



First Ever National Math Festival

The first ever National Math Festival drew more than 20,000 children and adults to the Smithsonian Museums in Washington, DC, on Saturday, April 18, 2015. There were games, puzzles, demos, performances, lectures, and more at the free public event.

The festival also addressed math policy, with a congressional briefing on the importance of teacher preparation in mathematics as well as a math education forum on college and career readiness standards.

More photos and a full report on the festival are in the center of the newsletter—[pages 6 and 7](#).

New Challenges in PDE: Blending Determinism and Randomness

Carl Mueller and Monica Visan

Over the past twenty years the theory of nonlinear dispersive partial differential equations and systems (PDE), on the one hand, and the theory of stochastic partial differential equations (SPDE), on the other, have developed into mature mathematical theories.

On the PDE side, the development of analytical tools in nonlinear Fourier and harmonic analysis to address nonlinear estimates, related deep functional analytic methods, and profile decompositions have fundamentally contributed to the study of the local and global (in time) well-posedness (that is, existence, uniqueness, and some form of stability for the Cauchy initial value problem) as well as singularity formation for dispersive equations and systems. The thrust of this body of work has focused primarily on deterministic aspects of wave phenomena. More recently, several publications have aimed at understanding the nondeterministic point of view as well.

On the SPDE side, similar questions about existence, uniqueness, and qualitative long-time behavior (existence and uniqueness of underlying stationary measure, mixing rates, etc.) have also been addressed for a large number of models by bringing together tools from statistical mechanics, dynamical systems, and probability theory. In recent years, the focus in both PDE and SPDE has thus shifted to trying to gain a more quantitative understanding of the nondeterministic long-time dynamical behavior in various regimes.

While recently there has been spectacular progress in both of these fields, the advances have taken place in a parallel fashion without substantial exchange of ideas between the PDE and SPDE communities. However,

(continued on page 8)

Below: Wave dispersion on the surface of water. Waves with short wavelength propagate faster than those with longer wavelength.



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The View from MSRI

David Eisenbud, Director

There's so much going on at MSRI that organizing my essay is hard! — so I thought I'd do it according to the Mission:

Science

As anyone following the doings of MSRI will know, we usually have two semester-long programs in the building at once, and we strive to choose programs that have some relation to each other. (We don't always succeed, but once in a while there is spectacular cross-fertilization; one of the most famous such was Vaughan Jones's discovery of the Jones polynomial, coming from the confluence of a program on von Neumann algebras with a program on knot theory.) Unusually, we have Jumbo programs this semester and for the next two. These are topics so big and diverse that they deserve to occupy all our space, instead of sharing the building. This semester's Jumbo resulted from the marriage of a program on stochastic PDE and a program on dispersive PDE. It has already proven to be a happy marriage, with many people enjoying the borderland between the subjects.

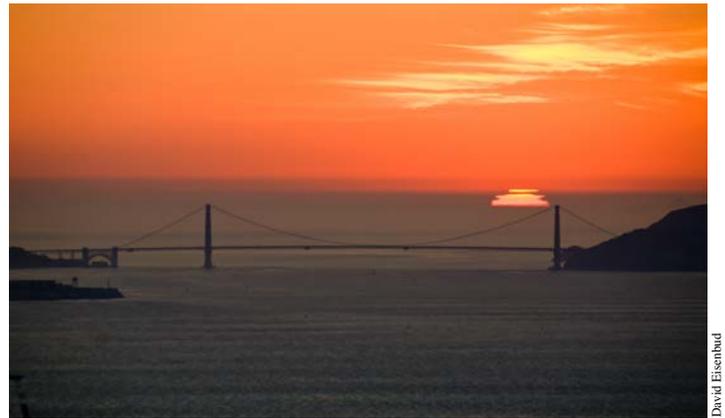
The major programs are not our only scientific activity: as I write, a workshop on the Theory of Neural Computation is in progress. It grew from the enthusiasm of the participants in last fall's Breaking the Neural Code workshop. The current topic is the development of theory for understanding neural networks in machines and (an even harder problem) understanding the neural networks in our brains. The workshop started with a day of lectures surveying the big problems of the field, sharp and provocative. We will of course videotape everything, and I think it will be a valuable resource for many beyond the over two hundred people who signed up to attend.

Another event coming soon is a workshop on combinatorial games, in honor of Elwyn Berlekamp. Elwyn has been a devoted participant and supporter of MSRI since before there was an MSRI — he went along with the founders to see the Chancellor and convince him that MSRI was a good idea, sometime around 1978.

Development of Talent

I cannot celebrate enough the work of the mathematicians who run our program for talented undergraduates from underserved backgrounds, MSRI-UP. They bring 18 students to MSRI for a six-week summer program each year, and then give them extensive mentoring through the transition to graduate school and beyond. MSRI-UP is supported by the NSF (outside MSRI's regular grants) and the NSA, and has been very productive: there is now a stream of graduates, doing very well. For example, the first Ph.D. coming from the program, Talea Mayo, became a postdoc at Princeton. One of the most recent, Bobby Wilson, graduated from the University of Chicago last spring and now holds a named postdoc at MSRI (see his profile on page 3). He has written that "... participating in the MSRI-UP program was a career-changing experience. Without it, I may not have pursued a Ph.D. in pure mathematics."

Other important programs in this area make special arrangements for women with young families and focus on the mentoring and development of postdocs and graduate students from across the nation and around the world.



