

Elementary Introduction to the Langlands Program. IV

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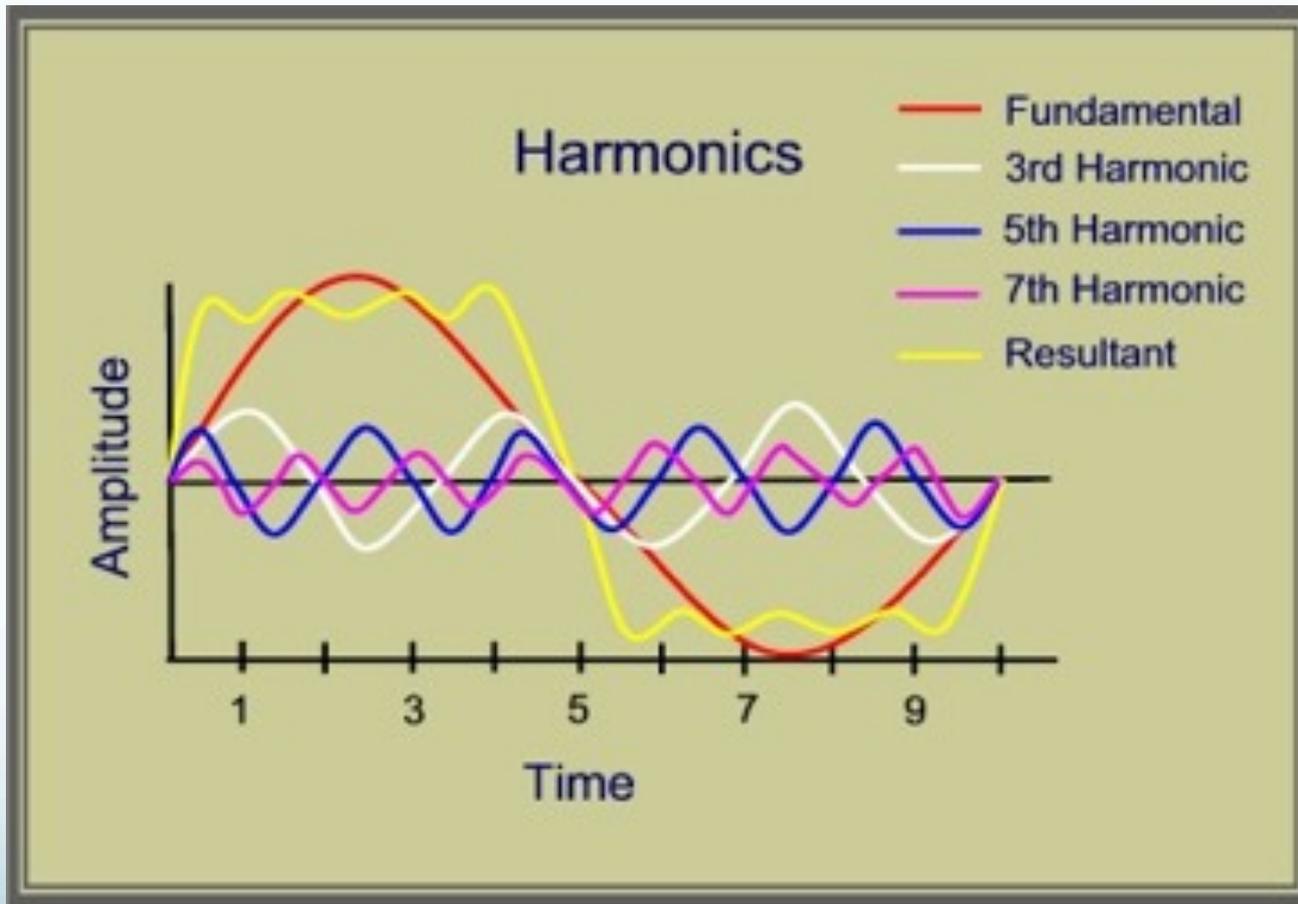


