Spring 2015

EMISSARY

Mathematical Sciences Research Institute

www.msri.org

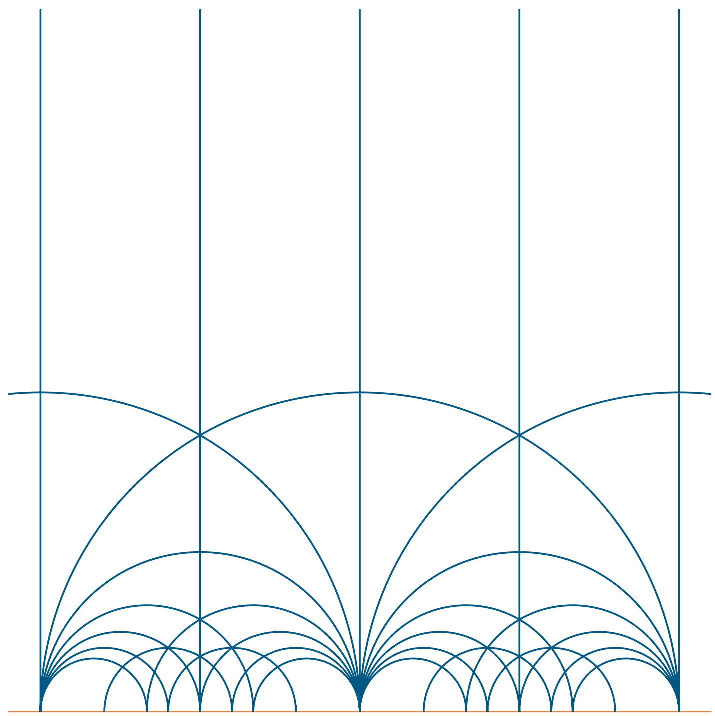
Dynamics on Moduli Spaces of Geometric Structures

Bill Goldman and François Labourie

This spring's program concentrates on dynamical systems arising from the classification of locally homogeneous *geometric structures* on manifolds.

What is a Geometric Structure?

Geometry concerns spatial relationships and quantitative measurements, whereas *topology* concerns the loose organization of points. Every geometric space has an underlying topological structure. Given a topological manifold Σ, and some geometry modeled on a homogeneous space X, can the local geometry of X (invariant under G) be put on the topology of Σ? If so, in how many ways? How does one understand the different ways of locally imparting the G-invariant geometry of X into Σ? The resulting *moduli space* (roughly speaking, the space of geometric structures on Σ) often has a rich geometry and symmetry of its own, and may be best understood, not as space but rather as a dynamical system. Here is a familiar example: *The sphere S^2 has no Euclidean geometry structure.* In other words, there is no metrically accurate world atlas. Therefore the moduli space of Euclidean structures on S^2 is empty. In contrast, the 2-torus T^2 has many Euclidean structures. The corresponding moduli space naturally identifies with the quotient of the upper halfplane H^2 by $PGL(2,\mathbb{Z})$, the group of integral homographies, as depicted in the tiling figure on the left. This quotient enjoys a rich and well-studied *hyperbolic geometry* of its own, which had been described in the mid-nineteenth century.



$PGL(2,\mathbb{Z})$-invariant tiling of the upper halfplane H^2.

Some Historical Background

The subject's roots indeed go back to the nineteenth century. Following Sophus Lie and Felix Klein's work on continuous groups of symmetry, the *Erlangen program* focused on the idea that a classical geometry (such as Euclidean geometry or projective geometry) is just the study of the G-invariant objects on a homogeneous space X. For instance, Euclidean geometry occurs when $X = \mathbb{E}^n$ and G is the group $\mathsf{Isom}(\mathbb{E}^n)$ of Euclidean isometries. In classical differential-geometric terms, this structure agrees with the notion of a *flat Riemannian metric*. Projective geometry concerns $X = \mathbb{R}P^n$ and

(continued on page 8)

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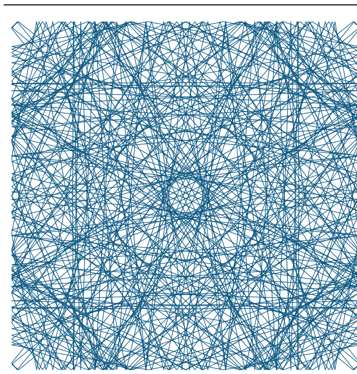


Geometric and Arithmetic Aspects of Homogeneous Dynamics. Do you recognize this curve? Can you see the circle in the picture? A Socratic dialogue about the spring program will lead you to a deeper understanding: page 4.

The View from MSRI

David Eisenbud, Director

As always, there is a great deal of excellent mathematics at MSRI. Most of it is done by wonderful researchers who don't win big prizes, but once in a while it's fun to "keep score": Last semester Peter Scholze, number-theory star, gave a course as our Chancellor's Professor (the youngest ever?); Ngô Bảo Châu was one of the organizers of the program; and Elon Lindenstrauss, a second Fields Medalist, also took part. This semester we have again two Fields medalists: Lindenstrauss is one of the organizers, and Maryam Mirzakhani will play an important part.

More important, by far, is the content: Geometric and Arithmetic Aspects of Homogeneous Dynamics (see page 4; don't tell your congressman that it's about billiards!) and Dynamics on Moduli Spaces of Geometric Structures (see page 1). As usual, MSRI presents a panorama of mathematics.

NSF Renewal

Surely the most significant event for MSRI in the past few months has been the NSF approval (as I write, still only in the "recommended stage") of the five-year NSF grant that supports more than half of MSRI's activities. IMA (University of Minnesota), IPAM (UCLA), MBI (Ohio State University) and ICERM (Brown University) were also up for renewal, with varying results.

MSRI did as well as any, and as well as I expected, but not as well as I hoped. The DMS is recommending us for funding at the same level as for the last five years. This represents a significant decrease in spending power, but fortunately, we have built up a modest endowment over these years: it stands now at $17 million. At least for the time being the endowment will just about fill the gap; but it is clear that continued excellence and growth will depend on continued fundraising outside the NSF core grant.

The National Math Festival

MSRI is partnering this spring with the Institute for Advanced Study to create the first-ever National Math Festival in Washington, DC, April 16–18. I wrote about this effort in an opinion piece in the March AMS Notices. We have been working on the organization for more than a year! We have established a cooperation with the Smithsonian Institution (a relatively rare honor) that will place lecturers and events of the Festival in several museums, as well as filling the big outdoor Enid A. Haupt Garden (behind the Smithsonian Castle). Events on math education and a gala dinner — "Act from Thought: the Case for Basic Science Research" — are among the other events. There is more information at mathfest.org.

Mythical? Mathematical? No, Mathical!

Mythical? . . . certainly not. Mathematical? . . . not quite. It's Mathical! — A prize, in five age categories, tots to teens, for books related to math. Fifty publishers submitted 175 books to compete for this award, which MSRI, in collaboration with the Children's Book Council, will give annually. The honors will be handed out, for the first time, at the National Math Festival this spring. Perhaps even more important than the prizes is the list of books ("Mathical honor books") that we will curate and distribute widely to librarians, teachers, and of course bookstores. To help these books get into the hands of kids in need, we are partnering with First Book, an organization that will retail the books at deep discounts through a very large network of people who work in underserved areas. Of course, someone has to pay the difference — we will be joining First Book in fundraising for this enterprise. See the winners (after April 17) at mathicalbooks.org.

"Fixing" Presentations Using a Computer

When I started out in mathematics (I got my Ph.D. in 1970) the community was learning how to give talks using overhead projectors. For quite a few years (I thought) they spoiled nearly every talk where they were used: writing too small (illegible); too big (only a few words per slide); changes too fast; lecturer read the slides verbatim; these were just a few of the problems. Slowly, mathematicians got better at the art; and many auditoriums came to have two projectors, so that one could still see the previous slide even as the presenter went on. That is still not as good (in my opinion) as seeing the mathematics evolving over six blackboards, but it's an improvement.

Then, as all my readers will know, our community began to use computer projectors. They are even less flexible than the overheads were (hard to correct or change on the fly!), *and* we are back to seeing just one slide of material at a time. I've cursed many such a "beamer" talk that I had hoped to enjoy. . .