Public Understanding of Math

MSRI has long held that math is not for mathematicians alone — it's for everyone. We continue to develop programs for young and old, face-to-face and through media, old and new.

For little kids to very big kids, there's the Mathical Book Prize (see page 7 for a poster showing the winners), after-school recreational math with Math Circles (see page 3 for an update), and the Bay Area Math Olympiad. For grown-ups and kids alike, there's a growing network of MoSAIC art/math mini-festivals around the country, the National Math Festival in alternate years in Washington, DC, and our own local Celebration of Mind (with a Lewis Carroll/Halloween theme on October 31 this year) — and that's to say nothing of the Numberphile YouTube channel (1.38 million subscribers and counting!).

For adults, there's Guerilla Science's award-winning Fire Organ and other traveling Sound Sculptures, new and soon-to-debut films for public television ("Counting from Infinity" on Yitang Zhang and the twin prime conjecture; "Navajo Math Circles" on a particular confluence of cultures), and many local math lectures for general audiences (this year's Not on the Test series at Berkeley City College features four dynamic women speaking on disparate topics: Internet voting, Pixar animation, knots and DNA, and the Black Girls Code movement).

I invite you to join us at msri.org/public to learn more — and play along!

Support

Gathering support for mathematics does not appear in the Mission statement, but all the rest presupposes it. The biggest piece still comes from the National Science Foundation, which has been our wonderful supporter since day one. I'm happy to say that our five-year core grant from the NSF was just renewed (start date, September 1, 2015), in a process lasting almost two years from the time I started writing the proposal. The NSA contributes further support for junior and senior members and for workshops. Private fundraising also goes well; the contributions of the mathematical community make us effective in seeking the contributions from those capable of larger gifts, so that in 2014 almost half of MSRI's budget came from private individuals and foundations too many to list here (there will be a comprehensive list, as always, in the spring Emissary). To them my — and our — warmest thanks! They make possible wonderful scientific and public activity! 

Gamelin Postdoc

Bobby Wilson is the Fall 2015 Gamelin Endowed Postdoctoral Fellow as a member of the New Challenges in PDE program.

Bobby received his undergraduate degree from Morehouse College in 2010. In June of 2015, he obtained his Ph.D. from the University of Chicago under the supervision of Wilhelm Schlag. Starting in January of 2016, Bobby will be a Moore instructor at MIT.



Bobby Wilson

Generally, Bobby is interested in the dynamics of dispersive PDEs and geometric measure theory. In particular, he has demonstrated frequency stability of periodic solutions to the multidimensional, periodic nonlinear Schrödinger equation with a general polynomial nonlinearity. The existence of this type of stability is interesting, considering the demonstration of frequency instability of the origin in this regime by many mathematicians beginning with Colliander, Keel, Staffilani, Takaoka, and Tao in 2008.

The Gamelin postdoctoral fellowship was created in 2014 by Dr. Ted Gamelin, Emeritus Professor of the UCLA Department of Mathematics. The Gamelin Fellowship emphasizes the important role that research mathematicians play in the discourse of K-12 education.

NAMC Kicks Off New Mentor/Partner Program

Diana White, Brandy Wieggers, and Anna Lane

“It’s easy to say that you want to help children learn mathematics and problem solving in a different way. The workshop is practically a complete bridge between talking about it and doing it.”

— MC-MAP Participant

The National Association of Math Circles (NAMC) recently launched its new Math Circle–Mentorship and Partnership (MC-MAP) Program. This is a year-long program to train novice Math Circles that we hope to repeat each year with a new cohort.

The program kicked off with an intensive three-day Math Circle training workshop in mid-September at the University of Colorado Denver (CU Denver). A group of 39 mathematicians and educators gathered from around the country to participate in the workshop, which trained 14 novice Math Circle leadership teams on the academic and administrative components of leading Math Circles.

Then, to give the novice teams hands-on training with facilitating questioning and exploration, NAMC and CU Denver co-sponsored the first Julia Robinson Math Festival held in Colorado. Throughout the festival—which brought together 324 students from six schools across the greater Denver metro area, as well as one school from Steamboat Springs—students explored mathematical problems under the guidance of the novice leadership teams as well as 39 faculty members, graduate students, and undergraduates from CU Denver. Workshop participants found that working with the middle school students was helpful for their development as Math Circle leaders.

In addition to the festival, MC-MAP participants also participated in a new Math Circle observation experience. Math Circle novice leaders observed a live stream of one of the monthly Rocky Mountain Math Teachers’ Circle sessions. Longtime NAMC contributors and outreach specialists Josh Zucker and Dr. Amanda Serenevy each led half the session, providing two different styles of problems and various facilitation techniques for the participants to observe. One participant noted the “great opportunity” this provided: “We almost never get to sit outside a learning environment to watch, learn, and discuss.”

Complementing these hands-on training experiences, participants spent time planning with their leadership teams, receiving feedback, and learning about resources available to them from NAMC and other organizations whose missions overlap with ours. This included the newly developed NAMC Math Circle Handbook.

As the MC-MAP program continues to grow, we will continue to provide support throughout the academic year through a listserv and monthly webinars, as well as by having mentors and novice Math Circles visit each other to exchange ideas. 