There are different continents
of Math

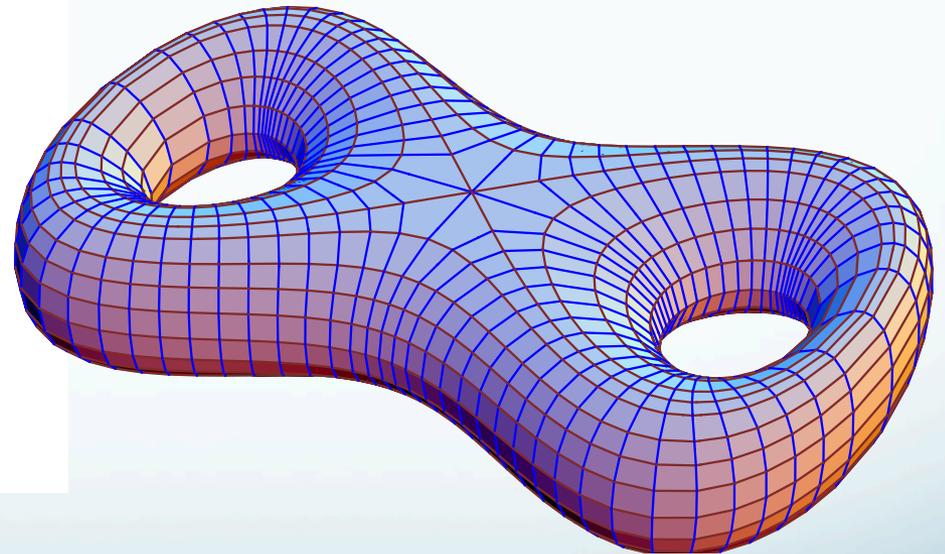
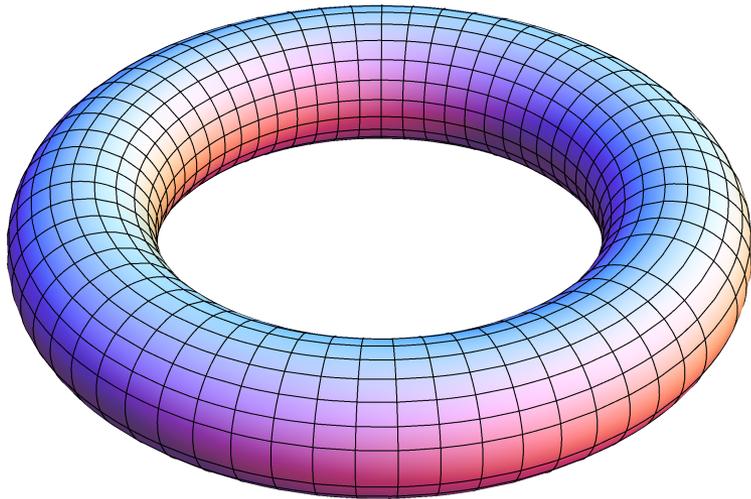
Number Theory

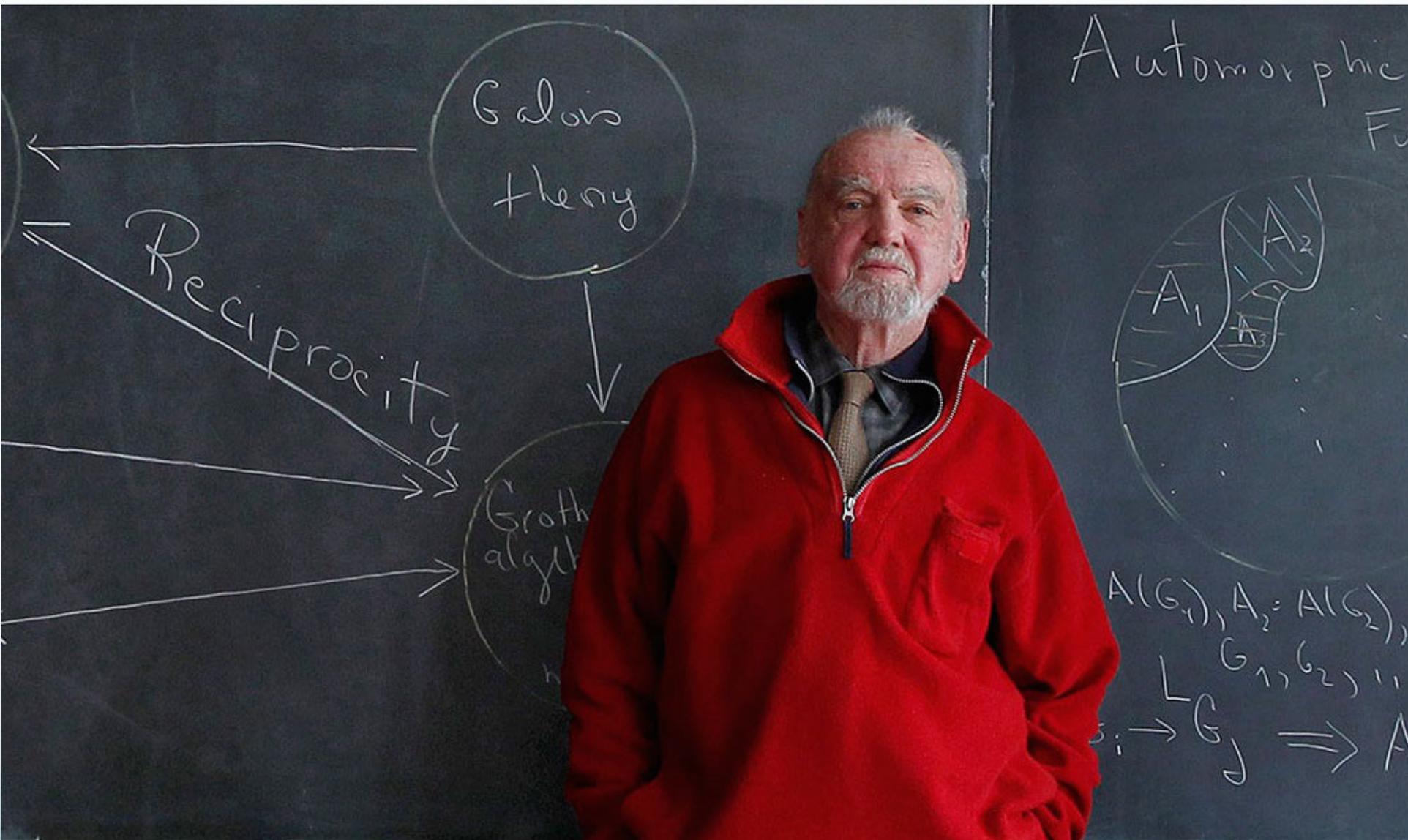


Harmonic Analysis



Geometry





Robert Langlands at his office, 2014 (photo: Toronto Star)

Professor Weil:

In response to your invitation to come and talk I wrote
the ^{enclosed} ~~following~~ letter. After I wrote it I realized there was hardly
a statement in it of which I was certain. If you are willing
to read it as pure speculation I would appreciate that; if not -
I am sure you have a waste basket handy.