But of course you *can* have more than one computer projector; it's just that the auditoria that are equipped with both the independent projectors and the software to make them work are rare, and some systems have required special efforts from the presenters. You generally have to have your presentation loaded on a special machine, with more than one video card; and the cost of installing such systems is high, much higher than the cost of a couple of extra projectors, now commodity items.

MSRI is about to install a system that is, as far as I'm aware, unique. In the Simons Auditorium, you'll soon be able to project a pdf file from your own machine and have the previous two images, as well as the current one, shown simultaneously and automatically as you page through your presentation. The trick is a new open-source software system called QED that was designed as a gift to the community by a Silicon Valley software engineer and friend of MSRI named Harshavardhana: three independent commodity PCs, following instructions from the QED server, will drive three independent projectors in sequencing the slides. I hope very much that the community will improve the system further (that's what open software is for!) and adopt it widely — so that in future we will have fewer talks "seen through a porthole" as they are at present.

Three New MSRI Faces

Claude Ibrahimoff joined MSRI in May 2014 as the Executive Assistant to Dr. Hélène Barcelo and International Scholar Advisor. Prior to MSRI, Claude worked for several years for a non-profit organization as a personal assistant to a filmmaker and activist who traveled around the world. Claude concurrently worked as a film editor mostly on independent and documentary films around her interests in the arts (documentaries on photographers Imogen Cunningham and Dorothea Lange), and as a translator. She paints as much as time allows while working and raising children.

Claude Ibrahimoff

Laura Montoya joined MSRI in January as the Assistant for Scientific Activities. She brings with her a diverse background working in the fields of insurance, hospital administration, regulatory affairs, and research. She graduated from Eastern Michigan University with a bachelor of science in Biology, Physical Science, and Human Development & Diversity; her research interests include psychology and human behavior. New to the Bay Area, most of her free time is spent exploring the city and getting to know her new home. Her favorite pastimes include volunteer work and community organizing for alternative lifestyle groups.

Laura Montoya

Diana White, the new Director of the National Association of Math Circles, earned her Ph.D. in commutative algebra from the University of Nebraska at Lincoln and held a postdoctoral position at the University of South Carolina before joining the faculty at the University of Colorado Denver, where she is now an associate professor. She specializes primarily in teacher training, mathematical outreach, and preparing mathematicians to educate teachers. She has been involved with Math Circles since the summer of 2008, and is the Founding Director of the Rocky Mountain Math Teachers' Circle and its associated Math Students' Circles.

Diana White

Endowed and Named Positions for Spring 2015

MSRI is grateful for the generous support that comes from endowments and annual gifts that support faculty and postdoc members of its programs each semester.

Eisenbud and Simons Professors

Dynamics on Moduli Spaces of Geometric Structures
Francis Bonahon, University of Southern California
Marc Burger, ETH Zurich
Richard Canary, University of Michigan
David Dumas, University of Illinois at Chicago
William Goldman, University of Maryland
Ursula Hamenstädt, Universität Bonn
Alessandra Iozzi, ETH Zurich
François Labourie, Université de Nice Sophia Antipolis
Howard Masur, University of Chicago
Anna Wienhard, Universität Heidelberg

Geometric and Arithmetic Aspects of Homogeneous Dynamics
Nalini Anantharaman, Université de Strasbourg
Yves Benoist, Université Paris-Sud (Orsay)
Alex Eskin, University of Chicago
Dmitry Kleinbock, Brandeis University
Elon Lindenstrauss, Hebrew University
Shahar Mozes, Hebrew University
Hee Oh, Yale University
Jean-François Quint, Université de Bordeaux 1
Nimish Shah, The Ohio State University
Ralf Spatzier, University of Michigan

Named Postdoctoral Fellows

Dynamics on Moduli Spaces of Geometric Structures
Cha–Chern: Qiongling Li, Rice University
Huneke: Sara Maloni, Brown University
Viterbi: Guillaume Dreyer, University of Notre Dame

Geometric and Arithmetic Aspects of Homogeneous Dynamics
Gamelin: Han Li, University of Texas

Clay Senior Scholarships

The Clay Mathematics Institute (www.claymath.org) has announced the 2015–2016 recipients of its Senior Scholar awards. The awards provide support for established mathematicians to play a leading role in a topical program at an institute or university away from their home institution. Here are the Clay Senior Scholars who will work at MSRI in 2015–2016:

New Challenges in PDE: Deterministic Dynamics and Randomness in High and Infinite Dimensional Systems (Fall 2015)
Martin Hairer, The University of Warwick
Pierre Raphaël, Université de Nice Sophia Antipolis

Differential Geometry (Spring 2016)
Tobias Colding, Massachusetts Institute of Technology

Geometric and Arithmetic Aspects of Homogeneous Dynamics

Yves Benoist

X. This spring at MSRI, one of the two programs focuses on *homogeneous dynamics*. Can you explain what the participants in this program are interested in?

Y. Homogeneous dynamics is the study of the stochastic properties of the action of Lie groups on their homogeneous spaces.

X. This does not look very concrete to me.

Y. Just the opposite! One of the aims of this topic is to solve concrete questions coming from arithmetic or geometry by using abstract tools that find their roots in ergodic theory.

X. What kind of tools?

Y. Ergodic theorems, mixing properties, invariant measures, entropy, and so on. All these tools are applied to an element g of a group G acting on a homogeneous space G/Γ.

X. This is very abstract! Can you show me a simple example?

Continued Fraction Expansion

Y. The first example is the *continued fraction expansion*. Start with an irrational real number x_0 and write $x_0 = [a_0, a_1, a_2, a_3, \ldots]$, where this sequence of integers a_i is constructed as follows: let a_0 be the integer part of x_0, let x_1 be the inverse of the fractional part of x_0, let a_1 be the integer part of x_1, and so on.

X. I remember that Euler and Lagrange proved that this sequence is periodic if and only if one can write $x_0 = a + b\sqrt{d}$ for some non-square positive integer d and some rational numbers a and b. For instance, one has $\frac{1}{2} + \frac{1}{2}\sqrt{5} = [1, 1, 1, 1, \ldots]$, $1 + \sqrt{2} = [2, 2, 2, 2, \ldots]$, $\frac{1}{2} + \frac{1}{2}\sqrt{3} = [1, 2, 1, 2, \ldots]$.

Y. You are right. But here is an open question. Let d be a non-square positive integer: do there exist two rational numbers a and b for which *the continued fraction of* $a + b\sqrt{d}$ *contains only 1's and 2's*? Can you guess the answer for $d = 7$?

X. One moment please… Using my computer, I find that $\frac{5}{8} + \frac{3}{8}\sqrt{7}$ has a periodic continued fraction whose period is $(1, 1, 1, 1, 1, 1, 1, 2, 1, 2)$. But I do not see any homogeneous dynamics in this question.

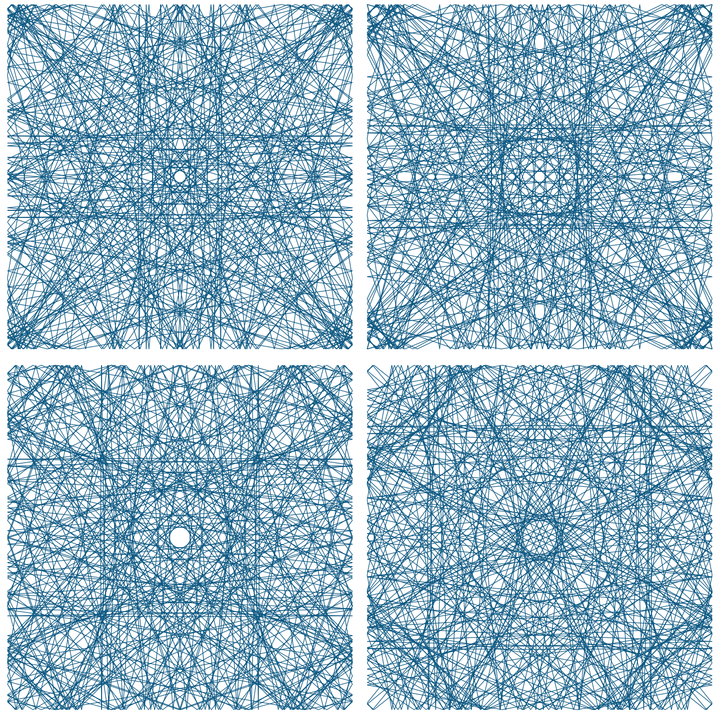
Y. This question is related to the excursions of the geodesic flow on the modular surface. This flow is one of the main sources of inspiration in *homogeneous dynamics*.