Call for Membership

MSRI invites membership applications for the 2016–2017 academic year in these positions:

Research Members by December 1, 2015

Postdoctoral Fellows by December 1, 2015

In the academic year 2016–2017, the research programs are:

Geometric Group Theory, Aug 15–Dec 16, 2016

Organized by Ian Agol, Mladen Bestvina, Cornelia Drutu, Mark Feighn, Michah Sageev, Karen Vogtmann

Analytic Number Theory, Jan 17–May 26, 2017

Organized by Chantal David, Andrew Granville, Emmanuel Kowalski, Philippe Michel, Kannan Soundararajan, Terence Tao

Harmonic Analysis, Jan 17–May 26, 2017

Organized by Michael Christ, Allan Greenleaf, Steven Hofmann, Michael Lacey, Svitlana Mayboroda, Betsy Stovall, Brian Street

MSRI uses **MathJobs** to process applications for its positions. Interested candidates must apply online at www.mathjobs.org. For more information about any of the programs, please see www.msri.org/scientific/programs.

Focus on the Scientist: Luis Vega



Luis Vega

Luis Vega is a member of the New Challenges in PDE program. He is the scientific director of the Basque Center for Applied Mathematics (BCAM), and holds a professor position at the University of the Basque Country UPV/EHU.

Luis's first contribution to mathematics came in the field of harmonic analysis. As a student of Antonio Cordoba at the Universidad Autónoma de Madrid, Luis was amongst the

first generations of Spanish mathematicians obtaining their Ph.D.s in Spain and later becoming leaders in their fields of research. His work on harmonic analysis had several landmarks, starting from his first paper on the maximal Schrödinger operator, and including successful collaborations with Adela Moyua, Ana Vargas, Terence Tao, and others. Tao, Vargas, and Vega made one of the major advances on the restriction conjecture by developing the “bilinear machinery” and using it effectively to obtain new linear estimates. The restriction conjecture remains one of the most important open problems in modern harmonic analysis.

Luis's harmonic analysis background influenced his later work in partial differential equations. In fact, his main contribution there was in the class of nonlinear dispersive PDE, whose analysis has been invigorated since the late eighties by deep input from harmonic analysis. Soon after meeting in Chicago in 1988, Luis started, with Carlos Kenig and Gustavo Ponce, one of the most successful and influential collaborations in dispersive PDE. They have produced more than forty celebrated works

to date, on topics ranging from low-regularity well-posedness of semilinear dispersive PDE to quasilinear Schrödinger equations. More recently, the Kenig–Ponce–Vega trio developed, in collaboration with Escauriaza, a robust approach to Hardy uncertainty principles, and used it to obtain new unique continuation results for the Schrödinger equation, even in non-homogeneous settings (as in the presence of a time-dependent potential).

Luis's other important contributions include his work with Frank Merle on minimal blowup solutions and profile decomposition for L^2 -critical nonlinear Schrödinger equations, as well as his work with Perthame on the Helmholtz equation. Most recently, Luis pioneered, with Valeria Banica, a very nice direction of research investigating the dynamics of vortex filaments with corners using dispersive PDE tools. Luis is continuing this direction of research as a mathematician *and* a scientist, performing theoretical, numerical, and experimental investigations.

In addition to his research portfolio, Luis has supervised several successful graduate students and postdocs, including Nicola Visciglia, Luca Fanelli, and Albert Mas. Throughout his career, he has held several visiting professor positions and earned many awards and honors, including the Pascal Medal from the European Academy of Sciences in 2015. He is a fellow of the American Mathematical Society, a member of the European and Spanish Academies of Science, and was an invited speaker at the 2006 ICM. Recently, Luis received two very prestigious grants, namely the senior ERC grant from the European Research Council and the Spanish Severo Ochoa grant.

Talking to Luis, one directly feels his high level of energy and enthusiasm for research. He has high ambitions for mathematics in Spain, and particularly in the Basque Country, where he sees his position at the BCAM institute as a great opportunity to promote mathematics as a fundamental tool for scientific progress.

— Zaher Hani

Call for Proposals

All proposals can be submitted to the Director or Deputy Director or any member of the [Scientific Advisory Committee](#) with a copy to proposals@msri.org. For detailed information, please see the website www.msri.org.

Thematic Programs

The Scientific Advisory Committee (SAC) of the Institute meets in January and November each year to consider pre-proposals for programs. Proposals for special events or conferences outside the programs are considered in a much shorter time frame. The deadlines to submit proposals of any kind for review by the SAC are **October 15** and **December 15**. Successful proposals are usually developed from the pre-proposal in a collaborative process between the proposers, the Directorate, and the SAC, and may be considered at more than one meeting of the SAC before selection. For complete details, see <http://tinyurl.com/msri-progprop>.

Hot Topics Workshops

Each year MSRI runs a week-long workshop on some area of intense mathematical activity chosen the previous fall. To be considered for the spring or fall of year $n+1$, a proposal should be received by **October 15** of year n . See <http://tinyurl.com/msri-htw>.

Summer Graduate Schools

Every summer MSRI organizes several 2-week long summer graduate workshops, most of which are held at MSRI. To be considered for the summer of year $n+1$, proposals must be submitted by October 15 of year $n-1$. See <http://tinyurl.com/msri-sgs>.

Congratulations to the New MAA President-Elect

We are happy to extend our warm congratulations to Professor Deanna Haunsperger, who has recently been elected President-Elect of the Mathematical Association of America. After one year in this role, Professor Haunsperger's term as president will be effective for two years. We are very fortunate to have Deanna on our Human Resources Advisory Committee, which she joined in 2014. In 2015, she was elected, along with Professor Lloyd Douglas, co-chair of this committee.