Yours truly,
R Langlands

Cover page of Langlands' letter to Weil, 1967
(from the archive of the Institute for Advanced Study)

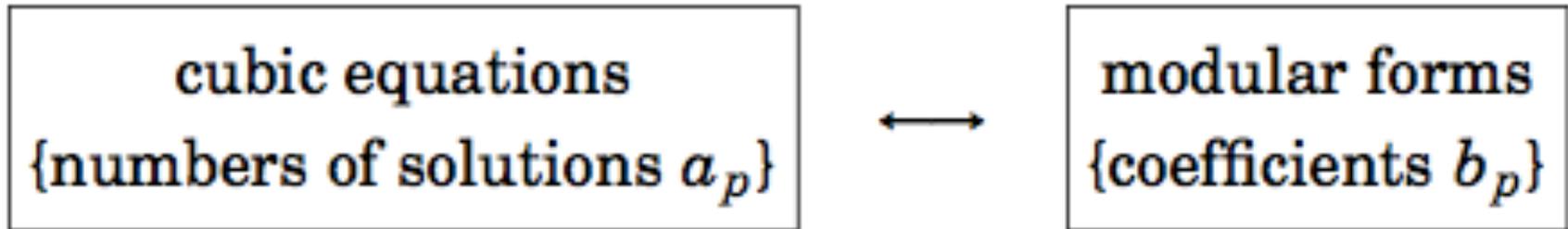
Langlands Program

- **Langlands Correspondence** — a vast generalization of the Shimura–Taniyama–Weil conjecture.
- On one side: “representations” of Galois groups
- On the other side: “automorphic functions”

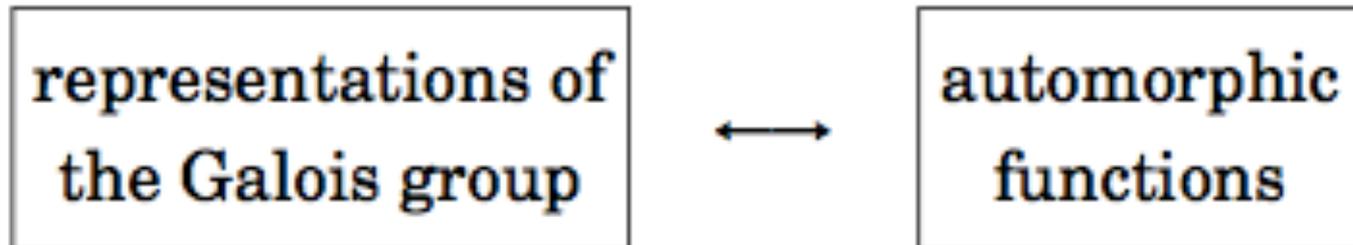
and

- **Langlands Functoriality** — conjectural relations between automorphic functions