X. Do you have an example with nice pictures?

Counting of Integer Points

Y. The second example is the *counting of integer points*. Do you recognize these four curves? (They're shown at the top of the next column.)

X. *????* Those are both nice and messy. The equation might be both subtle and complicated :o(.



Can you see the circle in these four pictures?

Y. Not at all. Each one of these four curves is just a circle, whose radius R is approximately 50! I overlaid all that one sees through a square window of side length 1 successively centered at the integer points in the plane :o) .

X. This reminds me that Gauss proved that the number of integer points inside this circle is approximately πR^2 with an error term bounded by $2\pi R$.

Y. You are right. But it is not known whether this error term is $O(R^{\frac{1}{2}+\varepsilon})$. One knows that these circles become equidistributed in the square and one controls their speed of equidistribution.

X. Does this equidistribution property help to find the best error term in Gauss approximation?

Y. Not quite. But for the analogous question with tilings in the hyperbolic space, the equidistribution of large spheres also allows one to obtain *counting results similar to those of Gauss*. The proof relies on the ergodic properties of the horocyclic flow which is the second main source of inspiration in *homogeneous dynamics*.

X. Do you have a simpler example?

The "$\times 2 \times 3$" Question

Y. The third example is the "$\times 2 \times 3$" question. One starts with an irrational number x and one denotes by $\{x\}$ its fractional part which is x minus its integer part. One looks at the n^2 points $\{2^p 3^q x\}$ where p and q vary between 1 and n. Here is an open question: Do *these sets of points become equidistributed in the interval* $[0, 1]$ for large n?

X. The word *equidistributed* seems important. What does it mean?

Y. Here it means that the proportion of points in these sets that belong to a given interval $I \subset [0,1]$ is approximatively equal to the volume of I, with an error term going to 0 when n grows.

X. You mean the length of I, not the volume?

Y. Correct! But in the interval $[0,1]$ the words *length*, *volume*, *mass*, and *probability* are synonymous! Don't tell that to a physicist!

X. This reminds me of a theorem of Borel: almost all real numbers x are *normal*, which means that the sets of points $\{10^p x\}$ with $p \leqslant n$ become equidistributed in the interval $[0,1]$ for large n.

Y. Here we insist on the equidistribution being true for all irrational numbers x. This question is also an important source of inspiration in *homogeneous dynamics* where pairs of commuting transformations often occur.

X. I guess that in your example the pair is the two maps $x \mapsto \{2x\}$ and $x \mapsto \{3x\}$. But these two maps are not invertible transformations of the circle $\mathbb{R}/\mathbb{Z}$. Isn't that a problem?

Y. We force them to be invertible. We replace this circle by a *solenoid*: each point x of the circle is replaced by the Cantor set of all its possible predecessors!

X. You mean that this guy x knows its future but has forgotten its past, and you force him to remember it!

Y. Kind of. Mathematically this solenoid is a compact homogeneous space G/Γ where G is the product $G = \mathbb{R} \times \mathbb{Q}_2 \times \mathbb{Q}_3$, the $\mathbb{Q}_p$'s being the p-adic fields and Γ the diagonal subgroup $\mathbb{Z}[\frac{1}{6}]$. The use of these local fields is another important feature in *homogeneous dynamics*.

X. Do you have another example with nice pictures?

Equidistribution of Lattices

Y. The fourth example is *the equidistribution of lattices*.

X. By a lattice you mean a subgroup of $\mathbb{R}^d$ generated by a basis of $\mathbb{R}^d$ as for instance the lattice $\mathbb{Z}^d$ of integer points in $\mathbb{R}^d$?

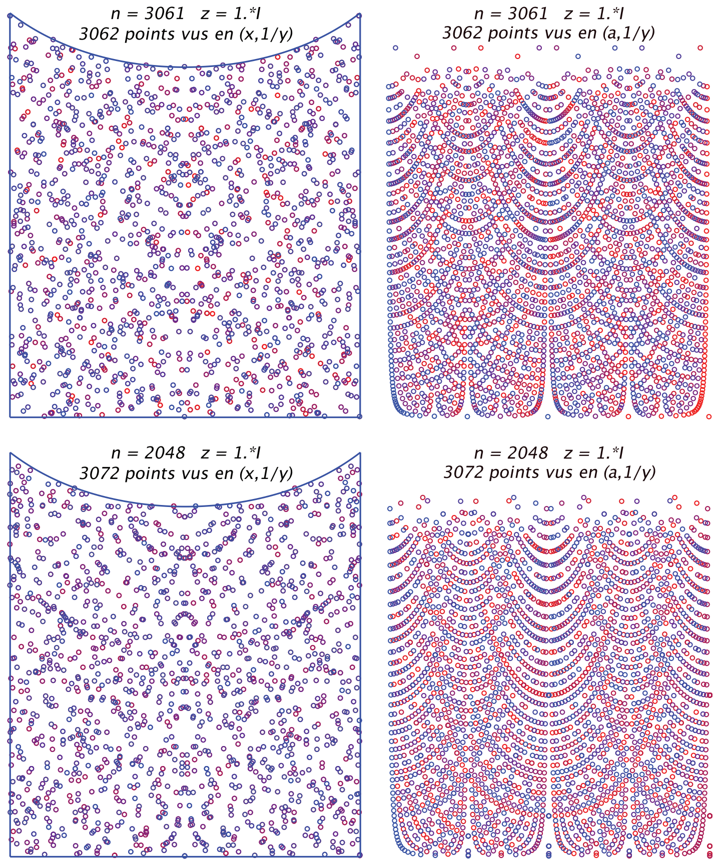
Y. Exactly. We will assume $d = 2$ to make it simple. We will focus on the shape of the lattices, not on their size. Thus we consider as equal two lattices which are images of one another by a homothety. Do you know how to parametrize the set X of these lattices Λ?

X. Yes, one has $X = \mathrm{SL}(2,\mathbb{R})/\mathrm{SL}(2,\mathbb{Z})$. After a homothety and a rotation by the angle $a \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, one can assume that the vector $(1,0)$ is one of the shortest non-zero vectors of Λ and let (x,y) with $y > 0$ be a shortest non-horizontal vector of Λ. This vector is in the strip given by $|x| \leqslant \frac{1}{2}$ and $x^2 + y^2 \geqslant 1$.

Y. Very good. We will use (a, x, y') with $y' = 1/y$ as parameters for Λ so that our set of parameters is bounded. Now we consider the set F_n of sublattices Λ of index n in $\mathbb{Z}^2$. This is a finite set. Do you know why?

X. Yes, Λ must contain $n\mathbb{Z}^2$ and is determined by its image in $(\mathbb{Z}/n\mathbb{Z})^2$.

Y. The theorem is that *these sets F_n become equidistributed in X for large n*, and, more precisely, one can bound effectively the error term.



Equidistribution of finite index lattices.

X. What does *equidistributed* mean here?

Y. The same as before: the proportion of points of F_n that belong to a given ball B in X converges to $m(B)/m(X)$, where m is the measure $m = da\,dx\,dy'$. This is illustrated in the two upper pictures where n is the prime number $n = 3061$. First for the parameters (x, y'), then for (a, y').

X. Why did you draw almost the same pictures twice?

Y. The lower pictures show that this equidistribution occurs also with a non-prime integer n. Here $n = 2048$ is a power of 2. To be precise, we have only drawn here the sublattices $\Lambda \in F_n$ that are not included in $2\mathbb{Z}^2$.

X. Can you explain the nice structures on the right-hand side pictures?

Y. Here is a hint: the vectors $\sqrt{ny'}(\cos a, \sin a)$ belong to $\mathbb{Z}^2$.

X. I guess the homogeneous dynamics hidden in this example is again the geodesic flow on the modular surface…

Y. Not quite! The homogeneous space here is $\mathrm{SL}(2,\mathbb{A})/\mathrm{SL}(2,\mathbb{Q})$ where $\mathbb{A}$ is the ring of adèles of $\mathbb{Q}$.