Deanna is a professor of mathematics at Carleton College in Minnesota, where she has taught since 1994. She received her B.A. from Simpson College, and M.A. and Ph.D. from Northwestern University. Since her own undergraduate days, Deanna has been interested in increasing the number of students who pursue advanced degrees in mathematics, in particular those from underrepresented groups. That passion has guided her as a former co-editor for *Math Horizons* (the MAA's magazine for undergraduates) and as co-founder and co-director of Carleton's Summer Mathematics Program for Women, which was named by the American Mathematical Society as one of the two 2014 Mathematics Programs that Make a Difference. She received the 2012 M. Gweneth Humphreys Award for Mentoring Undergraduate Women from the Association for Women in Mathematics. ♀



Deanna Haunsperger

Named Positions for Fall 2015

Eisenbud and Simons Professors

Sandra Cerrai, University of Maryland
Arnaud Debussche, École Normale Supérieure de Rennes
Martin Hairer, University of Warwick
Herbert Koch, Rheinische Friedrich-Wilhelms-Universität Bonn
Jonathan Mattingly, Duke University
Andrea Nahmod, University of Massachusetts, Amherst
Natasa Pavlovic, University of Texas
Jeremy Quastel, University of Toronto
Gigliola Staffilani, Massachusetts Institute of Technology
Luis Vega, University of the Basque Country UPV/EHU

Named Postdoctoral Fellows

Viterbi: Dana Mendelson, University of Chicago (from Sep 2016)
Berlekamp: Georg Menz, University of California, Los Angeles
Gamelin: Bobby Wilson, Massachusetts Institute of Technology

MSRI is grateful for the generous support that comes from endowments and annual gifts that support faculty and postdoc members of its programs each semester.

Chancellor's Professor

Pierre Raphaël, Université Nice Sophia Antipolis

The UC Berkeley Chancellor's Professorship award is open to nominees from MSRI only. Chancellor's Professors are top researchers known for excellent teaching.

Clay Senior Scholars

Martin Hairer, University of Warwick
Pierre Raphaël, Université Nice Sophia Antipolis

The Clay Mathematics Institute awards its Senior Scholar awards to support established mathematicians to play a leading role in a topical program at an institute or university away from their home institution.

Viterbi Postdoc



Dana Mendelson

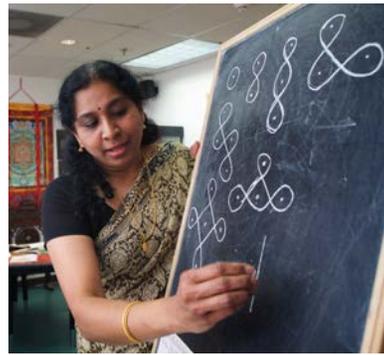
Dana Mendelson is the Fall 2015 Viterbi Endowed Postdoctoral Fellow as a member of the New Challenges in PDE program.

Dana did her undergraduate studies at McGill University in Montreal, Canada. In the fall of 2010, she began her doctoral studies at MIT, which she completed under the supervision of Gigliola Staffilani in June 2015. Following her appointment at

MSRI, Dana will be a member of the Institute for Advanced Study for the Spring 2016 semester, and an L.E. Dickson Instructor at the University of Chicago.

Dana's research has focused on the study of the long-time behavior and qualitative dynamics of solutions to nonlinear dispersive and Hamiltonian equations, particularly via probabilistic techniques. She has worked on questions about the almost sure global existence and asymptotic behavior of solutions to supercritical nonlinear dispersive equations. More recently, she has also been working on problems concerning symplectic non-squeezing for Hamiltonian PDEs, which are examples of infinite dimensional symplectic flows.

The Viterbi Endowed Postdoctoral Scholarship is funded by a generous endowment from Dr. Andrew Viterbi, well known as the co-inventor of Code Division Multiple Access (CDMA) based digital cellular technology and the Viterbi decoding algorithm, used in many digital communication systems.



First National Math Festival Draws Over 20,000 Attendees

The first ever National Math Festival drew more than 20,000 children and adults to the Smithsonian Museums in Washington, DC, on Saturday, April 18, 2015, for lectures, games, puzzles, demos, performances, and more.

Public Celebration of Math

As part of the free public event, festival-goers rode the *Square-Wheeled Tricycle* and walked Maze Mats, courtesy of the National Museum of Mathematics (MoMath). They adorned themselves with wearable mathematics: geometric balloon hats ([see the photo on page 1](#)) and folded polyhedra bracelets, necklaces, and earrings, thanks to the Bridges Organization. They strove to “beat the house” with solved games, tied and untied knots, and watched oobleck dance to music, all courtesy of Guerilla Science.

Festival-goers also played early learning math games with Gracie & Friends thanks to First 8 Studios WGBH, explored math research data trends with the help of NOVA Labs, and built

“It’s difficult to foresee where the next big and small wave of technological innovation will come from, but what we know is that it will spring out of an environment... where fundamental research in math, and more generally, science, is the highest priority of program design.”

— Mario Draghi

a Hyperbolic Star with the help of Zome-tool. They played the board game Amazons, built structures out of Space Chips, and solved puzzles thanks to the Elwyn and Jennifer Berlekamp Foundation.

And they queued for lectures: about firefly synchronicity, cosmic shadows, drag racing, tiny sea creature mobility, Rubik’s cubes, and many, many more subjects! Several Smithsonian museums contributed programs as well.

Policy Points

In addition to the free public festival, the event also featured a congressional briefing on the importance of teacher preparation in mathematics. Senators Lamar Alexander and Patty Murray (chair and ranking member of the Education Committee), Harry Reid, Charles Schumer, and Al Franken, as well as House Minority Leader Nancy Pelosi, all made remarks at the briefing.