Shimura-Taniyama-Weil Conjecture



Langlands Correspondence



Symmetry in Geometry

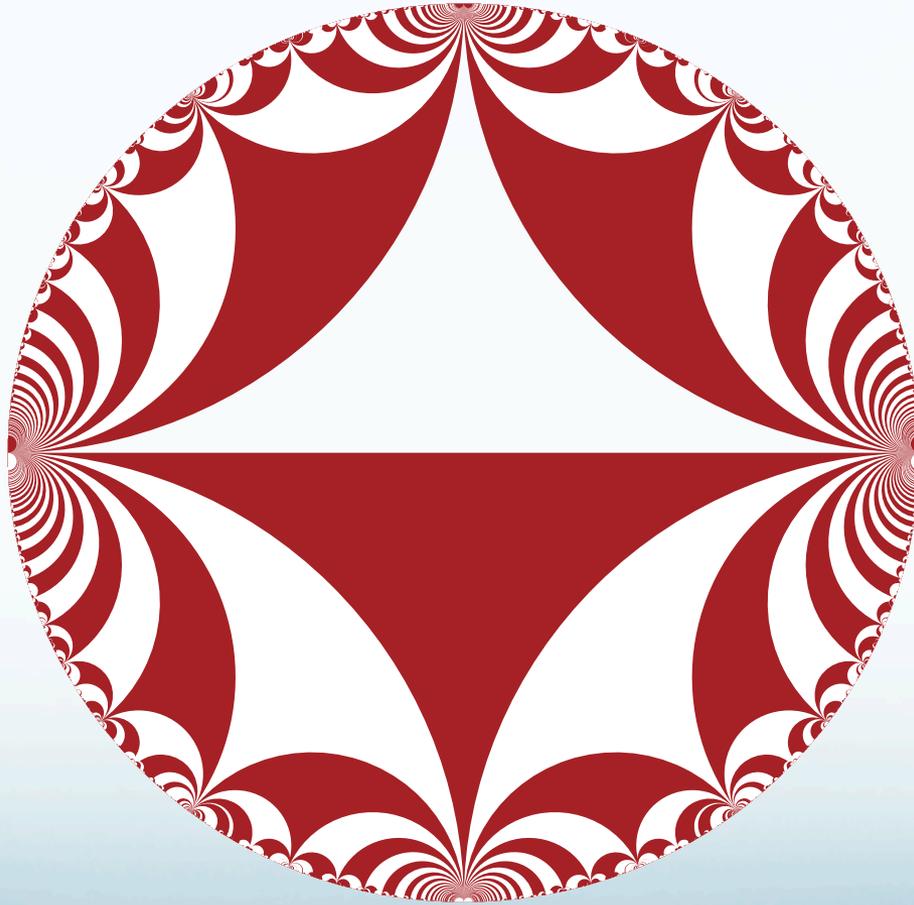


Symmetry in Number Theory

- Galois theory



Symmetry in Harmonic Analysis



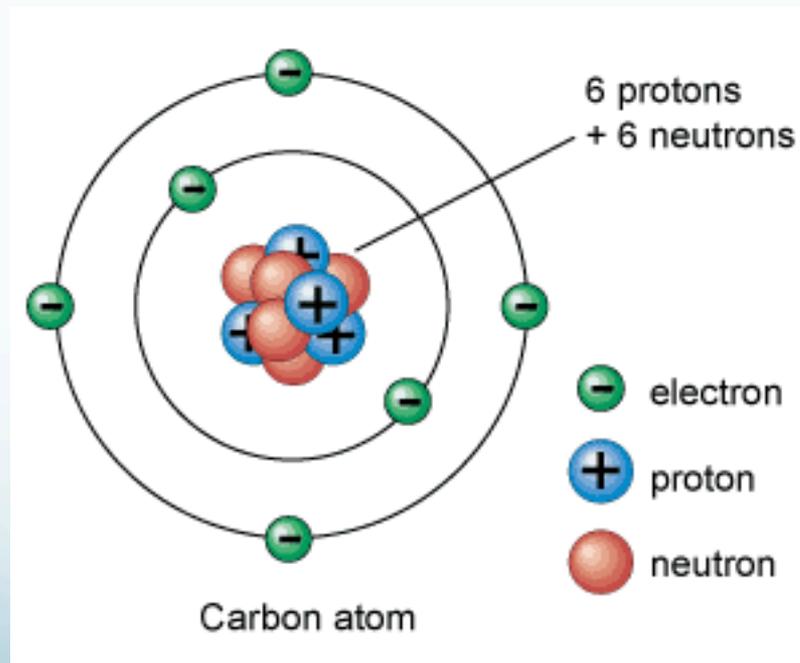
Langlands Correspondence

- Vast generalization of the Shimura–Taniyama–Weil Conjecture (whose proof implies Fermat's Last Theorem)
- A bridge between Number Theory and Harmonic Analysis
- These ideas have propagated to other continents, such as Geometry, and even to Quantum Physics