X. Why does one need such a strange ring?

Y. The ring of adèles is a very natural object: it is a locally compact ring that contains $\mathbb{Q}$ as a discrete subring and such that the quotient $\mathbb{A}/\mathbb{Q}$ is both compact and connected.

X. It behaves like the field $\mathbb{R}$ of real numbers for the ring $\mathbb{Z}$ of integers!

GAAHD *(continued from previous page)*

Y. Precisely. Another key tool in the proof is the *uniform mixing* property, also called *spectral gap* or *uniform decay of matrix coefficients*.

X. This looks tough... Do you have a simpler example?

Normal Subgroup Theorem

Y. The fifth example is the *normal subgroup theorem*. I will just describe a special case of this theorem. Consider a finite dimensional division algebra L over $\mathbb{Q}$ whose center is equal to $\mathbb{Q}$.

X. You mean like the quaternion algebras over $\mathbb{Q}$.

Y. The quaternion algebras are those L for which $\dim_{\mathbb{Q}} L = 4$. The dimension $\dim_{\mathbb{Q}} L$ is always a square d^2. Here we will assume $d \geqslant 3$.

X. But the quaternion algebras are the only examples I know!

Y. Yet, there are many others. Indeed, these division algebras L are described by the so-called Brauer group of $\mathbb{Q}$.

X. The very Brauer group which plays a role in the class field theory?

Y. Yes. Now choose a basis of L in which the multiplication of L has integer coefficients, and let Γ be the multiplicative subgroup of $L \smallsetminus \{0\}$ whose elements and their inverses have integer coordinates.

X. This group Γ is a non-commutative analogue of the group of units in a number field. Is this group Γ infinite, as in Dirichlet's units theorem?

Y. Yes, for $d \geqslant 3$. Indeed, Γ is a discrete cocompact subgroup of the group $G = SL(d, \mathbb{R})$. One wants to describe the normal subgroups of Γ.

X. This group Γ cannot be simple because a congruence condition like being equal to 1 modulo n defines a finite index normal subgroup of Γ.

Y. Exactly. The theorem says that *the normal subgroups of Γ are either finite or have finite index in Γ*.

X. You mean Γ is almost simple! What happens for the quaternion division algebras?

Y. In this case the group Γ is either finite or a finite extension of the fundamental group of a higher genus surface. It has lots of normal subgroups.

X. I guess the homogeneous space in this example is G/Γ.

Y. Yes. But another important homogeneous space in this context is the so-called flag variety $\mathcal{F}$ of G. One of the key points in the proof is to classify the Γ-invariant sub-σ-algebras of the Lebesgue σ-algebra of $\mathcal{F}$.

X. Do you have another example with nice pictures?

Further Examples

Y. Yes, many! The Apollonian circles, the integer points on spheres, the gaps in $\sqrt{n}$ modulo one, the random walks on tori, the space of quasicrystals, the irrational quadratic forms, ... But we are running out of time.

X. Thanks for your answers. How can I learn more on this topic?

Y. It's up to you to decide. Some Ph.D. students first study either the Margulis arithmeticity theorem or the Ratner classification theorem. Others focus directly on one of the many concrete remaining open questions.

X. Like the ones you explained to me. Where did you find these five examples?

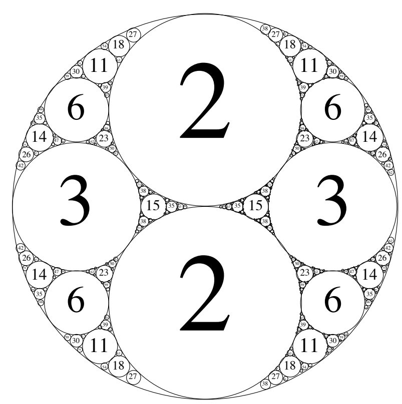
Y. The first one is due to McMullen, the second and fifth to Margulis, the third to Furstenberg, and the fourth to Clozel, Oh, and Ullmo.

Gamelin Postdoc

Han Li, a postdoctoral scholar in the Geometric and Arithmetic Aspects of Homogeneous Dynamics, is the Spring 2015 Gamelin Endowed Postdoctoral Fellow.

Han Li

Han completed his undergraduate studies at Nankai University in China and completed his Ph.D. at Yale University under the supervision of Gregory A. Margulis. He joined The University of Texas at Austin as a Bing instructor in Fall 2014. Han, in broad terms, is interested in dynamics on homogeneous spaces and applications to number theory. More specifically, he has investigated application of effective methods in homogeneous dynamics to the study of quadratic forms in two joint works with Gregory Margulis. These investigations, in particular, have lead to an affirmative solution of the so called Small-Height Equivalence Conjecture of David Masser. The Gamelin postdoctal fellowship was created in 2014 by Dr. Ted Gamelin, Emeritus Professor of the UCLA Department of Mathematics. The Gamelin Fellowship emphasizes the important role that research mathematicians play in the discourse of K-12 education.



Another aspect of homogenous dynamics: Counting circles in an Apollonian packing, as described in the focus on scientist Hee Oh on the next page.

Focus on the Scientist: Hee Oh

A native of Korea, Hee Oh briefly considered a career in social work before changing her mind in favor of mathematics. Following undergraduate studies at Seoul National University, she got her Ph.D. in 1997 at Yale University under the guidance of Margulis, and then held faculty positions at Princeton, Caltech, and Brown before returning to her alma mater, where she is currently the Abraham Robinson Professor of Mathematics. Her recent honors include: a talk at the 2010 ICM, an invited address to the AMS–MAA joint meeting in 2012, and a Ruth Lyttle Satter Prize of the AMS in 2015. This semester she is one of the organizers of the Geometric and Arithmetic Aspects of Homogeneous Dynamics program at MSRI.

Hee has made many important contributions to the study of Lie groups, their discrete subgroups, and group actions on homogeneous spaces, an area that pursues deep connections between ergodic theory, geometry, and number theory. A substantial part of her work has been related to decay of matrix coefficients of unitary representations of semisimple linear groups over various local fields. Applied to the quasi-regular representation of a group G on the space of functions on the quotient G/Γ, where Γ is a lattice in G, this makes it possible to control the rate of mixing of the G-action on G/Γ. Exponential decay of matrix coefficients is instrumental in a number of Hee's results, such as: explicit calculation of Kazhdan constants of semisimple groups, non-existence of compact quotients of non-Riemannian homogeneous spaces, effective equidistribution of Hecke points (joint with Clozel and Ullmo), and a proof of Manin's conjecture on rational points of bounded height (with Gorodnik and Maucourant), to name just a few.

Another theme of Hee's work has been unipotent dynamics on homogeneous spaces. In the early 1990s Marina Ratner proved a set of deep conjectures regarding orbit closures and invariant measures of unipotent flows. Ratner's orbit closure theorem was one of the main ingredients of Hee's Ph.D. thesis, where she established arithmeticity of certain discrete subgroups of algebraic groups. Her later research repeatedly involved unipotent dynamics and its applications, for example, to counting representations of integers by an invariant polynomial (with Eskin) and for the proof of equidistribution of semisimple adelic periods (jointly with Gorodnik).

Hee Oh

Hee's more recent mathematical achievements are described in the 2015 Satter Prize citation as follows: "Her work brings together in a beautiful way dynamics on homogeneous spaces, the geometry and topology of 3-dimensional manifolds, and various subtle number-theoretic phenomena, for example the distribution of primes." This refers to a series of papers, joint (in various combinations) with Kontorovich, Lee, Lim, Margulis, Mohammadi, and Shah, which approach counting problems related to Kleinian groups via dynamics on infinite volume homogeneous spaces. One such problem involves counting circles in an Apollonian packing, like the one shown in the figure at the bottom of the previous page. Studying dynamics on Kleinian manifolds and searching for possible generalizations of Ratner's theorems for those infinite volume spaces constitutes one of the hottest challenges in homogeneous dynamics these days, and Hee Oh is one of the main players there.

Hee surely would have been a terrific social worker. Yet we are all very fortunate that she has chosen mathematics instead!

