hydrodynamic scale of integrable many-body systems

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Toda lattice $H_N = \sum_{j=1}^{N} \left(\frac{1}{2} p_j^2 + e^{-(q_{j+1}-q_j)} \right)$

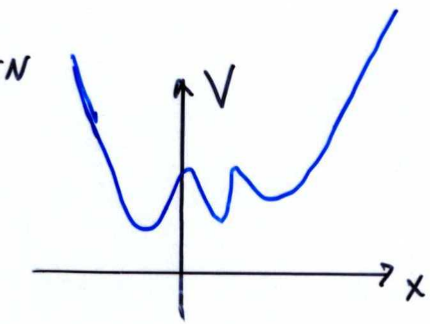
b.c. $q_{j+N} = q_j + \ell N$, $\ell \in \mathbb{R}$, confining

Flaschka $a_j = e^{-(q_{j+1}-q_j)/2}$

Lax matrix

$$L_N = \begin{pmatrix} p_1 & a_1 & & 0 & a_N \\ a_1 & \ddots & \ddots & & \\ & \ddots & \ddots & \ddots & \\ 0 & & \ddots & \ddots & \\ a_N & & & & p_N \end{pmatrix}$$

eigenvalues $\lambda_1, \ldots, \lambda_N$



Gibbs measure

$$\frac{1}{Z_N} e^{-\mathrm{tr}\, V(L_N)} \prod_{j=1}^{N} a_j^{-1+2P} dp_j da_j \quad \text{on } (\mathbb{R} \times \mathbb{R}_+)^N, \ P > 0, \ \text{parameter } P, V$$

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empirical DOS $\rho_{DOS,N} = \frac{1}{N} \sum_{j=1}^{N} \delta_{\lambda_j} \xrightarrow[N\to\infty]{} \rho_{DOS}$ a.s.

$$\mathcal{F}(\rho) = \int V(w)\rho(w) - P \int \rho(w)\rho(w') \log|w-w'| + \int \rho(w) \log \rho(w)$$

$\rho \geq 0$, $\int \rho(w) = 1$, unique minimizer $\rho^*(P)$

Theorem

$$\rho_{DOS} = \partial_P \left(P \rho^*(P) \right)$$