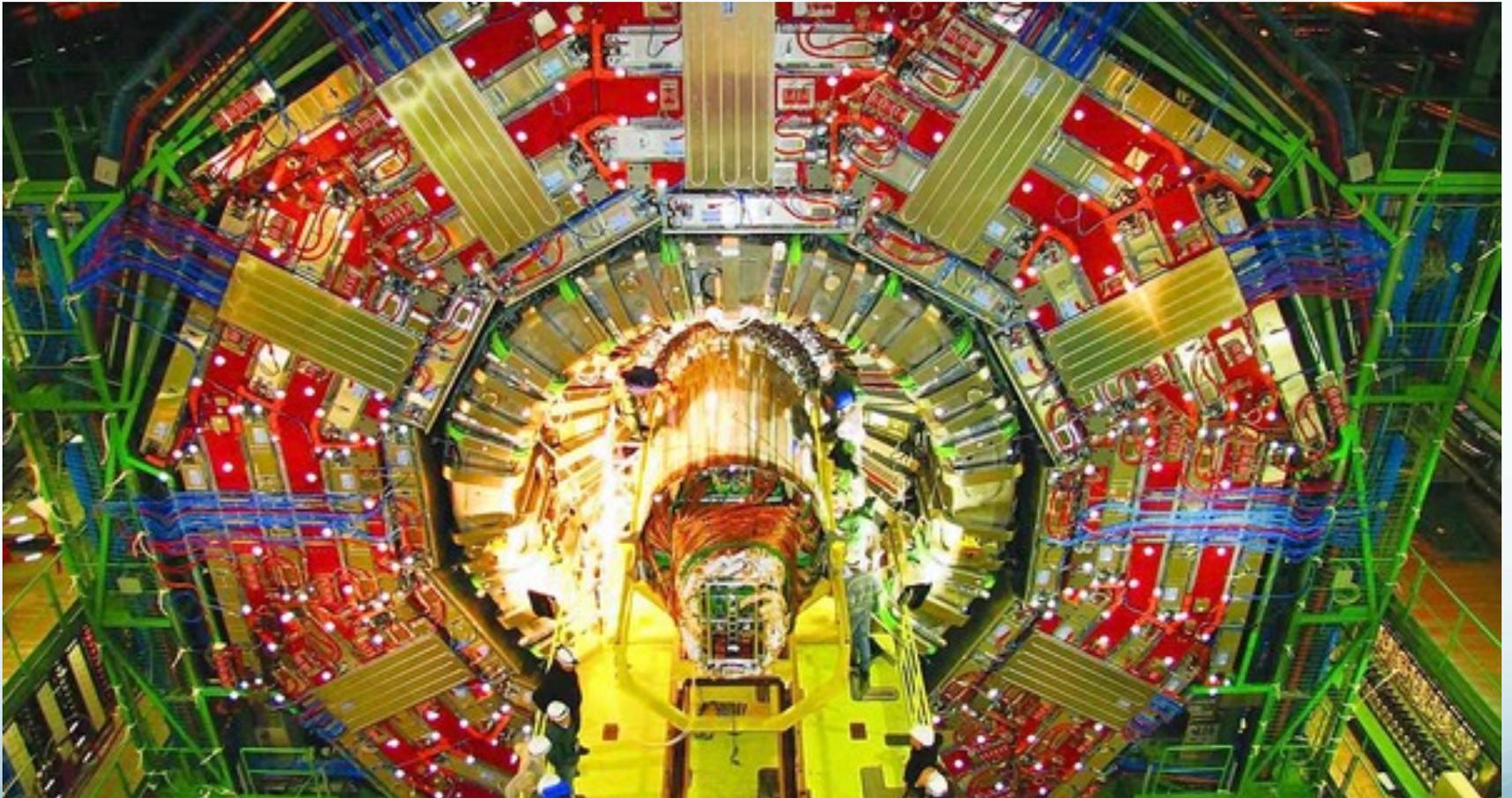
Symmetry in Quantum Physics

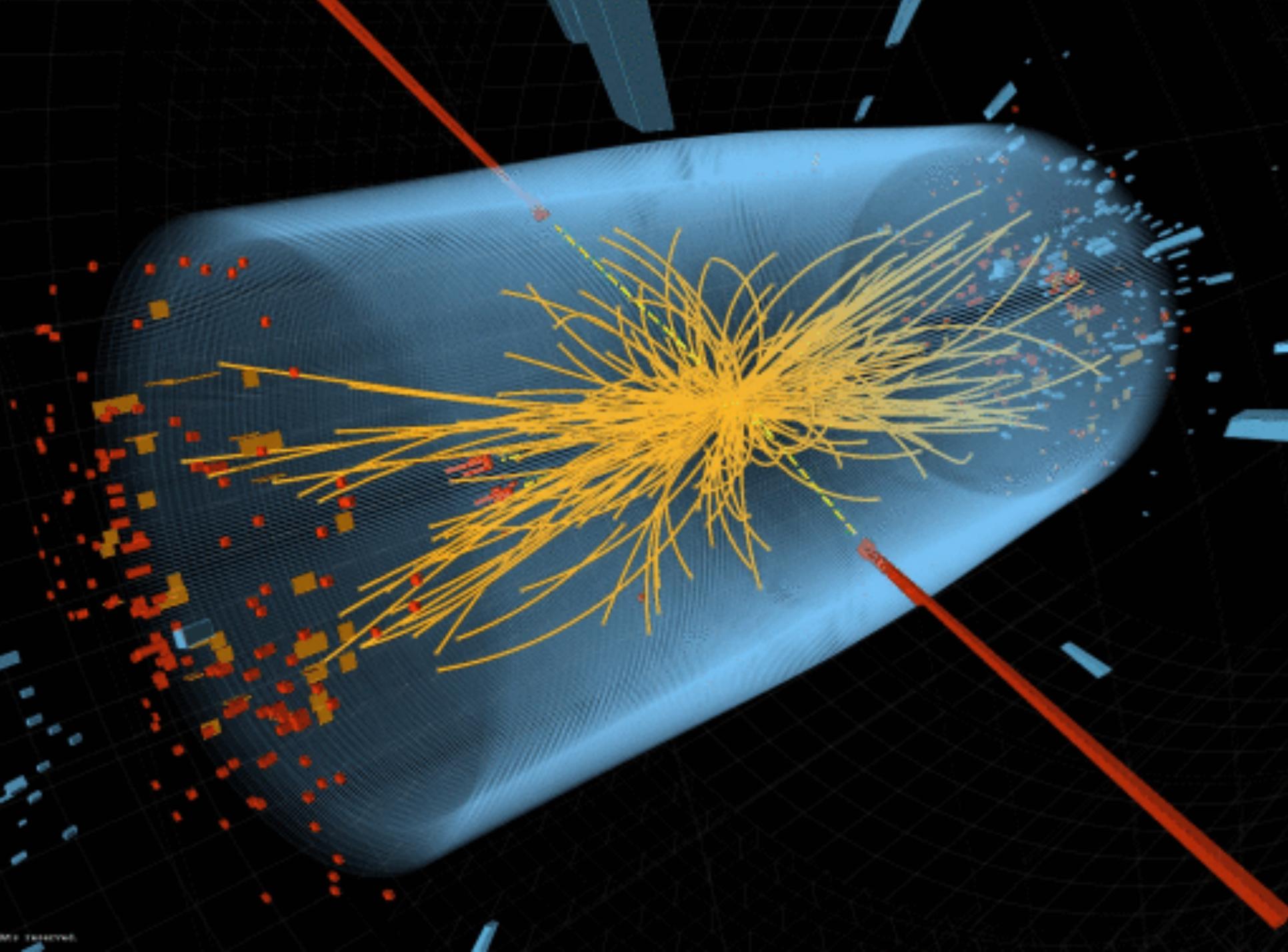
Our Universe is built from elementary particles

Each atom consists of a nucleus and electrons orbiting around it.



