Apart from this one complaint, I was extremely pleased with this book. It is a well-written and carefully planned introduction to a rapidly growing subject, and it takes the reader from the basics up to at least part of the current research frontier.

Algebraic Geometry. An Introduction to Birational Geometry of Algebraic Varieties. By Shigeru Iitaka. Graduate Texts in Mathematics, Vol. 76, Springer-Verlag, New York, 1982. $x + 357$ pp.

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It may be that it is easier to study and practice algebraic geometry now than it has been at any previous time this century. One reason is the stabilization of the foundations and the return of interest to the classical concrete problems. A less fundamental reason, important to the prospective student, is the current luxuriant bloom of new textbooks, at all levels, including the book under review.

Before coming to that book I would like to sketch some history and to mention a few of my favorite texts, as a basis for comparison. It will be a rather idiosyncratic tour. I hope it will be useful to the prospective student. To make this review reasonably short, I will in any case omit mention of related texts on complex analysis, number theory, algebraic groups, and the many local questions like singularity theory and commutative algebra, as well as most of the really elementary books suitable for an undergraduate audience.

A large proportion of the results that we now know as the core of algebraic geometry, those on projective curves and surfaces and the many varieties (Jacobians, Grassmannians,...) associated to them in one way or another, was already known by the 1920's. But there were many gaps in the theory, including a lack of rigorous justification of the powerful "principles" that were used both implicitly and explicitly. Some of the famous men of the time were able to avoid employing the principles in objectionable ways; others used them to perpetrate serious errors.

The inadequacy of the foundations was clearly perceived by Zariski, van der Waerden, and others, who began to extend the algebra pioneered partly by Hilbert and Kronecker to the level necessary to treat the highly developed geometry. The book of van der Waerden [1939], still a useful elementary text, and that of Hodge and Pedoe [1947], with its still-standard account of Grassmann varieties and Schubert cycles, give an idea, on the textbook level, of the direction taken. The central objects of study in this period were projective varieties, always implicitly and often explicitly given through a particular projective embedding, over a particular field, initially the complex numbers.

Weil's classic *Foundations* [1946] represented a transition to a more intrinsic point of view, with varieties represented as unions of affine varieties, without explicit or implicit projective embeddings, and points having coordinates in variable subfields of a fixed "universal domain." The rationale for this transition was the difficulty of embedding certain very natural varieties of higher dimensions, such as the Jacobians of curves. The principal textbook version embodying Weil's viewpoint is surely that of Lang, whose series of books, beginning with [1958], treats many aspects of the theory.

It was not long before the foundations began to shift again, as Serre introduced sheaves, which already played an important role elsewhere in geometry, as the formalism through which the "gluing" of affine varieties was to be done. The fundamental work of Serre [1955] remains today an outstanding introduction to the use of sheaves, accessible with surprisingly little preparation.

Serre's techniques proved well suited to the major foundational overhaul undertaken by Grothendieck, embodied in the shift to *schemes* as the fundamental objects. Very crudely, these are patched together from local pieces with sheaves, just as in Serre, but the local pieces are allowed to be much more general (spectra of arbitrary commutative rings, where, in Grothendieck's terms, Serre only allows domains finitely generated over an algebraically closed field).

In the period of development that followed, the old theory was very greatly extended; but in the years between, say, 1958 and 1970 there were few textbooks. The enormous work of Dieudonné and Grothendieck [1960–1967] was supposed to become a complete textbook of the theory, though it rapidly became too encyclopedic for any but the most intrepid readers. Moreover, the basic theory was felt to be unsettled. Mumford's useful "Red" notes [1960] was perhaps the leading introductory text of the period, though it actually represents only an aborted attempt at a textbook and Mumford once said to me modestly that its trouble is that it contains "no theorems."

W. H. Auden has said that "a poem is never finished, only abandoned," and so it seems also with foundations in Mathematics. The building period ended with the abandonment by Grothendieck of the project. (Perhaps the complete incorporation of Artin's algebraic spaces, more general then schemes, would have been one of the next steps—see Knutson [1971].) Today, Grothendieck is, to many young algebraic geometers,

> ... No more a person
> now but a whole climate of opinion
> under whom we conduct our different lives:
> Like weather, he can only hinder or help...

(from Auden's poem "In Memory of Sigmund Freud." reprinted in *Auden* [1976]).

After the shift away from Grothendieck's foundational work, interest in algebraic geometry was again focussed on the classical questions. (To be fairer, a large part of Grothendieck's work was aimed at some concrete classical questions: Deligne's brilliant work on the "Weil conjectures," the background to which was recently exposed in textbook form by Milne [1980], represents the success of the ideas in one of these directions.) As the techniques have come to seem less strange, quite a number of people have set to work writing interesting texts.

