{Z}_p$ independently with probability p. The metric is inherited from $\mathbb{Z}$, so that $d(j, k) = |j - k|$.

Question: are there $p, q \in (0, 1)$ such that with positive probability, $\mathbb{Z}_p$ and $\mathbb{Z}_q$ are roughly isometric? Equivalently, is it the case that for *every* $p, q \in (0, 1)$, $\mathbb{Z}_p$ and $\mathbb{Z}_q$ are roughly isometric with probability 1? (Also equivently, the same questions about Poisson processes on $\mathbb{R}$.)

background:

The concept of rough isometry was introduced in group theory by Gromov in the late '70s, and studied in a wider context by Kanai in '85. Miklós Abért asked whether the infinite clusters of two percolations on the Cayley graph of a finitely-generated group are roughly isometric; Balász Szegedy then asked more specifically about site percolation on (all of) $\mathbb{Z}^2$, then $\mathbb{Z}$. For more information and partial results, see Ron Peled's article in *Ann. Appl. Prob* **10** #2 (2010), pp. 462–494, http://arxiv.org/abs/0709.2383.