Large Hadron Collider





“Who ordered them?”

- In the 60s physicists discovered dozens of elementary particles called hadrons
- Quantum Physics was “turning into botany” (W. Pauli)
- Physicists desperately needed an explanation of this plentitude, which was provided by...

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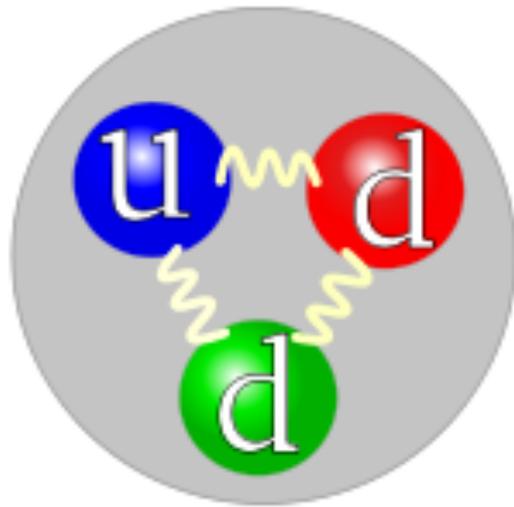
Group Theory!

Murray Gell-Mann

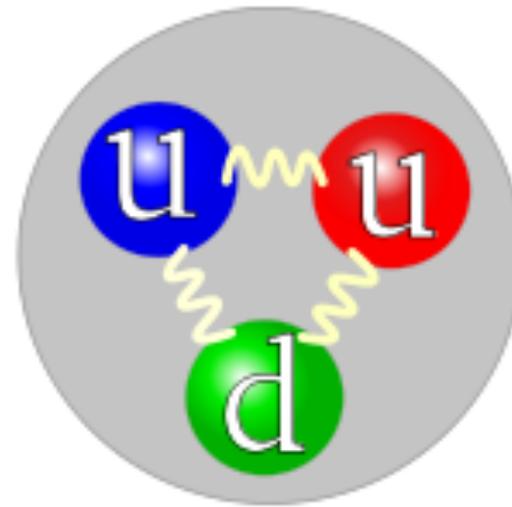
- 1961: Used the group **SU(3)** to classify hadrons
- 1964: This led him to predict the existence of **quarks**