There was also a math education forum on college and career readiness standards, co-sponsored with the Council of Chief State School Officers (CCSSO). The forum featured William McCallum (one of the lead writers for the Common Core State Standards), Deborah Ball (University of Michigan), as well as other math educators.



From left to right: Keynote speaker Mario Draghi, President of the European Central Bank, speaking with Jim Simons, Chair, Simons Foundation; Sen. Charles Schumer in conversation with Rep. Steny Hoyer during the congressional briefing; MSRI Director David Eisenbud welcomes House Minority Leader Nancy Pelosi to the congressional briefing.



“...We need to train amazing young minds, we need to fund the best ideas—because, at least so far, no one has invented any system that has had a greater impact on human welfare.”

— Eric Lander

A gala dinner in the Great Hall of the Library of Congress’s Thomas Jefferson Building spotlighted basic science research. Keynote speakers were former co-head of the Human Genome Project Eric Lander and President of the European Central Bank Mario Draghi.

The National Math Festival will run every two years, so it will return to Washington in April 2017. The dates and venue will be announced in the spring of 2016. Stay tuned! 🍌

More photos and videos of the Festival can be found at mathfest.org. Videos of selected speeches are posted at msri.org/mathfestival.

The National Math Festival was organized by MSRI and the Institute for Advanced Study with the generous support of: Carnegie Corporation of New York; Google; Howard Hughes Medical Institute; Simons Foundation; The Charles and Lisa Simonyi Fund for Arts and Sciences; Alfred P. Sloan Foundation; The Kavli Foundation; Gordon and Betty Moore Foundation; Research Corporation for Science Advancement; and IBM.

Mathical

Books for Kids from Tots to Teens

Inspiring youth of all ages to cultivate a love of mathematics in the world around them



First Mathical Awards. Also in April, MSRI and the Children’s Book Council announced the first winners of the new Mathical youth book prize. The prize honors the most inspiring math-related fiction and nonfiction books for readers in grades Pre-K–12: mathicalbooks.org.

New Challenges in PDE: Determinism and Randomness

(continued from page 1)

many fundamental questions remain open in both fields, and some of these questions are being explored in a deep and synergetic fashion this fall at MSRI.

What is Dispersive PDE?

In the physical sciences, dispersion represents the phenomenon of waves with different frequencies (or light of different colors) traveling with different speeds. A familiar manifestation of dispersion is the splitting of colors in water drops to produce a rainbow. This also occurs in a glass prism, which was used by Isaac Newton to first explain the rainbow. Water waves give us another example. From [the image on the cover](#), one can clearly see that waves with short wavelength propagate faster than those with longer wavelength.

Dispersive equations are ubiquitous in nature: dispersion is the norm for waves in materials — light in vacuum is exceptional for traveling at a single universal speed. Dispersive equations model phenomena spanning from linear and nonlinear optics to Bose–Einstein condensates, Heisenberg ferromagnets, relativistic electrons, magnetism in solids, and water waves. Correspondingly, solutions to these equations exhibit a wealth of behaviors ranging from asymptotically decaying solutions that can be modeled in the distant future by the underlying linear flow, to solitons (non-decaying solutions that maintain their profiles for all times), to solutions that blow up in finite time. Note that finite-time blowup can occur despite the presence of conservation laws for these equations. In this setting, the image that blowup should conjure is that of a Big Crunch, rather than a Big Bang — the solutions concentrate most of their mass on ever smaller sets; for an example, see the figure showing the pseudo-conformal ground state in the next column.

Constant coefficient linear dispersive equations can be understood via Fourier analysis. This leads one to consider solutions with characters $e^{i\xi \cdot x}$ as initial data, which in turn lead to solutions in the form of plane waves:

$$u(t, x) = \exp(i x \cdot \xi - it\omega(\xi)) = \exp(i\xi \cdot [x - t \frac{\omega(\xi)\xi}{|\xi|^2}]).$$

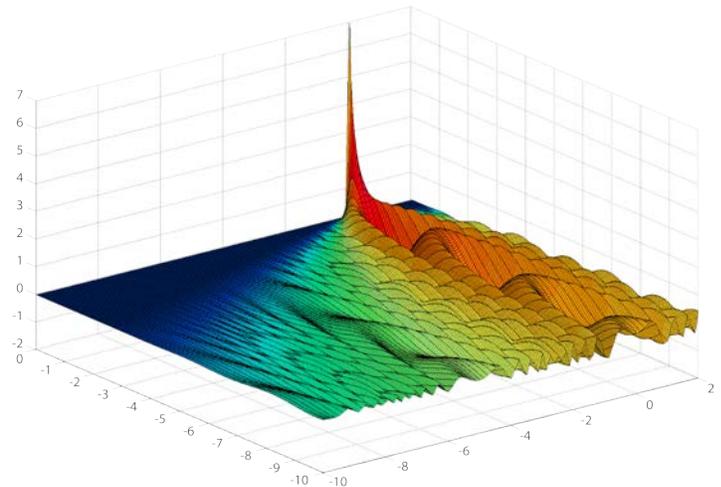
The function $\omega(\xi)$ is called the dispersion relation and dictates how the speed of the wave varies with the wavenumber ξ . Note that there are two natural notions of velocity: The velocity of the plane wave, called the phase velocity, is given by $\frac{\omega(\xi)\xi}{|\xi|^2}$. The group velocity is given by $\nabla\omega(\xi)$, and it represents the velocity of the amplitude of a localized wave packet with characteristic wavenumber ξ .

