Naomi Feldheim (Bar-Ilan University) - research member

Warm-up:

Let $\{f(k)\}_{k\in\mathbb{Z}}$ be a **Gaussian**, **stationary** random sequence.
What is the order of

$$P_f(N) := \mathbb{P}\big(f(1) > 0, f(2) > 0, \ldots, f(N) > 0\big) \asymp \ ?$$

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Guess: "typically" $P_f(N) \asymp e^{-\theta N}$.

Simple examples:

$$\{X_n\} \text{ i.i.d.} \qquad \Rightarrow P_X(N) = 2^{-N}$$

$$Y_n = X_{n+1} - X_n \qquad \Rightarrow P_Y(N) = \frac{1}{(N+1)!} \asymp e^{-N\log N}$$

$$Z_n \equiv Z_0 \qquad \Rightarrow P_Z(N) = \mathbb{P}(Z_0 > 0) = \frac{1}{2}$$

Paradigm: Explained by the spectral behavior near the origin.

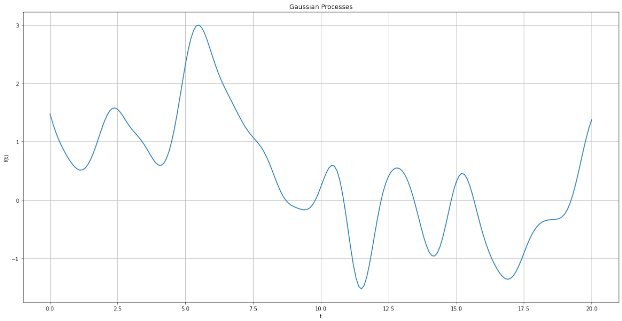
Zeroes of Gaussian Stationary Functions

Let $\{f(t)\}_{t\in\mathbb{R}}$ be a **Gaussian**, **stationary** random functoin.

$$r(t) = \mathbb{E}[f(0)f(t)] = \int_{\mathbb{R}} e^{-i\lambda t} d\rho(\lambda).$$

We study

$$N(T) = \#\{x \in [0, T] : \ f(x) = 0\}.$$



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- $\mathbb{E}N(T) = \frac{1}{\pi}\sqrt{\frac{-r''(0)}{r(0)}}$ [Rice (1944)]
- $\mathrm{var} N(T) \asymp T \int_0^T \left(r(t) - \frac{r''(t)}{r''(0)} \right)^2 dt$, if $\lim_{t\to\infty} r(t) = 0$
  [Assaf-Buckley-F. ('21+)]
- $\mathbb{P}\big(N(T) = 0\big) \asymp ?$ [F.-Feldheim et al. ('15-'21+)
- $\mathbb{P}\big(\,|N(T) - \mathbb{E}N(T)| > \delta T\big) \leq e^{-cT}$ if ...
  [Basu-Dembo-F.-Zeitouni ('20)]
- high-dimensions, non-Gaussian, repulsion, phase transitions...