Proton and Neutron consist of quarks



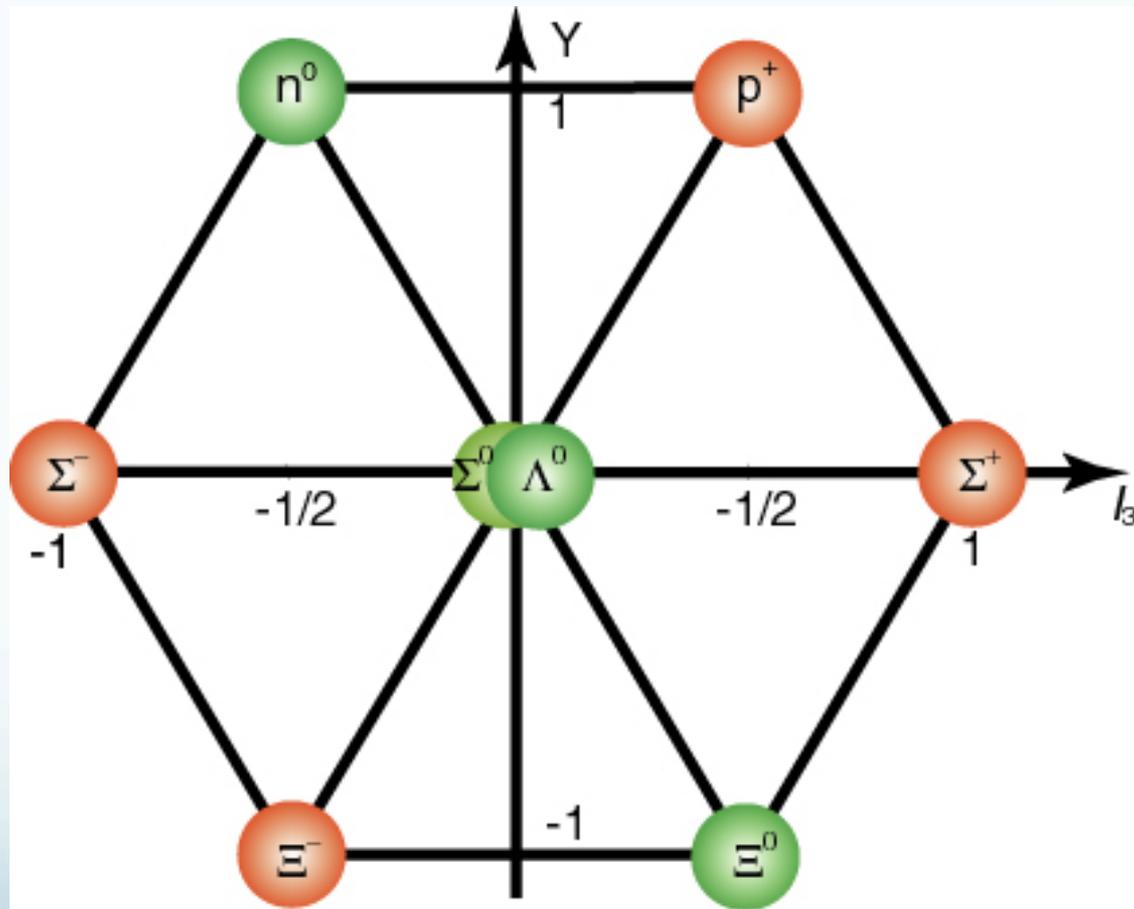
NEUTRON
Quark structure



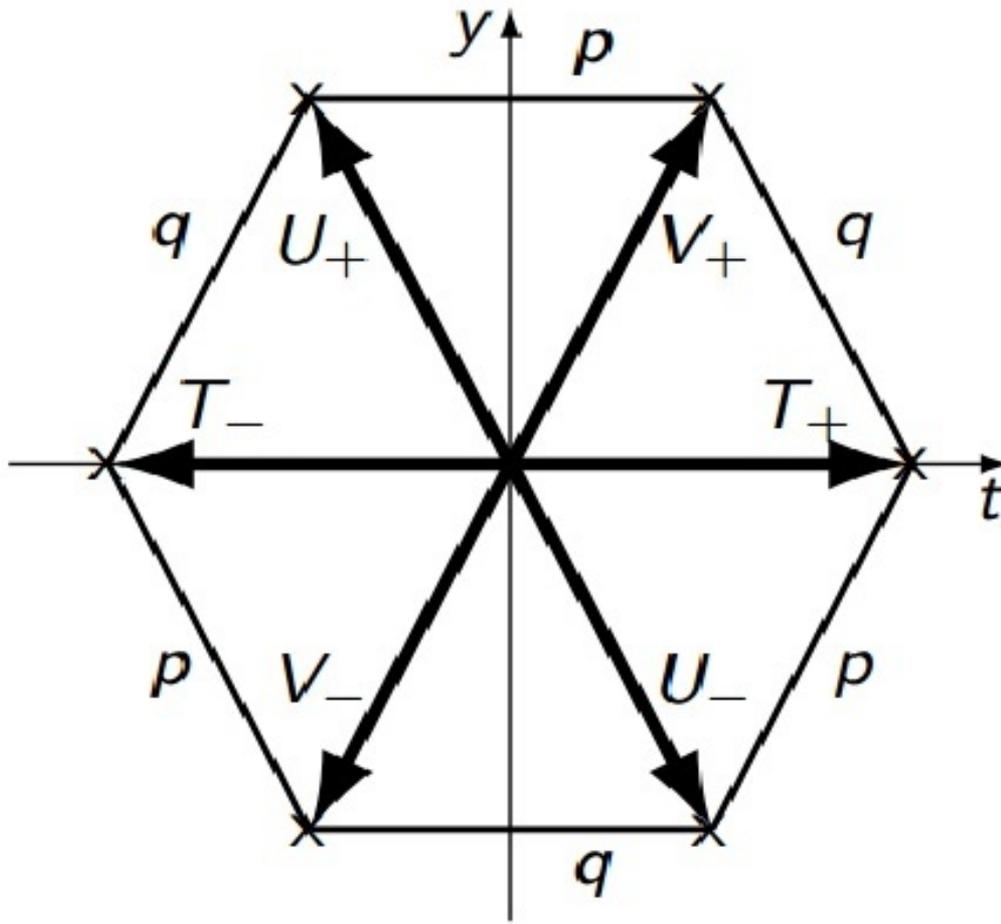
PROTON
Quark structure

- Other elementary particles (hadrons) also consist of quarks.
- Murray Gell-Mann came up with a crucial insight: hadrons can be organized into families: **octets** and **decuplets**, because they obey certain **symmetry** (namely, the group **SU(3)**). He called this “eightfold way” (referencing “noble eightfold path of Buddha”).
- Similar ideas were proposed by Kazuhiko Nishijima, Yuval Ne’eman, and George Zweig.

Example of an Octet



Representation of SU(3)



“What could be more mysterious, what could be more awe–inspiring, than to find that the structure of the physical world is intimately tied to the deep mathematical concepts, concepts which were developed out of considerations rooted only in logic and the beauty of form?”

—C.N. Yang (Nobel Prize winner)

Standard Model

- The quantum theory of all known forces of nature, except gravity. Spectacular experimental success (most recently: discovery of the Higgs boson at LHC)
- Electromagnetic, strong, and weak forces
- These are **Gauge Theories**

Gauge Theories and Lie Groups

- Electromagnetic: **U(1)** (the circle group!)
- Weak: **SU(2)**
- Strong: **SU(3)**

Electromagnetic Duality

- Maxwell's equations (in vacuum) do not change if we switch electric and magnetic fields.
- Is there an analogous duality in quantum electromagnetism?
- Yes, at least in the maximal *supersymmetric* extension of the electromagnetism (“well-balanced” theory between bosons and fermions)
- Under this duality, “electron” becomes “magnetic monopole”, and vice versa. $e \rightarrow 1/e$

Borscht



Duality in other gauge theories

- In 1970s, physicists discovered that more general supersymmetric gauge theories exhibit an analogous duality.
- But there is an important twist:
- The dual of the gauge theory with gauge group G is the gauge theory with gauge group...