Famous examples of dispersive equations include the Korteweg–de Vries equation, for which the dispersion relation is given by $\omega(\xi) = -\xi^3$; the Schrödinger equation, for which $\omega(\xi) = \frac{1}{2}|\xi|^2$; and the wave equation, for which $\omega(\xi) = |\xi|$. The careful reader will notice that according to the definition given above, the wave

equation is not truly dispersive; indeed, all waves travel with unit speed, albeit in different directions.

This brings us to the following empirical definition: the field of dispersive equations is delineated by its common body of techniques. In this sense, this field has expanded dramatically in recent years, spilling into geometric partial differential equations (such as wave maps, Maxwell–Klein–Gordon, Yang–Mills, and general relativity) and into fluid models (such as water waves and the Euler–Maxwell system).

Much of the field’s advance has been driven by a successful synthesis of classical PDE techniques with the tools of the Calderon–Zygmund school of harmonic analysis. Work on Stein’s restriction conjecture has played a central role: Strichartz inequalities are used as the basic expression of dispersion; they say that linear solutions disperse rapidly enough that high powers are integrable in space-time. Moreover, the recent development of bilinear and multilinear analogues provides the simplest, most compact expression of the fact that waves with different wavenumbers can have little interaction. This is very intuitive: such waves travel with different speeds and so spend little time in one another’s presence.



The Big Crunch: the pseudo-conformal ground state concentrates all its mass at the origin.

Some Goals in Dispersive PDE

As noted above, many dispersive equations arise from physical theories. A basic requirement for the verifiability of a physical theory is that it be well-posed. A problem is called well-posed if it has a solution, the solution is unique, and the solution depends continuously on the initial data. Several outstanding problems in this field are of this nature, such as well-posedness of the Yang–Mills equation and that of the Landau–Lifshitz model of ferromagnetism (also known as Schrödinger maps).

Even the well-posedness of very simple looking equations can pose a formidable challenge. One example is the defocusing septic wave equation in three space dimensions:

$$\partial_{tt}u - \Delta u + u^7 = 0 \quad \text{for } (t, x) \in \mathbb{R} \times \mathbb{R}^3.$$

Focus on the Scientist: Martin Hairer



Martin Hairer

Martin Hairer is spending two months at MSRI as Clay Professor during the New Challenges in PDE: Deterministic Dynamics and Randomness in High and Infinite Dimensional Systems program.

Martin's work is mostly concerned with stochastic (partial) differential equations arising in statistical physics, the analysis of algorithms, or quantum field theory. After obtaining his Ph.D. in the Physics Department of the University of Geneva under the supervision of Jean-Pierre Eckmann in 2001, he moved to the University of Warwick where he has been since, interrupted only by one year as Associate Professor at NYU in 2009. Currently he is Regius Professor of Mathematics in Warwick. It is very difficult to keep track of the large number of honors and awards Martin has received; most prominently, he was awarded the Fields medal in 2014.

Martin has worked on profound existence and uniqueness problems for stochastic partial differential equations driven by a very irregular stochastic forcing. These equations had previously been studied in the physics literature via formal expansions organized

in Feynman diagrams. In his theory of "Regularity Structures," Martin developed a new notion of regularity, which is based on these expansions and which allowed him to define and rigorously construct solutions. This solved several longstanding open questions, including the construction of solutions to the KPZ equation and the construction of a reversible dynamics for the Euclidean Φ_3^4 quantum field theory.

Earlier important contributions concerned the uniqueness of invariant measures for stochastically forced Navier–Stokes equations and infinite dimensional stochastic systems in general. It had been known previously that solutions to the two dimensional stochastic Navier–Stokes system equilibrate in a statistically invariant state in the long time limit, but the uniqueness of these invariant states was an important open question. In joint work with Jonathan Mattingly, he was able to establish this uniqueness, even in the most difficult case where the noise only acts on a small spectrum of frequencies, thereby developing methods that are among the most powerful to study ergodicity of infinite dimensional stochastic systems.

Besides his mathematical work, Martin is an excellent computer programmer. Based on a project he developed for a student science contest in Switzerland, he developed the sound editing software Amadeus, which he commercializes under the name HairerSoft.

— Hendrik Weber

Just like the famous Navier–Stokes system, this equation is supercritical with respect to all known conservation laws (such as energy). Answering the global regularity question for this equation — that is, whether smooth initial data lead to smooth global solutions — would be a significant advance in the field.

More subtle properties of solutions also constitute central themes in the theory. We illustrate this with two outstanding conjectures. First, the soliton resolution conjecture asserts that generic initial data leads to solutions that resolve into a sum of solitons (traveling at different speeds) and a radiation term in the form of a solution to the underlying linear problem. Next, the asymptotic stability of black holes is a conspicuous mathematical problem of clear physical origin. It states that initial data that are a close approximation to a black hole in the Kerr family lead to a global solution to the vacuum Einstein equations that relaxes to another black hole (with possibly different parameters), at least in the observable region of space-time.

Introducing Noise: Stochastic PDE

For more than thirty years, stochastic partial differential equations (SPDE) has been a quite active area of interdisciplinary research in mathematics, at the crossroads of several of its pure and applied branches, such as stochastic analysis, partial differential equations, probability, dynamical systems, statistics, and scientific com-

puting. In addition to being a fascinating object of mathematical study, the theory and the practice of SPDE also represents a fundamental tool in applied fields, such as fluid dynamics, statistical physics, geophysics, financial modeling, super-processes, and non-linear filtering. And it is from the motivations arising from concrete problems in applications and the important contributions to their rigorous formulation and resolution that the area of SPDE has flourished so much in these years.