The first major book to appear was that of Shafarevich [1972]. It is a leisurely, cultured, altogether charming book which aims to do algebraic geometry without—or at least with a minimum of—algebra. Part I (about 200 pages) makes an excellent introduction to quasiprojective varieties. The part on schemes is perhaps too brief to be useful, but the last part, on real and complex varieties, with some analytic theory, is again quite valuable. It is a good book to read first, especially for someone with little background in algebra and other kinds of geometry. Unfortunately, it doesn't include enough, in any direction, I think, to give the reader access to the literature. I think that this is a fault in a serious book of over 400 pages, though at the time the book was written, it seemed very brief by comparison with the Dieudonné-Grothendieck volumes [1960–1967].

Soon after, the far shorter text by Mumford [1976] appeared. Like that of Shafarevich, it had as its goal the exhibition of some of the interesting objects of algebraic geometry without serious use of algebra (sheaves and schemes), but there the resemblance ends. The difference in styles of writing will be apparent to the most casual browser. As for content, Mumford has as his goal to show how many different approaches to complex projective varieties there are, how many different kinds of geometric tools serve in the study. For this reason, and because of its relative brevity, I think Mumford's text is the best starting point for someone who knows only a little commutative algebra, but who is already at home in a basic way with manifolds and geometry in the differentiable setting. The book has no exercises as such, but most people find reading Mumford to be quite an active enterprise.

My own favorite textbook, for those with a suitable algebraic background, is that of Hartshorne [1977]. The first chapter seems to me an extraordinarily good introduction to the subject, culminating with the equivalence of smooth curves and "algebraic function fields in 1 variable." It should provide motivation for the two rather technical chapters (schemes, sheaves, and cohomology), which go, perhaps, a little far toward abstraction for those without either a previous background or an active guide. (The prospective reader might consider Serre [1955] as an inoculation against "schematitis" before beginning those chapters.) Chapter 4, on curves, is once again an exposition of the greatest simplicity and beauty. Very little of the hard chapters 2 and 3 is required; one could, with only a little help, read this after an elementary book on curves such as Fulton [1969] or even Walker [1950] (the theory of Jacobians is unfortunately omitted here; the reader could turn to Serre's book [1959] (whose first chapter should be read last) to fill the gap). Hartshorne's final chapter, on surfaces, is somewhat less satisfying, if only because the field is so vast, but it is a good beginning. The book contains a very large number of excellent and fascinating exercises, many of which are fairly easy applications of material or techniques in the text. One of the advantages of this book is that it suffices for the understanding of many current papers and seminars in the field.

The last of the modern "general introductions" I wish to mention is that of Griffiths and Harris [1978]. This book—"collection of essays" would be more descriptive, since the chapters are rather independent—is much further toward the analytic side than the others here: schemes are not mentioned, though sheaves, treated in a simple way, abound. The book suffers from an unusually large number of errors, both serious and merely annoying. But it also has far more of the ravishingly beautiful topics from "synthetic" projective geometry than any of the other texts, and it very effectively exploits its restriction to varieties over the complex numbers to come quickly to grips with such central notions as the Jacobian of a curve or the Chern classes of a bundle. It is, I think, the book of choice for an experienced reader looking for that part of the subject which made it exciting 50 or 100 years ago, and keeps it exciting today. My personal advice to prospective readers is to read chapters 2 and 4 first (maybe in the opposite order, skipping bits that seem less interesting) picking up the theory of the Grassmannian (Chapter 1.5) and the rest as necessary.

In addition to the four basic texts above, there are a number of more specialized, "second level" texts which are now or are about to be available, and which bridge, to some extent, the gap between the books already mentioned and the ongoing research of various schools. I can mention here only a few favorites. An early but important one of these is Mumford's book on families of curves on surfaces [1966], which contains a textbook account of Hilbert and Picard schemes. Also very interesting is the same author's book on Abelian varieties [1974] (which, alas, ignores Jacobians).

On the matter of curves, Mumford's little survey [1975] should not be missed by any admirer (either of curves or of Mumford). A large book by Arbarello, Cornalba, Griffiths, and Harris will presumably make available, in a year or so, an account in detail of a number of topics from the classical theory which have seen intense recent development.

As for surfaces, the short volume of Beauville [1978] (soon to be available in English) goes somewhat further than Hartshorne; a larger and long awaited work by Barth, Peters and Van de Ven is now in press.

Two books on other topics which deserve mention are those by Okonek, Spindler and Schneider [1980] (to which, I think, commutative algebraists interested in module theory should pay more attention) and a forthcoming book by Fulton on intersection theory, which will fill a gap in an important area.

Let us turn, at last, to the book of Iitaka under review. It is clear that it faces a lot of competition.