— Dmitry Kleinbock and Shahar Mozes

Call for Membership Applications

MSRI invites membership applications for the 2016–2017 academic year in these positions:

Research Professors	by October 1, 2015
Research Members	by December 1, 2015
Postdoctoral Fellows	by December 1, 2015

In the academic year 2016–2017, the research programs are:

Geometric Group Theory, Aug 15 – Dec 16, 2016
Organized by Ian Agol, Mladen Bestvina, Cornelia Drutu, Mark Feighn, Michah Sageev, Karen Vogtmann

Analytic Number Theory, Jan 17 – May 26, 2017
Organized by Chantal David, Andrew Granville, Emmanuel Kowalski, Philippe Michel, Kannan Soundararajan, Terence Tao

Harmonic Analysis, Jan 17 – May 26, 2017
Organized by Michael Christ, Allan Greenleaf, Steven Hofmann, Michael Lacey, Svitlana Mayboroda, Betsy Stovall, Brian Street

MSRI uses **MathJobs** to process applications for its positions. Interested candidates must apply online at www.mathjobs.org after August 1, 2015. For more information about any of the programs, please see www.msri.org/scientific/programs.

Dynamics on Moduli Spaces of Geometric Structures

(continued from page 1)

$G = \mathsf{PGL}(n+1, \mathbb{R})$ with a similar differential-geometric description.

The theory of crystallographic groups and their classification by Bieberbach is another historical source. This is equivalent to the classification of flat Riemannian manifolds, and, in turn, to the classification of discrete groups of Euclidean symmetries.

Yet another source arose from integration of analytic differential equations, which related to conformal mappings of plane domains, as studied by Schwarz, Klein and Poincaré, and many others.

This was part of a larger development of the theory of connections by Ricci, Levi-Civita and É. Cartan, which generalized classical surface theory. Einstein's theory of relativity used these ideas and also was a major contribution.

Some of the most important examples arise from geometric structures on surfaces. In higher dimensions, the moduli spaces are often finite sets, since the fundamental group is *overdetermined* in this case. For instance, by the Mostow rigidity theorem a manifold of dimension greater than two admits at most one hyperbolic structure. In dimension two, the moduli spaces often admit *symplectic structures,* and natural Hamiltonian flows constructed out of the topology of the surface and the invariants of G provide ways of navigating around the moduli space.

Geometric Structures and their Models

Start with a geometry in the sense of Lie and Klein, that is, a homogeneous space X upon which a Lie group G acts transitively. In other words, we restrict to "geometries" in which neighborhoods of all points "look the same." We model a manifold M locally on X as follows. Choose an atlas of *coordinate charts* on M, mapping coordinate patches $U \subset M$ by homeomorphisms $U \xrightarrow{\psi} \psi(U) \subset X$. We require that on overlapping coordinate patches, the *coordinate change* locally lies in G. Therefore the G-invariant geometry on X is transplanted locally to M. For example, a Euclidean structure defines notions of distance, angles, lines, and area locally satisfying Euclidean rules. Similarly, a projective structure on M defines notions of lines, etc. which satisfy rules of projective geometry, such as Pappus's theorem.

The notion of local coordinates in X on Σ and the above *notion of a* (G, X)*-structure* was first explicitly defined by Charles Ehresmann in the 1930s. This notion was rejuvenated in the 1970s when Thurston formulated his geometrization program for 3-manifolds in the context of (G, X)-structures. In this theory, hyperbolic geometry plays the prominent role.

One convenient way to globalize the coordinate atlas of a geometric structure involves the *universal covering space* $\widetilde{M}$ of the geometric manifold M. If M is already simply connected, a geometric structure boils down to an immersion of M into the model space X. In general, one can describe the geometric structure in terms of a *developing map* $\widetilde{M} \xrightarrow{\mathsf{dev}} X$ and a compatible *monodromy* (or *holonomy*) representation $\pi_1(M) \xrightarrow{\rho} G$. The developing map globalizes the coordinate charts and the monodromy representation globalizes the coordinate changes.

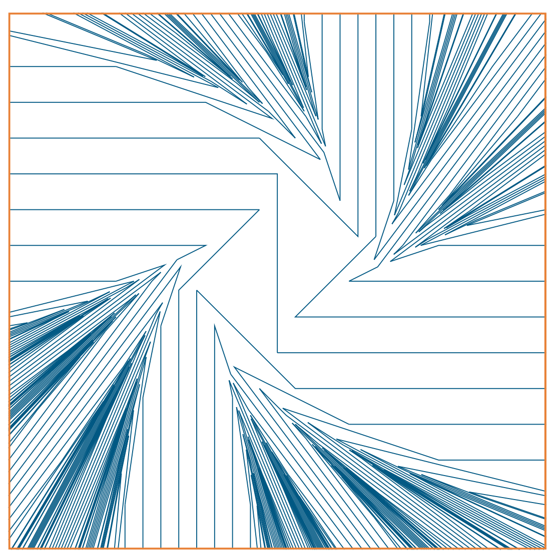
While some developing maps are bijective and identify M with a quotient X/Γ, others may just identify M with quotients of proper domains $\Omega \subset X$. Others may not even be covering spaces of domains, and may wildly wrap $\tilde{M}$ onto all of X in an extremely complicated way. In the nicest cases the holonomy image Γ may be a discrete subgroup of G, but in the wildest cases it may even be dense. The developing map/holonomy representation itself displays potentially complicated dynamical behavior.

Let us now describe two important sources of examples: (1) tilings and discrete symmetries, as epitomized by Euclidean crystallographic groups and flat Riemannian manifolds; and (2) monodromy of differential equations, as epitomized by projective structures on Riemann surfaces (that is, $\mathbb{C}\mathsf{P}^1$-manifolds) and classical Kleinian groups and uniformization.

Tilings and Discrete Symmetries. Regular tilings in Euclidean space led to the notion of a *crystallographic group,* which in modern parlance, is just a *lattice* Γ (a discrete subgroup of finite covolume) in the group $\mathsf{Isom}(\mathbb{E}^n)$.

Gradually the point of view changed, as the role of the transformations between the tiles became more prominent: *The shape of the tile is less relevant than the motions of the tiler.*

The Bieberbach theorems gave an effective classification of crystallographic groups Γ, as finite extensions of *lattices* $\Lambda \subset \mathbb{R}^n$ of translations. Geometry arises through the quotient $M = \mathbb{E}/\Gamma$, which is often a manifold with a Euclidean structure. That M is actually a quotient of the model space X — that is, dev is a homeomorphism — relates to the metric nature of the structure, and we will see more complicated phenomena later. Nevertheless, this example underscores the intimate relationship between geometric structures and discrete subgroups of Lie groups.



Cross-section of a tiling arising from a properly discontinuous group of affine Lorentz isometries in dimension three.

An interesting development is the understanding of the analog of the Bieberbach theorems in *indefinite metric*, and in particular the construction and classification of geodesically complete spacetimes when there are two space dimensions and one time dimension. The figure at the bottom of the previous page depicts one type of example, a so-called *Margulis spacetime,* whose fundamental group is a free group of rank two.

Monodromy of Differential Equations. Another precursor of the theory of geometric structures on surfaces is the study of differential equations on complex domains. Even on $\mathbb{R}$, solutions of periodic linear differential equations on $\mathbb{R}$ may not be periodic. The solutions f of

$$f'(z) = a(z)f(z),$$

when $a(z+T) = a(z)$ (where T is the period), are not necessarily periodic, but satisfy

$$f(z+T) = \lambda f(z)$$

for some λ. The solution f looks like a developing map for a geometric structure, where λ generates the monodromy.

Similarly, on the unit disc Δ, Hill's equation

$$w''(z) + q(z)w(z) = 0$$

leads to a Riemann surface M with a projective structure (a $\mathbb{C}P^1$-structure). If $w_1(z), w_2(z)$ is a basis of the space of solutions on Δ, then the *projective solution*

$$f(z) := w_1(z)/w_2(z)$$

defines a map $\Delta \to \mathbb{C}P^1$. If, furthermore, $q(z)$ is periodic with respect to a Fuchsian group Γ, then f defines a developing map for a $\mathbb{C}P^1$-structure on the quotient Δ/Γ.

