The pointwise convergence of Fourier Series near $L^1$.
Historical evolution, main questions, recent developments, implications.

Abstract: In our talk we will discuss the old and celebrated question regarding the pointwise behavior of Fourier Series near $L^1$. This presentation will include

- the resolution of Konyagin’s conjecture (ICM, Madrid 2006) on the pointwise convergence of Fourier Series along lacunary subsequences;
- the $L^1$-strong convergence of Fourier Series along lacunary subsequences.
- recent progress on the $L^1$-strong convergence of (the full) Fourier Series.

We end with several considerations on the relevance/impact of the above items on the subject of the pointwise convergence of Fourier Series.