2013 Summer Graduate Workshop, Cortona, Italy:
Mathematical General Relativity

Working Sessions

The purpose of the working sessions is to have some freely structured time to work on problems with other students, ask questions on lectures and/or background material, and/or read and discuss articles in groups. The "official" time slot runs until 5:00 PM, but of course you are welcome to work longer, or, say, walk up to town with your group and continue over a coffee or gelato! We may occasionally have an additional presentation during this hour, as needed.

We were planning to leave the time unstructured, but if we feel the need to impose some structure, we can form official groups and have problem solution presentations. If you have comments or suggestions, please let us know!

Problem Sets

Several problem sets have been emailed and/or posted to
http://www.msri.org/summer_schools/718

The current list (we will continue to add/update) is as follows:

- Basic Geometry Problems
- Scalar Curvature Problems
- Schwarzschild Problems
- PDE/Laplace problems
- Problems from Fernando Schwartz
- Problems from Lan-Hsuan Huang

It is important for you to familiarize yourself with the Schwarzschild geometry, to struggle with some of the scalar curvature problems, to understand the statement of the variation of area formulas (see Professor Huang's set), and to understand what the first eigenfunctions of the Laplacian on the sphere are (see the PDE/Laplace problems). PLEASE ask the lecturers and TAs questions on problems, and/or show us if you solve a problem that you are particularly proud of!!

Articles

We will suggest several articles suitable for reading, and we encourage you to read them with peers and discuss them together. Everyone should pick one article, find some folks to work with, and ask when you have questions! If you are having trouble finding a reading partner, please let us know!

1. *On isoperimetric surfaces in general relativity*, Justin Corvino, Aydin Gerek, Michael Greenberg, Brian Krummel. This article appeared in the *Pacific Journal of Mathematics* in 2007. I can print out a preprint version for you to read. The first half of the paper discusses isoperimetric surfaces in spaces related to relativity. It's a good article for those who are just getting started doing geometry calculations. If

you get through the first half and are interested in a space-time analogue, I can get you a copy of *On isoperimetric surfaces in general relativity II*, by Farhan Abedin, Justin Corvino, Shelvean Kapita and Haotian Wu, *Journal of Geometry and Physics,* 2009.

If you really get excited, we recommend further related reading:
A volume comparison theorem for asymptotically hyperbolic manifolds, by Simon Brendle and Otis Chodosh, arXiv:1305.6628 [math.DG]

Constant mean curvature surfaces in warped product manifolds, by Simon Brendle, arxiv:1105.4273 [math.DG].

2. *The graphs cases of the Riemannian Positive Mass Theorem and Penrose Inequalities in all dimensions*, by. M.-K. George Lam, arXiv:1010.4256 [math.DG].

3. *Constant mean curvature solutions of the Einstein constraint equations on closed manifolds*, by James Isenberg, *Classical and Quantum Gravity*, 1995. This is a great place to see the set up of the conformal method to solve the Einstein constraint equations. If you want to see applications of elliptic PDE and the super- and sub-solution method, this paper is a really good introduction to motivate these topics in a gravitational context. If you finish this and want a bit more, I can send a copy of *A set of nonconstant mean curvature solutions of the Einstein constraint equations on closed manifolds*, by James Isenberg and Vincent Moncrief, *Classical and Quantum Gravity*, 1996.

4. *Ricci-flow conjugated initial data sets for Einstein equations,* by Mauro Carfora. This appeared in *Advances in Theoretical and Mathematical Physics* in 2011, and is available at the arXiv:1006.1500 [gr-qc]. It's a long article with beautiful figures, and you have the opportunity to discuss it with the author!

5. *Hyperbolicity of the 3+1 system of Einstein equations,* by Yvonne Choquet-Bruhat and Tommaso Ruggeri, *Communications in Mathematical Physics*, 1983. If you're interested in the connection between the constraints and the evolution problem, you may enjoy reading this paper, even though Professor Ruggeri left for Sicily!

6. If you have a strong background in elliptic PDE and are interested in geometric gluing constructions for scalar curvature, you could read *Deformation of scalar curvature and volume*, by Justin Corvino, Michael Eichmair, and Pengzi Miao, to appear in *Mathematische Annalen*. You can fetch it from arXiv:1211.6168 [math.DG].

7. If you want to read a survey article on the Einstein constraint equations, have a look at *Scalar curvature and the Einstein constraint equations*, by Justin Corvino and Daniel Pollack, arXiv:1102.5050 [math.DG].

8. *Unique isoperimetric foliations of asymptotically flat manifolds in all dimensions* arXiv:1204.6065 [math.DG], and *On large volume preserving stable constant mean curvature surfaces in asymptotically flat manifolds* arXiv:1102.3001 [math.DG], by Michael Eichmair and Jan Metzger. Isoperimetric surface techniques and constant mean curvature foliations tie into the notions of mass and center of mass of isolated systems, part of the theme of Michael Eichmair's topics lectures.