Summer Graduate Workshop: Toric Varieties

The MSRI summer graduate workshop on toric varieties, organized by David Cox of Amherst College and Hal Schenck of the University of Illinois at Urbana-Campaign, took place June 15–26, and brought together a diverse group of 45 participants, ranging from first- through fifth-year graduate students, with backgrounds in combinatorics, algebraic and symplectic geometry, and commutative algebra. Toric varieties are a class of algebraic varieties (roughly speaking, objects which look locally like the zeroes of a system of polynomial equations) which lie at the interface of geometry, combinatorics and algebra. The class of toric varieties is both large enough to include a wide range of phenomena and concrete enough to provide an excellent computational environment. This atypical combination leads to applications in many other fields including string theory, coding theory, approximation theory and statistics. Toric varieties also provide a wonderful vehicle for teaching algebraic geometry.

Geometrically, a toric variety is an irreducible algebraic set in which an algebraic torus forms a dense open subset, such that the action of the torus on itself extends to an action on the entire set. Combinatorially, a normal toric variety is determined by a fan; the cones in the fan yield affine varieties and the intersection of cones provide gluing data needed assemble these affine pieces together. Algebraically, an embedded toric variety corresponds to a prime binomial ideal in a polynomial ring. More generally, a toric variety can be described by a multigraded ring together with an irrelevant ideal. The importance of toric varieties comes from this dictionary between algebraic spaces, discrete geometric objects such as cones and polytopes, and multigraded commutative algebra.

Because of the wide range of backgrounds, the workshop had a very intense schedule. In the evenings, there were background lectures on basic material in algebraic geometry (ranging, for example, from valuation rings to vector bundles to sheaf cohomology). Each morning, there were two one hour lectures on interpreting algebro-geometric concepts in the toric setting. After lunch, participants were presented with several different sets of problems, ranging from very computational (compute the Picard group of a Hirzebruch surface) to more theoretical (prove a lemma stated during the morning lecture). Participants broke up into small groups of six or seven people, helped when needed by the organizers and two very able TAs (Dustin Cartwright and Daniel Erman) from Berkeley. At the end of the afternoon, the groups presented their results to the whole workshop.

During the latter part of the second week, three guest speakers spoke on topics related to toric geometry: David Eisenbud on the cone of betti tables; Matthias Beck on normality and semigroups; and Sam Payne on toric vector bundles. Participants really enjoyed seeing research talks on topics they had just studied. Among other participant comments:

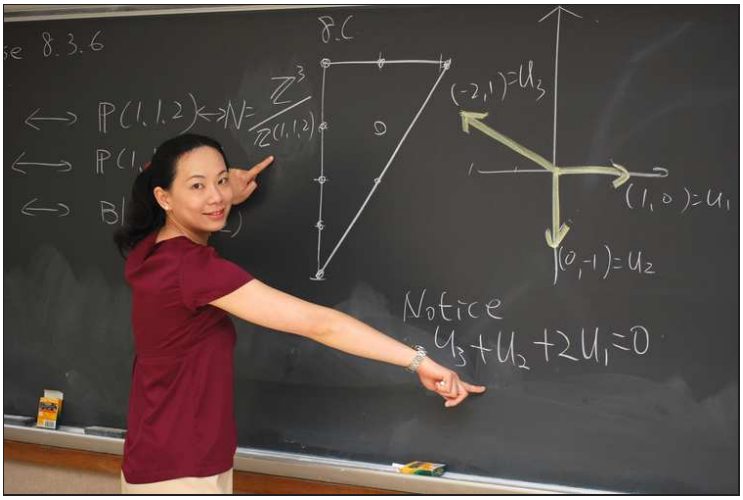
"The workshop was a truly amazing experience. The only way to improve it would be to make it longer!"

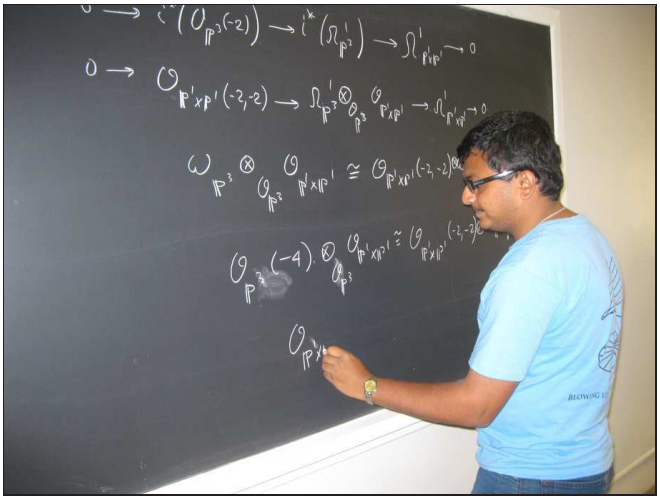
"I can't believe how much I learned in these short two weeks."

"Excellent workshop. The problem session and presentation setup was very conducive to working together and understanding the material. Intensive but also fun."

"The format of the workshop, although incredibly intensive, was very effective. Although there was no way for me to have digested everything, I learned a lot. Also, I really enjoyed the problem sessions because it encouraged us to meet each other and socialize."

"The morning lectures gave us the big picture. The afternoon problem sessions filled in the details of the picture; I particularly enjoyed the group work. The evening lectures helped prepare us for the next day's topics. This was an awesome experience."





XXXX and XXX lecturing.

Workshop lectures are available on streaming video at http://www.msri.org/calendar/sgw/WorkshopInfo/455/show_sgw. A draft of the forthcoming AMS book *Toric Varieties* by Cox, Little and Schenck is available at http://www.cs.amherst.edu/ dac/toric.html.