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G^v — the **Langlands dual group** of G !

What's the connection between
the Langlands Program and
the Electromagnetic Duality?

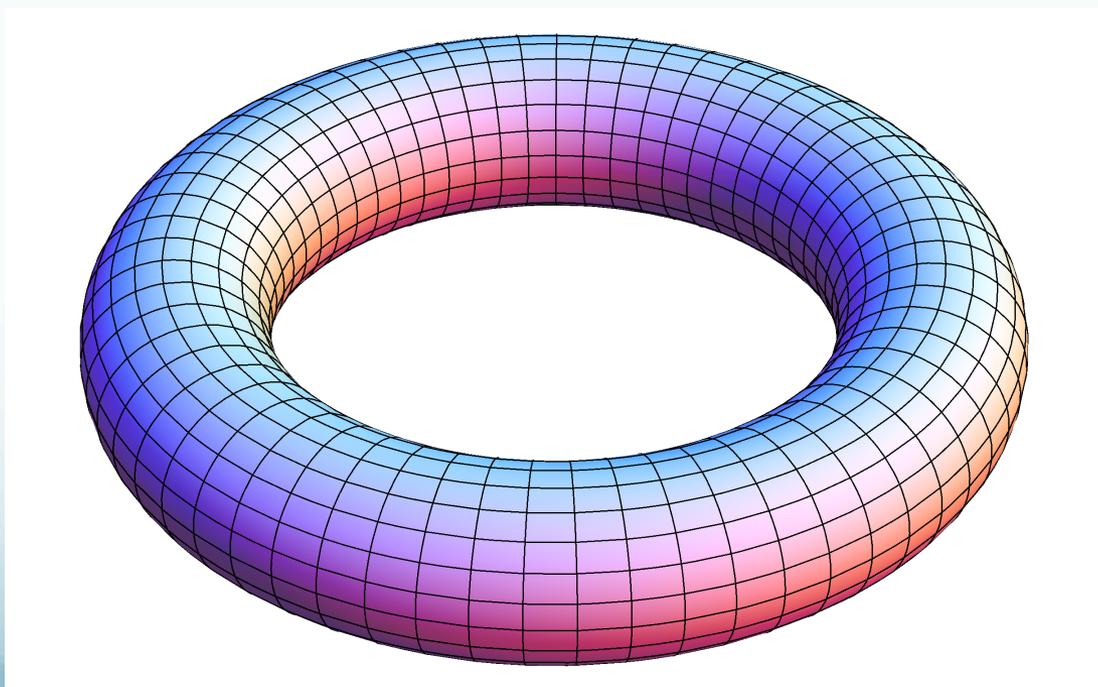
Edward Witten

- In joint works with his collaborators (including myself) established links between the Langlands Program and Electro–Magnetic duality.
- 2014: **Kyoto Prize**



There is a lot more...

In the same cubic equation, let x and y be complex numbers. Then the solutions form a torus — that's how we make a link to geometry.



“Unreasonable effectiveness of
mathematics in the natural sciences”

—**Eugene Wigner (Nobel Prize winner)**

“Mathematical concepts form an objective reality of their own, which we cannot create or change, but only perceive and describe.”

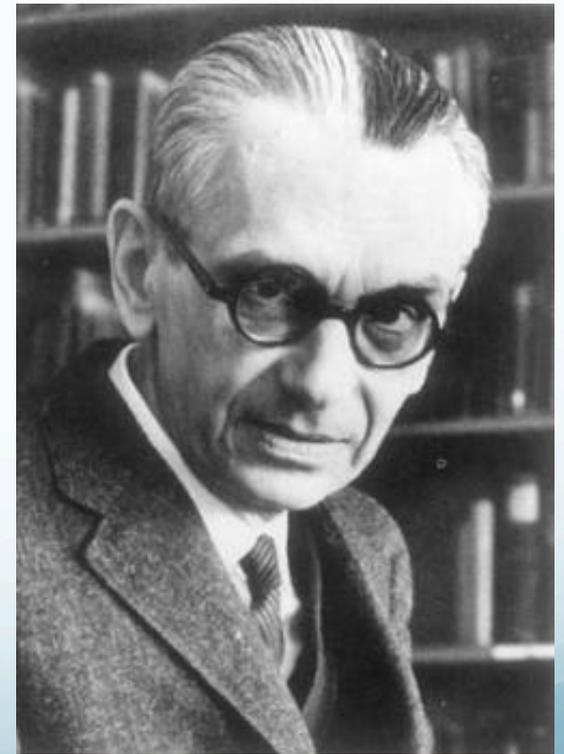
—Kurt Gödel



“Mathematical concepts form an objective reality of their own, which we cannot create or change, but only perceive and describe.”

“The human mind infinitely surpasses the powers of any finite machine.”

—Kurt Gödel



Undiscovered Ocean of Truth

“To myself I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.”

—Isaac Newton