Stochastic partial differential equations are used to model many physical, biological, and economic systems that are affected by the influence of noise. Such a noisy influence may be intrinsic, due to inherent features of the system, or extrinsic, because of environmental influences and random user input. Moreover, SPDE arise when considering deterministic models from random initial conditions, or as tractable approximations to complex deterministic systems. In many cases the presence of noise leads to new phenomena, both at the mathematical and the phenomenological level.

The most widely studied noise in SPDE has been Gaussian white noise, motivated by the universality of the Gaussian given by the central limit theorem. But there are many types of noise which occur in nature, and most of them have been much less studied in the context of SPDE. For example, many natural systems experience sudden shocks which could be modeled by Levy noise. There is already a literature on such equations, but much more remains to be done.

PDE *(continued from previous page)*

Random perturbations of PDE are also important in order to understand how the presence of noise can affect deterministic systems. Especially in fluid dynamics, well-posedness can be a very serious issue, and well-posedness is still an open question.

In this vein, it is worth noting the well-posedness of the 3D Navier–Stokes equations, one of the millennium Clay Institute problems. The lack of well-posedness may occur both as lack of uniqueness or as creation of singularities. In both cases, adding a noise to a PDE and seeing if the corresponding SPDE shows any better behavior is of great interest. As a matter of fact, the investigation of the occurrence of regularization by noise in fluid dynamic models is still at its beginning, and only very little has been understood.

The most widely studied SPDE are of parabolic type. But on the PDE side, there has been spectacular recent progress in nonlinear dispersive equations such as the wave, Korteweg–de Vries, and Schrödinger equations. The effect of noise on such equations is largely an untouched area. For example, one of the most well studied phenomena for the Korteweg–de Vries equation is traveling waves, but the effect of noise on such waves is not well understood.

As with PDE, there are still many unsolved questions about the occurrence of blowup of solutions in SPDE, especially when we move beyond parabolic equations. If the unperturbed equation has a solution which blows up in finite time, how is such solution affected by the presence of noise?

Open Problems in SPDE

Traditionally, mathematicians have focused on proving existence and uniqueness for general classes of SPDE, but more recently their attention is shifting to the detailed study of the properties of solutions for equations associated with some specific physical model.

Nevertheless, there are still some stubborn questions remaining about existence and uniqueness of solutions for some relevant models. For example, a model for population densities is described by the equation for the superprocess, or Dawson–Watanabe process:

$$\partial_t u = \Delta u + |u|^\gamma \dot{W}(t, x),$$

where $\dot{W}(t, x)$ is multiparameter white noise, $x \in \mathbf{R}$, and $\gamma = 1/2$. If we look at $\gamma \in (0, 1)$, it was recently shown by Mytnik and Perkins, and Mueller, Mytnik, and Perkins, that uniqueness holds in the almost sure sense if $\gamma > 3/4$, but does not hold if $\gamma < 3/4$. The situation at the critical value $\gamma = 3/4$ is unknown. Since the superprocess equation is a model for population densities, it is natural to focus on nonnegative solutions. The leading problem in the area is currently: When $1/2 \leq \gamma \leq 3/4$, do we have almost sure uniqueness among nonnegative solutions of the superprocess equation?

Another important and extremely challenging model is given by the Kardar–Parisi–Zhang (KPZ) equation. The KPZ equation is an SPDE which arises naturally in the study of surface growth:

$$\partial_t u = \Delta u + |\nabla u|^2 + \dot{W}(t, x).$$

This equation can only be rigorously posed using renormalization, such as with the recent theory of regularity structures by Martin

Hairer. One of the most important questions related to KPZ is universality. The last two decades the KPZ equation has emerged as a continuum object that is believed to describe the limiting behavior of a wide class of particle systems, known as the KPZ universality class. Several such examples have been studied, but a complete theory is still lacking.

Finally, we would like to mention another area of intense activity related to the analysis of the stochastic Navier–Stokes equation:

$$\begin{aligned} \partial_t u + u \cdot \nabla u &= \nu \Delta u + \dot{F}(t, x); \\ \operatorname{div} u &= 0. \end{aligned}$$

Here, $\dot{F}(t, x)$ is a Gaussian noise that is white in time and smooth enough for the solution to exist in a reasonable function space. It is believed that fluid flows are unstable at small scales, so there is good motivation for adding noise to the Navier–Stokes equation. Much attention here has focused on the cascade of energy between different scales. One way to study this phenomenon is in terms of the invariant measure for the equation, and there are many unsolved questions about the detailed properties of this measure. 

Berlekamp Postdoc

Georg Menz, a member of the New Challenges in PDE program, is the Fall 2015 Berlekamp Endowed Postdoctoral Scholar.



Georg Menz

Georg received his Ph.D. in 2011 under the supervision of Felix Otto from the University of Bonn, Germany. Then he was a postdoc at the Max-Planck-Institute for Mathematics in the Sciences in Leipzig and became, in January 2013, Szegő Assistant Professor at Stanford University. After his stay at MSRI he will join the faculty of UCLA as an assistant professor in probability.