These classical examples are both basic and extremely rich. For $q = 0$, the developing map is the embedding of the disc Δ into the projective line $\mathbb{C}P^1$. If q is sufficiently small, the developing map embeds Δ as a domain bounded by a fractal curve (a *quasicircle*). (See the quasi-Fuchsian and a quasicircle figures in the next column.) However, at this stage the developing map remains injective. As q increases, the developing map does not embed $\widetilde{M}$, nor admit a boundary. The asymptotic behavior of the monodromy as $q \longrightarrow \infty$ is an active subject of research related to mathematical physics.

Moving from ordinary differential equations to partial differential equations produces a wealth of new examples of fundamental importance, such as Yang–Mills equations, harmonic maps, and Hitchin systems.

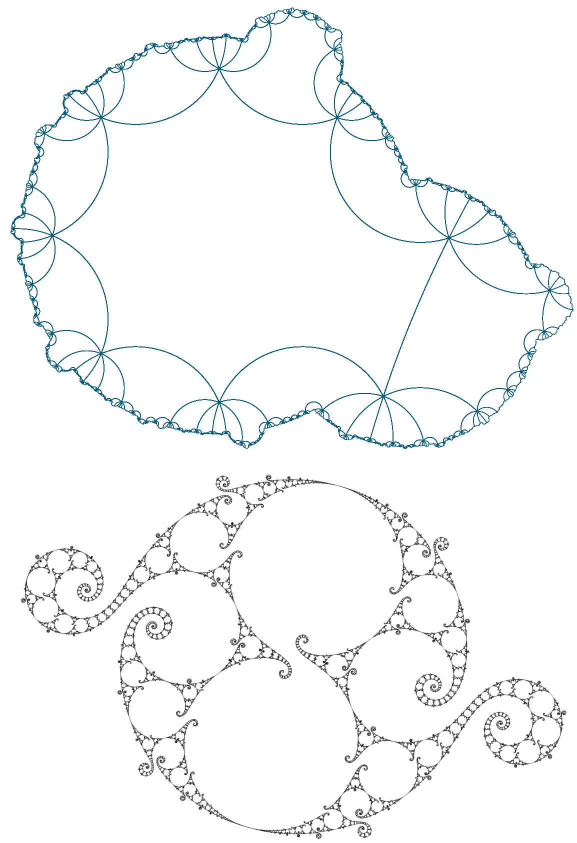
Moduli Spaces

Isomorphism classes of (G, X)-structures on Σ locally modeled on X form a space, called the *moduli space*, which enjoys its own interesting geometry and dynamics.

Here are two examples: (1) moduli of Euclidean tori; and (2) projective triangle tilings. In the first case, the moduli space of unmarked structures is the quotient space $H^2/PGL(2,\mathbb{Z})$, and in the second case, the moduli space of (either marked or unmarked structures) is the half-open interval $[0,\infty)$, parametrized by a cross-ratio invariant.

Moduli of Euclidean Tori. Euclidean structures on T^2 form a space enjoying hyperbolic geometry. If M is a Euclidean manifold homeomorphic to T^2, then the geometric structure identifies M as a quotient $\mathbb{E}^2$ by a lattice $\Lambda \subset \mathbb{R}^2$. In this context, a marking of M is just a basis of $\pi_1(M)$ which identifies with Λ.

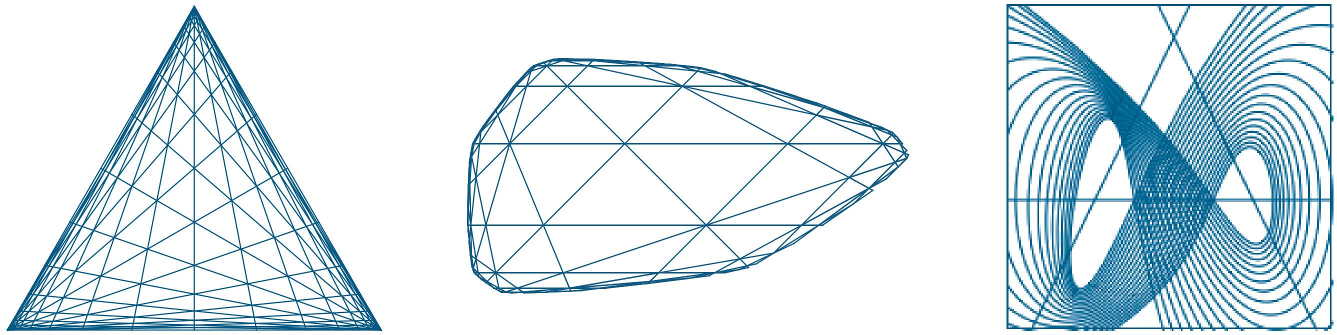
The *moduli space* of unit-area marked Euclidean structures identifies with the upper halfplane in $\mathbb{C}$ as follows. A *marked Euclidean structure* is then just the choice of a parallelogram of area 1 with one horizontal side. Corresponding to the other side of the parallelogram is a complex number τ with positive imaginary part, which is a point in the Poincaré upper halfplane H^2.



A quasi-Fuchsian $\mathbb{C}P^1$-structure (top) and a quasicircle limit set.

Changing the marking amounts to applying an element of $GL(2,\mathbb{Z})$ to the parallelogram, which corresponds to applying the associated integral linear fractional transformation of $PGL(2,\mathbb{Z})$ to $\tau \in H^2$. Therefore the corresponding *moduli space* of unit area *unmarked* Euclidean tori identifies with the quotient $H^2/PGL(2,\mathbb{Z})$, as depicted in the tiling figure on the cover. Note that the group associated with changing the markings preserves the hyperbolic geometry of the upper halfplane H^2.

Projective Triangle Tilings. Now let us move from tiling the Euclidean plane $\mathbb{E}^2$ by parallelograms to tiling domains in the projective plane by triangles. As $\mathbb{E}^2 \subset \mathbb{R}P^2$ is a domain, and isometries of $\mathbb{E}^2$ extend to projective transformations of $\mathbb{R}P^2$, every Euclidean structure is a *projective structure*. Now the familiar tiling of the Euclidean plane by equilateral triangles deforms projectively in a nontrivial way. The triangular figure (a) at the top of the next page depicts a projective deformation of this tiling. Here the developing map is not onto, but remains injective.



From left to right: (a) Projective deformation of tiling by Euclidean equilateral triangles; (b) Projective deformation of hyperbolic triangle group; (c) Exotic developing map for $\mathbb{R}\mathsf{P}^2$-structure on T^2.

Similarly, the Klein–Beltrami model embeds the hyperbolic plane H^2 in the projective plane $\mathbb{R}\mathsf{P}^2$, where the isometries of H^2 extend as projective transformations. The oblong figure (b) at the top of the page depicts a projective deformation of a triangle group in H^2. The new domain Ω is tiled by triangles. Furthermore, Ω is bounded by a C^1 convex curve which is nowhere C^2.

The symmetry group of each of these tesselations of domains Ω contains a finite-index subgroup Γ_0 such that Ω/Γ_0 is a surface with an $\mathbb{R}\mathsf{P}^2$-structure. These provide examples where the developing map is injective but not surjective. In contrast, the figure (c) at the top right of the page depicts an $\mathbb{R}\mathsf{P}^2$-structure on T^2 whose developing map is neither injective nor surjective.

Dynamics

To make the moduli space more tractable, it is often useful to introduce some extra topological structure, called a *marking*. Changing the marking leads to the action of a group which defines a dynamical system.

We saw that the moduli space of *unmarked* unit-area Euclidean structures on T^2 is the quotient of H^2 by $\mathsf{PGL}(2,\mathbb{Z})$, whereas H^2 is the moduli space of *marked* Euclidean structures. Although the $\mathsf{PGL}(2,\mathbb{Z})$-action is not free, the quotient has a nice structure. However, sometimes the moduli space of *unmarked structures* is not a *space in the classical sense*: it may not admit nonconstant continuous functions. Therefore studying the space of *marked structures*, together with the group action corresponding to *changing the marking*, is more natural. In two dimensions, this is described by the action of the *mapping class group* of Σ.

