

Roozbeh Gharakhloo



Postdoc: MSRI, Fall 2021 (Pavel Bleher), You can find me in Room 201.

Postdoc: Colorado State University, Aug 2019 - Aug 2021, Spring 2022 (Ken McLaughlin)

PhD: IUPUI, Aug 2019 (Alexander Its)

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OUTLINE OF ONGOING RESEARCH

1. Toeplitz+Hankel determinants (with A. Its) $\det_{0 \leq j, k \leq n-1} (\phi_{j-k} + w_{j+k})$.
 - **Motivations:** Ising Model on the zig-zag layered half-plane, Spectral theory of Hankel matrices, ...
 - **Results:** Formulation and nonlinear steepest descent analysis of the associated 4×4 RHP, asymptotics for the norms of the orthogonal polynomials
 - **Prospects of future work:** a) Properties of associated bi-orthogonal polynomials b) Solution of the model problem for wider classes of w and ϕ c) extension to Fisher-Hartwig symbols, and ...
2. $2j - k$ and $j - 2k$ determinants (with N. Witte) $\det_{0 \leq j, k \leq n-1} (\phi_{2j-k})$ and $\det_{0 \leq j, k \leq n-1} (\phi_{j-2k})$.
 - **Motivation:** Asymptotics of the moments of derivatives of characteristic polynomials $\Lambda_A(s) = \det(I - As)$, where $A \in USp(2N), SO(2N), O^-(2N)$.
 - **Results:** Four-term recurrence relations for the system of biorthogonal polynomials, multiple integral representations, Christoffel-Darboux identity.
 - **Prospects of future work:** a) Asymptotics, b) $p_j - q_k$ systems with co-prime integers p and q , c) In general: any analogue of the Toeplitz theory.
3. Bordered Toeplitz determinants (with E. Basor, T. Ehrhardt, A. Its, Y. Li)
 - **Motivation:** Asymptotics of the next-to-diagonal correlations $\langle \sigma_{0,0} \sigma_{N-1,N} \rangle$ in the 2D Ising model.
 - **Results:** Proof of Strong Szegő theorem for a large class of bordered Toeplitz determinants, rigorous justification of the magnetization in the next-to-diagonal direction in the low-temperature regime.
 - **Prospects of future work:** a) Extension of these results to Fisher-Hartwig singularities, b) next-to-diagonal correlations in the critical temperature and high temperature regimes, c) multi-bordered and multi-framed Toeplitz determinants.

OUTLINE OF ONGOING RESEARCH

4. Topological expansions of complex matrix models (with P. Bleher, K. McLaughlin)

- **Motivations:** Analytical description of the phase diagram for different potentials, Determination of the order of phase transitions for the free energy, Graph enumeration on Riemann surfaces of a given genus.
- **Results for the quartic potential:** Analytical description of the critical contours in the complex parameter plane where phase transitions between the one-cut, two-cut, and the three-cut regimes take place using the theory of quadratic differentials, proof of the $1/N^2$ expansion for the free energy in the entire one-cut regime, extension of existing results about the number $N_j(g)$ of 4-valent connected labeled graphs with j vertices on a compact Riemann surface of genus g . The leading order asymptotics of $N_j(g)$ as the number j of vertices of the 4-valent graphs tends to infinity, for an arbitrary genus g .
- **Prospects of future work:** a) The $1/N^2$ expansion for the free energy in the entire two-cut and three-cut regimes, b) Double scaling limits in the quartic case, c) Extension to other potentials d) Openness of the one cut regime for the general complex potential of even degree.