A counterexample to birational CY3 Torelli

which is found in joint work with John Christian Ottem
and also independently described by Borisov–Căldăraru–Perry

Jørgen Vold Rennemo

Postdoc
Enumerative Geometry Beyond Numbers

University of Oslo

- X smooth, projective Calabi–Yau 3-fold
- Hodge decomposition $H^3(X, \mathbb{Z}) \otimes \mathbb{C} = \bigoplus_{p+q=3} H^{p,q}(X)$

Question

If X and Y are deformation equivalent, and $H^3(X, \mathbb{Z}) \cong H^3(Y, \mathbb{Z})$ as Hodge structures, then is $X = Y$?

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New "stronger" counterexample

Consider $Gr(2,5) \subset \mathbb{P}^9$, choose generic $g \in GL(10,\mathbb{C}) \curvearrowright \mathbb{P}^9$. Let

$$X = Gr(2,5) \cap gGr(2,5) \text{ and } Y = Gr(2,5) \cap g^{-t}Gr(2,5).$$

Using "Homological Projective Duality", find

$$D^b(X) \cong D^b(Y) \rightsquigarrow H^3(X,\mathbb{Z}) = H^3(Y,\mathbb{Z}),$$

but X and Y not isomorphic (or even birational).