Iitaka's book is a revision of a text in Japanese which, I imagine, must have been aimed at helping students to the expertise necessary to work with the author and his colleagues in the interesting new area around the birational classification and maps of (possibly incomplete) varieties that they have pioneered and developed. At least a good part of it would therefore have fit into the "second level" category above. The author has attempted, in the revision, to make the book accessible to the beginner. Indeed, the jacket blurb announces that "This edition has been revised so that even the beginner can use it without referring to other books. Knowledge of the definition of Noetherian rings and the statement of the Hilbert Basis Theorem is all that is required."

Unfortunately, I cannot recommend this as a book for beginners. Indeed, the elementary definitions are carefully given, and many interesting results are proved from scratch. But despite the blurb quoted above, many of the central results are simply quoted from elsewhere (for example, from Hartshorne [1971], Dieudonné-Grothendieck [1960–67], or Mumford [1974]). The level of writing is quite uneven, there is no motivational treatment like that of the opening chapters of Hartshorne or Shafarevich to justify the plunge into the formalism of abstract schemes and commutative algebra, and there is a serious shortage of nontrivial (and, especially, nonrational) examples.

Here are some brief comments to justify each of these objections: The results, large and small, for which the reader is referred elsewhere include practically all the basic theorems on cohomology, the Zariski Connectedness Theorem, and the result that the only connected algebraic groups of dimension 1 are $\mathbb{A}^1$ and $\mathbb{G}_m$.

The variability of level is perhaps illustrated by the contrast between the first four examples of spectra—those of the trivial ring, the integers, a field, and the ring of polynomials in one variable—and the proof given for the basic properties of higher direct images, which I quote in full: "The next result is a consequence of the theory of derived functors." (P. 183.) I believe that this is the first mention of derived functors in the text.

The lack of motivation is illustrated by the first mention of the projective line which appears on page 63, *after* treatments of spectra, coherent and quasi-coherent sheaves, schemes, and open and closed immersions.

The lack of examples is typified by the treatment of curves, where one might expect finally to have reasonably many interesting examples. It does not start until page 208, and lasts, in any case, only 17 pages. The examples (broadly interpreted) that it contains are: The projective line (1 brief paragraph), hyperelliptic curves, treated in three brief remarks, and the Fermat plane curves $X^n + Y^n = Z^n$, treated in two paragraphs.

A more trivial, but rather pervasive problem is in style. For example, the verbal expression is often rather loose, and explication of the symbols frequently follows long symbolic expressions. More seriously, there is a heavy dependence on a casually selected notation, for which there is no index. For example, the notation spm (Y) (Y being a variety) appears without reference or hint on pages 314 and 315, in the statement of two propositions, and the proofs given do not make it (for me, anyway) especially easy to guess the meaning of the symbol. I feel that only the most devoted reader will remember the definition, on page 84, where it says that spm (Y) is the set of closed points of Y. I presume that the notation comes from the fact that closed points correspond to maximal ideals, so spm is the Maximal Spectrum (or at least the "spectre maximal"), but Iitaka does not comment on this. If there is a second printing of this work, a notational index would be a great help.

If we ignore the blurb and regard this as a "second level" text for someone, say, who has read Mumford [1976], Shafarevich [1972], or even sections of Griffiths-Harris [1978], and wants a view of schemes, or an introduction to an interesting and active group of topics, the book fares somewhat better. The ideal reader will bring with him his own motivation and examples, and will enjoy the many references to results proved elsewhere for the culture they bring, sighing perhaps with relief at the postponement of some hard work. He will certainly find items not available in other texts to reward him, from the notions of strict birational maps to the automorphisms of varieties of logarithmically general type. But Iitaka remarks in the Preface that he has had to cut the "high end" of the book, the sections on curves, surfaces, and logarithmic Kodaira dimension, to make room for more elementary material, and this hypothetical reader might, as I did, feel that the book stops only a little after the going becomes interesting.

In sum, I feel that Iitaka's book should stand not with the introductions to algebraic geometry, but with the more specialized works; and that by comparison with other works of this type, the book suffers, both through its style and use of notation, which make it hard to use for reference, and through the relatively small space given to advanced topics. The reader who braves the thicket does get, however, a taste to whet his appetite for more of the striking results from the author's distinguished school.

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LETTERS TO THE EDITOR

Material for this department should be prepared exactly the same way as submitted manuscripts (see the inside front cover) and sent to Professor P. R. Halmos, Department of Mathematics, Indiana University, Bloomington, IN 47405.

Editor,

In his paper, "Generalized Hyperbolic Functions" (this MONTHLY, vol. 89, pp. 688–691) A. Ungar discusses the circulant matrix $H_n(z)$ in which the elements of the first column are

$$\sum_{k=0}^{\infty} \frac{z^{nk+r}}{(nk+r)!} \qquad r = 0, 1, \ldots, n-1.$$