Here is an example of chaotic dynamics. A *marked complete affine structure* on T^2 is an identification of T^2 as a quotient of the affine plane $\mathbb{R}^2$ by a discrete group Γ of affine transformations. The moduli of marked complete affine structures on T^2 identifies with $\mathbb{R}^2$, where standard Euclidean structures on T^2 all correspond to the origin. Corresponding to changing marking is the standard linear action of $\mathsf{GL}(2,\mathbb{Z})$. In this case, the quotient space is non-Hausdorff and doesn't even support nonconstant continuous functions.

In this way, classification of geometric structures naturally leads to interesting dynamical systems. Here is an example related to a venerable subject in number theory.

The Dynamics of Markoff Triples. Sometimes a marked geometric structure on Σ identifies with its holonomy representation. Thus the moduli space of marked structures identifies with a subset of an algebraic set: the coordinates are matrix entries and the defining equations arise from the defining relations in $\pi_1(M)$.

A simple and fundamental example is the space of equivalence classes of pairs of matrices in $\mathsf{SL}(2,\mathbb{C})$, corresponding to representations of the free group on two generators. In this case Σ is the once-punctured torus. Since the nineteenth century, we know that such a pair of matrices is described (up to equivalence) by the traces x, y of the two generators and the trace z of their product. Thus the moduli space identifies with $\mathbb{C}^3$. Further imposing the natural boundary condition around the puncture leads to the cubic equation

$$x^2 + y^2 + z^2 - xyz = t,$$

where $t \in \mathbb{C}$ corresponds to the trace of holonomy around the puncture. This moduli space has a rich group of symmetries generated by polynomial automorphisms such as

$$(x, y, z) \longmapsto (x, y, xy - z)$$

and permutations, corresponding to changes of markings. When $t = 0$, this is the classical Markoff equation, arising from the classification of binary quadratic forms. For other values of t, the dynamics ranges from proper dynamics (with a *Hausdorff* quotient space) to dynamically interesting chaos.

Conclusion

Rooted in classical origins, our research area is thriving. The study of geometric structures involves many fields and diverse techniques: ergodic theory, geometric analysis, geometric group theory, Lie theory, and combinatorics. As a natural extension of classical Riemann surface theory and Lie theory, it relates to the interests of many mathematicians and theoretical physicists. Many more connections are expected, notably with the companion program currently running at MSRI, Geometric and Arithmetic Aspects of Homogeneous Dynamics. Both the NSF-funded GEAR Research Network and the European Research Council have supplemented the MSRI budget to spread intellectual benefits of our MSRI program to a broader group of mathematicians. This program has been instrumental in expanding, clarifying, and consolidating the general field.

Focus on the Scientist: Marc Burger

Marc Burger is a Clay Senior Scholar for this spring's Dynamics on Moduli Spaces of Geometric Structures program. He presented a 4-lecture minicourse at the introductory workshop and is co-organizing the research seminar for this program.

Marc has enjoyed a distinguished career, making fundamental contributions to Lie theory, analysis on manifolds, geometric group theory, and differential geometry. His work is characterized by its depth and breadth, and a striking ability to forge relationships between different fields.

Marc is enthusiastic, energetic, and truly enjoys discussing mathematics with researchers at all levels. He has mentored numerous students and postdocs, many of whom are participating in several of the programs this spring. He has directed 10 doctoral students and has three current students. His work has strongly influenced many young mathematicians.

His early work involved eigenvalues of the Laplacian on Riemann surfaces and locally symmetric spaces, in particular estimating the lowest eigenvalue. In this work he combines differential geometry, complex analysis, topology, and representation theory. Many of these results have important consequences in number theory. Harmonic analysis on graphs appeared already in this work on spectral theory in the late 1980s, but reappeared in later work in the mid-1990s. One of his most cited papers is his 1991 paper where he explicitly computes Kazhdan constants for the group $\mathsf{SL}(3,\mathbb{Z})$. Another highly cited paper initiates the study of the horocycle flow on infinite area hyperbolic surfaces.

His work on lattices and rigidity with Shahar Mozes led to new examples of finitely presented simple groups. More recently, Alessandra Iozzi, Anna Wienhard, Marc, and others have studied *maximal representations* of surface groups in Lie groups acting on Hermitian symmetric spaces. These representations have many special properties and enjoy a rich deformation theory, which is one of the key foci of the MSRI program. Burger's approach to these questions involved the use of the theory of *bounded cohomology,* a theory developed by Trauber and Gromov in the 1970s. Earlier work with his previous student Nicolas Monod constructed a space that greatly facilitated the computation of new examples. They applied these calculations to show that many lattices admit no nontrivial actions on the circle.

Marc Burger

Marc is currently Full Professor at ETH Zürich, a position he has held since 1997. From 1999 until 2009, he was the director of the Forschungsinstitut für Mathematik at ETH. He is a member of the National Research Council of the Swiss National Science Foundation. He received his Diplôme de Mathématicien in 1983 from the University of Lausanne, and his Habilitation in 1990 from Universität Basel. Since then, he has held positions at Stanford University, CUNY Graduate Center, the Institute for Advanced Study, and the University of Lausanne. He is a Fellow of the American Mathematical Society and an elected member of the Leopoldina, National Academy of Sciences, Germany. He served on the editorial board of *Inventiones Mathematicae* from 1994–2005, and he was an invited speaker to the International Congress of Mathematicians in Zürich in 1994.

— Bill Goldman

Call for Proposals

All proposals can be submitted to the Director or Deputy Director or any member of the Scientific Advisory Committee with a copy to proposals@msri.org. For detailed information, please see the website www.msri.org.

Thematic Programs

The Scientific Advisory Committee (SAC) of the Institute meets in January and November each year to consider pre-proposals for programs. Proposals for special events or conferences outside the programs are considered in a much shorter time frame. The deadlines to submit proposals of any kind for review by the SAC are **October 15** and **December 15**. Successful proposals are usually developed from the pre-proposal in a collaborative process between the proposers, the Directorate, and the SAC, and may be considered at more than one meeting of the SAC before selection. For complete details, see http://tinyurl.com/msri-progprop.

Hot Topics Workshops

Each year MSRI runs a week-long workshop on some area of intense mathematical activity chosen the previous fall. To be considered for the spring or fall of year $n+1$, a proposal should be received by **October 15** of year n. See http://tinyurl.com/msri-htw.

Summer Graduate Schools

Every summer MSRI organizes several 2-week long summer graduate workshops, most of which are held at MSRI. To be considered for the summer of year $n+1$, proposals must be submitted by October 15 of year $n-1$. See http://tinyurl.com/msri-sgs.

Viterbi and Huneke Postdocs — DMS Program

Guillaume Dreyer

Guillaume Dreyer is the Viterbi Endowed Postdoctoral Fellow at MSRI this spring. Guillaume grew up in the Alsace region of France and did his undergraduate studies at the University of Strasbourg. He then moved to the University of Southern California to complete his Ph.D. under the direction of Francis Bonahon in 2012. Prior to his semester at MSRI, he held a postdoctoral position as Visiting Assistant Professor at the University of Notre Dame. Guillaume works in the area of higher Teichmüller theory. He studies the geometric and dynamical properties of representations of surface groups, and, more precisely, for Hitchin representations. The Viterbi Endowed Postdoctoral Scholarship is funded by a generous endowment from Dr. Andrew Viterbi, well known as the co-inventor of Code Division Multiple Access (CDMA) based digital cellular technology and the Viterbi decoding algorithm, used in many digital communication systems.

Sara Maloni

Sara Maloni is the Spring 2015 Huneke Endowed Postdoctoral Scholar. Sara received her Ph.D. in 2013 at Warwick University under the supervision of Caroline Series and is currently a Tamarkin Assistant Professor at Brown University. Sara has made substantial contributions to the study of moduli spaces of hyperbolic 3-manifolds, dynamics on character varieties, and anti-de Sitter geometry. One of her beautiful results, joint with Jeffrey Danciger and Jean-Marc Schlenker, is that any hyperbolic metric on the sphere with n labelled cusps, and a distinguished equator and top and bottom polygon, can be uniquely realized as the induced metric on a convex ideal polyhedron in the anti-de Sitter space AdS^3. The Huneke postdoctoral fellowship is funded by a generous endowment from Professor Craig Huneke, who is internationally recognized for his work in commutative algebra and algebraic geometry.

