MSRI-UP 2020 Research Project Abstracts

June 25, 2020

Project #1

Title

Automorphism Groups and Monodromy of Classical Modular Curves

Group Members

Samuel Heard, Fabian Ramirez, and Vanessa Sun

Abstract

It is well known that all compact connected Riemann surfaces X of genus at least two are quotients of the extended upper-half plane $\mathbb{H}^*$ by a discrete subgroup Γ of $PSL_2(\mathbb{R})$. For example, when Γ is a classical congruence subgroup such as $\Gamma_0(N)$, $\Gamma_1(N)$ or $\Gamma(N)$, then the Riemann surfaces, namely $X_0(N)$, $X_1(N)$, and $X(N)$ are well-known. By projecting to the "j-line" $X(1) \simeq \mathbb{P}^1(\mathbb{C})$, we have a morphism $\beta : X \to \mathbb{P}^1(\mathbb{C})$ branched above 0, 1, ∞. In this project, we consider the monodromy groups $\mathrm{Mon}(\beta)$ and automorphism groups $\mathrm{Aut}(\beta)$ of such maps.

Project #2

Title

Explicit Constructions of Finite Groups as Monodromy Groups

Group Members

Ra-Zakee Muhammad, Javier Santiago, and Eyob Tsegaye

Abstract

In 1963, Greenberg proved that every finite group appears as the monodromy group of some morphism of Riemann surfaces. In this paper, we give two constructive proofs of Greenberg's result. First, we utilize free groups, which given with the universal property and their construction as discrete subgroups of $PSL_2(\mathbb{R})$, yield a very natural realization of finite groups as monodromy groups. We also give a proof of Greenberg's result based on triangle groups $\Delta(m, n, k)$. Given any finite group G, we make use of subgroups of $\Delta(m, n, k)$ in order to explicitly find a morphism π such that $G \simeq \mathrm{Mon}(\pi)$.

Project #3

Title

Dessin d'Enfants from Cartographic Groups

Group Members

Nicholas Arosemena, Yaren Euceda, and Ashly Powell

Abstract

A Belyĭ map is a morphism $\beta : S \to \mathbb{P}^1(\mathbb{C})$ of degree N defined on a compact, connected Riemann surface S which is branched above $\{0,\, 1,\, \infty\}$. The cartographic group associated with β is generated by permutations $\sigma_0,\, \sigma_1$, with $\sigma_\infty = (\sigma_0 \circ \sigma_1)^{-1}$ satisfying the relations ${\sigma_0}^m = {\sigma_1}^n = {\sigma_\infty}^k = \sigma_0 \circ \sigma_1 \circ \sigma_\infty = \mathbf{1}$. The associated Dessin d'Enfant $\Delta_\beta = \big(B \cup W,\, E\big)$ is that bipartite graph whose "black" vertices are $B = \beta^{-1}(0)$, "white" vertices are $W = \beta^{-1}(1)$, and edges $E = \beta^{-1}\big([0,1]\big)$. The monodromy group may be viewed as the cartographic group of this bipartite graph.

In this project, we work in the opposite direction. Say that we are given a triple $\big(\sigma_0,\, \sigma_1,\, \sigma_\infty\big)$ of permutations $\sigma_0,\, \sigma_1,\, \sigma_\infty \in S_N$ such that (i) $\sigma_0 \circ \sigma_1 \circ \sigma_\infty = \mathbf{1}$ and (ii) $G = \langle \sigma_0,\, \sigma_1,\, \sigma_\infty \rangle$ is a transitive subgroup of S_N. There exists a unique bipartite graph $\Delta \hookrightarrow S$ which can be drawn on a compact, connected Riemann surface in such a way that its cartographic group is G. This project focused on drawing such Dessin d'Enfants when the Riemann surface has genus 1 or greater by focusing on examples which appear in the L-Series and Modular Forms Database (LMFDB).

Project #4

Title

Visualizing Toroidal Belyĭ Pairs

Group Members

Deion Elzie, Mikaela Nishida, and Cameron Thomas

Abstract

In this project, we present a Mathematica program to write code which generates a 2D and 3D Dessin d'Enfant from an elliptic curve E and a Belyĭ map β. Following [5], we compute the elliptic logarithm $E(\mathbb{C}) \simeq \mathbb{C}/\Lambda$ using a modification of the arithmetic-geometric mean, then compose with the canonical correspondence $\mathbb{C}/\Lambda \simeq \mathbb{T}^2(\mathbb{R})$ between $\mathbb{C}$ modulo the period lattice of E and the standard torus. We provide sample outputs using Belyĭ maps which appear in the L-Series and Modular Forms Database (LMFDB).

Project #5

Title

Computing Monodromy of Toroidal Belyĭ Pairs

Group Members

Rebecca Lopez and Chidera Okenwa

Abstract

In this project, we present a Mathematica program which takes an elliptic curve and a Belyĭ map and returns the generators for the monodromy group. This program also outputs a set of points on the elliptic curve which can be used to visualize the monodromy. Our method employs solving a system of first-order differential equations. In this paper we discuss the history of the project, review necessary background information, outline our algorithm, and compute the monodromy group for several toroidal Belyĭ pairs appearing in the L-Series and Modular Forms Database (LMFDB).

Project #6

Title

To and From 2-Generated Groups and Origamis: Starting from Square One

Group Members

Sarai Gonzalez, Elisa Rodriguez, and William Sablan

Abstract

In this research, we focus on the geometric construction of origami in detail. Initially, we construct various origami, by considering different examples of 2-generated groups. Conversely, we begin with an arbitrary collection of squares, glued together to form an origami, and determine the corresponding transitive subgroup of S_n.