He is broadly interested in probability and PDE with applications in finance, physics, and materials science. His primary research interests are in equilibrium and non-equilibrium dynamics of spin systems, scaling limits, sampling of probability measures, and functional inequalities. In his Ph.D. thesis he solved, together with his former advisor Felix Otto, a conjecture on the log-Sobolev inequality for Kawasaki dynamics.

The Berlekamp Postdoctoral Fellowship was established in 2014 by a group of Elwyn Berlekamp's friends, colleagues, and former students whose lives he has touched in many ways. He is well known for his algorithms in coding theory and has made important contributions to game theory. He is also known for his love of mathematical puzzles.

Puzzles Column

Elwyn Berlekamp and Joe P. Buhler

1. Let p be a prime. Show that for every integer y in $\{1, 2, \dots, p-1\}$, there is some integer x such that $y \equiv x^x \pmod{p}$.

Comment: This problem is due to David Wilson, and the following problem was motivated by it.

2. Let p be an odd prime. Show that the mapping $f(x) \equiv x^x \pmod{p}$ from $\{1, 2, \dots, p-1\}$ to itself is not a one-to-one correspondence.

3. Find an obtuse triangle with sides of integer length and having two acute angles in the ratio 7 to 5.

Comment: We heard this from Stan Wagon, and it is due to Dick Hess, appearing in *All-Star Mathlete Puzzles*, 2009, Problem 60.

4. Let A be the 2^n by 2^n matrix where the i -th row is the n -bit binary expansion of i , for $0 \leq i < 2^n$. Let $B = AA^t$ (where A^t denotes the transpose of A) Find the largest submatrix of B consisting entirely of odd numbers. (A submatrix is obtained by intersecting a set of rows with a set of columns, and the size of a matrix is its number of entries.)

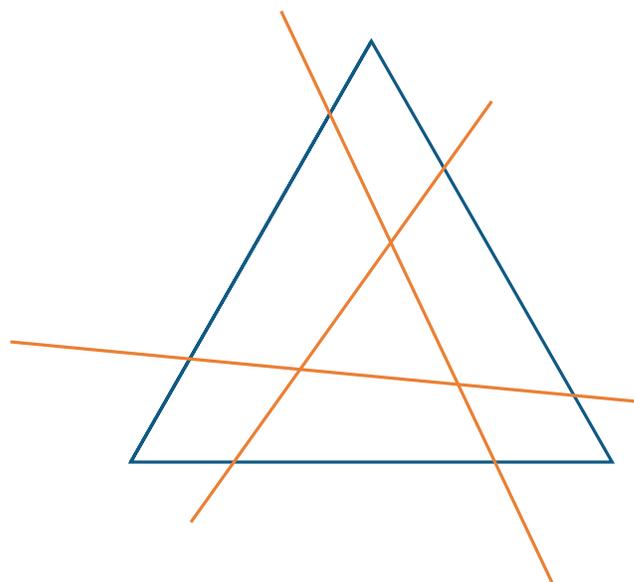
5. Prisoner A is brought into the warden's room and shown a face-up deck of 52 cards, lined up in a row in arbitrary order. She is required to interchange two cards, after which she leaves the room. The cards are then turned face down, in place.

Prisoner B is brought into the room. The warden thinks of a card, and then tells it to B (for example, "the three of clubs"). Prisoner B then sequentially turns over 26 cards. If the named card is among those turned over, the prisoners are freed immediately. Find a strategy that guarantees that the prisoners succeed. (If they fail, they must spend the rest of their lives in prison.)

Needless to say: The two prisoners have the game described to them the day before and are allowed to have a strategy session;

absolutely no communication between them is allowed on the day of the game. Notice that at no time does Prisoner A know the chosen card.

Comment: We heard this from Kiran Kedlaya, who heard it from Piotr Krason, and we do not know its ultimate source.



6. Let T be a triangle in the plane. Is it possible to draw 3 lines that partition T into 7 polygons — 6 quadrilaterals and 1 central smaller triangle (see figure) — such that all 7 polygons have the same area?

Comment: This question is due to our frequent contributor Gregory Galperin. So far, despite the elegance of the question, the only solution that we know about involves excessive algebraic exertion. 

Forthcoming Workshops

January 14–15, 2016: *Connections for Women: Differential Geometry*

January 18–22, 2016: *Introductory Workshop: Modern Riemannian Geometry*

February 10–12, 2016: *Critical Issues in Mathematics Education 2016: Observing, Evaluating and Improving Mathematics Teaching from the Early Grades through the University*

March 21–25, 2016: *Kähler Geometry, Einstein Metrics, and Generalizations*

March 28–April 1, 2016: *Hot Topics: Cluster Algebras and Wall-Crossing*

May 2–6, 2016: *Geometric Flows in Riemannian and Complex Geometry*

June 11–July 24, 2016: *MSRI-UP 2016: Sandpile Groups*

Summer Graduate Schools

May 30–June 11, 2016: *Séminaire de Mathématiques Supérieures 2016: Dynamics of Biological Systems (Canada)*

June 13–June 24, 2016: *Harmonic Analysis and Elliptic Equations on Real Euclidean Spaces and on Rough Sets*

June 20–July 01, 2016: *Mixed Integer Nonlinear Programming: Theory, Algorithms and Applications (Spain)*

July 11–22, 2016: *An Introduction to Character Theory and the McKay Conjecture*

July 18–29, 2016: *Electronic Structure Theory*

July 25–August 05, 2016: *Chip Firing and Tropical Curves*

For more information about any of these workshops, including the list of organizers for each, please see www.msri.org/scientific.



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