Forthcoming Workshops

May 11–15, 2015: *Advances in Homogeneous Dynamics*, organized by Dmitry Kleinbock (Lead), Hee Oh, Alireza Salehi Golsefidy, Ralf Spatzier

May 28–29, 2015: *Partnerships: a Workshop on Collaborations between the NSF and Private Foundations*, organized by Cynthia Atherton, Paulette Clancy, David Eisenbud (Lead), Thomas Everhart, Caty Pilachowski, Robert Shelton, Yuri Tschinkel

June 13–July 26, 2015: *MSRI-UP 2015: Geometric Combinatorics Motivated by Social Sciences*, organized by Federico Ardila, Duane Cooper (Lead), Herbert Medina, Ivelisse M. Rubio, Suzanne Weekes

June 15–26, 2015: *Séminaire de Mathématiques Supérieures 2015: Geometric and Computational Spectral Theory*, organized by Alexandre Girouard, Dmitry Jakobson, Michael Levitin, Nilima Nigam, Iosif Polterovich , Frederic Rochon

June 15–26, 2015: *Geometric Group Theory*, organized by John Mackay (Lead), Anne Thomas, Kevin Wortman

June 15–July 11, 2015: *CRM-PIMS Summer School in Probability*, organized by Louigi Addario-Berry (Lead), Omer Angel, Louis-Pierre Arguin, Martin Barlow, Edwin Perkins, Lea Popovic

June 29–July 10, 2015: *Mathematical Topics in Systems Biology*, organized by Steven Altschuler (Lead), Lani Wu

June 29–July 10, 2015: *NIMS Summer School on Random Matrix Theory*, organized by Jinho Baik (Lead)

July 06–17, 2015: *Berkeley Summer Course in Mining and Modeling of Neuroscience Data*, organized by Ingrid Daubechies, Bruno Olshausen, Christos Papadimitriou, Fritz Sommer, Jeff Teeters (Lead)

July 13–24, 2015: *Gaps between Primes and Analytic Number Theory*, organized by Dimitris Koukoulopoulos, Emmanuel Kowalski (Lead), James Maynard, Kannan Soundararajan

July 27–August 07, 2015: *Incompressible Fluid Flows at High Reynolds Number*, organized by Jacob Bedrossian, Vlad Vicol (Lead)

August 19–21, 2015: *Connections for Women: Dispersive and Stochastic PDE*, organized by Kay Kirkpatrick (Lead), Andrea Nahmod

August 24–28, 2015: *Introductory Workshop: Randomness and Long Time Dynamics in Nonlinear Evolution Differential Equations*, organized by Kay Kirkpatrick, Yvan Martel (Lead), Luc Rey-Bellet (Lead), Gigliola Staffilani

October 19–30, 2015: *New Challenges in PDE: Deterministic Dynamics and Randomness in High and Infinite Dimensional Systems*, organized by Jonathan Mattingly, Andrea Nahmod (Lead), Pierre Raphaël, Luc Rey-Bellet, Daniel Tataru

For more information about any of these workshops as well as a full list of all upcoming workshops and programs, please see www.msri.org/scientific.

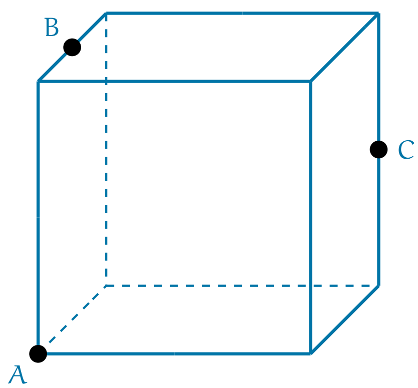
Puzzles Column

Elwyn Berlekamp and Joe P. Buhler

This year's BAMO (Bay Area Mathematical Olympiad) exam was given on February 24. This contest is supported by MSRI as well as through other grants and donations. It is divided into two parts, BAMO-8 and BAMO-12, roughly aimed at junior high and high school students respectively (though they overlap in two problems). The first problem below is from BAMO-8 and the second from BAMO-12.

1. Members of parliament serve on various committees. Each committee has at least two people, and every pair of committees has at least one person in common. Prove that it is possible to 3-color the members of parliament (for example, by giving each of them a red, white, or blue hat) such that no committee is monochromatic (that is, every committee has two members with distinct hat colors).

2. Let A be a corner of a cube as in the figure, and B and C be midpoints of their respective edges. Consider the polygon P that is the intersection of the (solid) cube and the plane containing the triangle ABC. How many sides does P have? What is the ratio of the area of P to the area of ABC?



3. Define a Random Digit Algorithm (RDA) to be an algorithm that takes a random string of input bits and (with probability 1) halts after a finite number of bits and emits a uniformly random digit. Here "uniformly random" means that each of the ten digits is equally likely.

An example: Look at 5 bits; if they are the binary expansion of an integer between 0 and 9, halt and output that integer; otherwise, look at the next 5 bits, and repeat. What is the expected number of bits required by this algorithm?

Find an RDA whose expected number of bits used is smaller.

Comment: This seems to be part of folk knowledge from the middle part of the last century. The optimal number of expected bits is 4.6.

4. Prisoner A is brought into the warden's office and shown a row of n coins. The warden points to one of the coins.

The prisoner is then required to turn over exactly one coin (that is, reverse the heads/tails of that coin). Prisoner A is escorted out of the room, and then Prisoner B is brought into the room and asked to point at a coin. If the prisoner points at the same coin that the warden pointed at earlier to Prisoner A, then both prisoners are released. Otherwise, one year is added to each of their sentences.

The "usual" rules are assumed: the prisoners have a strategy session the night before and know what will happen the next day, though of course they do not know what coin the warden will point to, or what the heads/tails orientation of the coins will be (so that they might as well assume that the initial orientation is random, that is, the orientation of each coin is determined by a flip of a fair coin). No further communication is allowed after the strategy session; for example, they do not communicate at all after Prisoner A is taken to the office.

Prove that a perfect strategy (that is, one in which the prisoners will always be released) exists if and only if n is a power of 2.

Comment: This problem apparently first appeared (with $n = 8$) in the 2007 International Tournament of Towns, and we first saw it (with $n = 64$) in Anany and Maria Levitin's *Algorithmic Puzzles*, problem 148.

Cha–Chern Scholar

Qiongling Li, a postdoctoral scholar in the Dynamics on Moduli Spaces program, is the Spring 2015 Cha–Chern scholar. Qiongling was an undergraduate at Nankai University before coming to Rice University in 2010; she earned her Ph.D. in 2014, writing a thesis advised by Michael Wolf. Following her Spring at MSRI, Qiongling will begin a three-year joint postdoctoral position at Caltech and the Centre for Quantum Geometry and Moduli Spaces at Aarhus University (Denmark). Qiongling's work centers on geometric-analytic aspects of moduli spaces of representations into semi-simple Lie groups of surface groups, one of the foci of the Dynamics on Moduli Spaces program. The Cha–Chern scholarship combines two funds that were established by Johnson Cha and the family of Shiing-Shen Chern. Shiing-Shen Chern was an outstanding contributor to research in differential geometry and was one of the three founders of MSRI. He acted as its first director from 1981–84. Johnson Cha served on MSRI's Board of Trustees from 2000–04 and is Managing Director of the Mingly Corporation, a Hong Kong investment company.

Qiongling Li

2014 Annual Report

We gratefully acknowledge the supporters of MSRI whose generosity allows us to fulfill MSRI's mission to advance and communicate the fundamental knowledge in mathematics and the mathematical sciences; to develop human capital for the growth and use of such knowledge; and to cultivate in the larger society awareness and appreciation of the beauty, power and importance of mathematical ideas and ways of understanding the world. This report acknowledges grants and gifts received from January 1 – December 31, 2014. In preparation of this report, we have tried to avoid errors and omissions. If any are found, please accept our apologies, and report them to development@msri.org. If your name was not listed as you prefer, please let us know so we can correct our records. If your gift was received after December 31, 2014, your name will appear in the 2015 Annual Report. For more information on our giving program, please visit www.msri.org.

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