Take a Good Look at the 2017 National Math Festival

The National Math Festival welcomed 20,000 visitors of all ages to explore the unexpected sides of mathematics this past April 22 at the Walter E. Washington Convention Center in Washington, DC.

The free, public event offered a full-day program of presentations by some of the most fascinating mathematicians of our time as well as dozens of hands-on demonstrations, art, films, performances, puzzles, games, athletic events, children’s book readings — and more — to inspire and challenge participants to see math in new and exciting ways. These young mathematicians are building a geometric sculpture with George Hart of the Bridges Organization.

Read news, enjoy more photos, and in short, take a good look at the Festival on pages 8 and 9! 📸

A Pearl in Combinatorial Convexity —
Tverberg’s Theorem

Imre Bárány and Florian Frick

During this fall MSRI is hosting a special semester on Geometric and Topological Combinatorics. Here we briefly showcase an influential result that has been central in combinatorial convexity, on the geometric as well as on the topological side of the field: Tverberg’s theorem on intersecting convex hulls of point sets.

Intersecting Convex Hulls

Birch showed that any $3n$ points in the plane can be split into $n$ triples that span $n$ intersecting triangles. Actually $3n - 2$ points suffice if we are willing to allow one or two of the sets to be edges or points instead, whose convex hulls all share a common point.

(continued on page 11)

Geometric Functional Analysis and Applications is also starring at MSRI this semester. Read about the volume of this convex body (shown in hyperbolic form) in the GFA program article on page 5.
The View from MSRI

David Eisenbud, Director

MSRI’s Mission has three parts, roughly summarized as: basic research, developing talent, and enhancing the public understanding of mathematics. Once in a while I like to take stock: Are we really doing worthwhile things in each of these areas? You be the judge:

Basic Research

This is for us the most important of the three parts, the one without which the others wouldn’t make sense. Most semesters we have two big programs, chosen by our Scientific Advisory Committee so that some interaction is possible. This semester, Geometric and Topological Combinatorics is paired with Geometric Functional Analysis. Seen from a great distance, what they share is the study of convex bodies. On the ground, there’s a lot of interaction, with talks in one program of great interest to some members of the other. Overload danger is real — but what a rich environment!

Next semester we’ll move on to Group Representation Theory and Applications, paired with Enumerative Geometry Beyond Numbers. A sign of the success of these programs is the increasing pressure to attend, with numbers of very strong applicants well beyond what we can accommodate. Fortunately, week-long workshops within the programs can accommodate many more.

On a different subject, our Hot Topics workshop this spring will treat an advance that came from amazing progress in one area leading to the solution of a slew of the best-known problems — open for more than 40 years — in another. The areas are $p$-adic Hodge theory, known as part of high-tech number theory, and commutative algebra, usually a rather low-tech subject. I would never have guessed . . .

Developing Talent

The ferment of mathematics and the great number of the senior experts who are present give us the substrate for a postdoctoral program that nurtures young researchers. Postdocs spend the semester in an environment where the activity in their field is concentrated as in no university. Simply putting young people and senior people together, without too many responsibilities, can be good; but we’ve learned from experience that the effect is enormously enhanced by a strongly specified mentoring program. In our implementation, the program organizers pair each postdoc (and postdoc-aged member) with a senior member who will be at MSRI for the whole semester. (Sometimes the mentoring responsibility is shared by researchers who together span the semester.) Each member of the pair is given a list of interesting topics to discuss, and told that we expect at least weekly meetings, unless both make other arrangements. We interview all the postdocs — and make any necessary adjustments — to make sure the system is functioning for all of them.

Sound rigid? It did to me as the program developed. But, again and again, I hear from the postdocs how much they appreciate it. On many days, Deputy Director Hélène Barcelo and I wish we had mentors, too.

Public Understanding of Mathematics

Our most penetrant project is Numberphile, now with 2.2 million subscribers and over 295 million views on YouTube. Where else can an MSRI postdoc expect an audience of half a million? Brady Haran, the former BBC video reporter who makes the videos, spends some time at MSRI each semester harvesting interesting stories. The videos are easily accessible to high school kids (and some seven year olds); but I learn something that interests me from nearly every one.

In other news: The Mathical Book Prize (honoring imaginative children’s books related to math; see page 14) will soon enter its fourth cycle; 20,000 children and adults took part in the National Math Festival last spring in DC — we’re planning and taking part in more math festivals — see the article on pages 8–9; and our feature documentaries Navajo Math Circles and Counting from Infinity are showing here and there on public television. Navajo Math Circles, in particular, has been in nine festivals, and has great reviews! 🌟
New Della Pietra Fellowship and . . .

MSRI provides unique opportunities for postdoctoral fellows by bringing them together with leading researchers in their fields in an environment that promotes creativity and the effective interchange of ideas and techniques. Thirty-four postdoctoral fellows spent a semester or more at MSRI during academic year 2016–17; eight of them were supported by privately funded fellowships.

A donation by brothers Stephen and Vincent Della Pietra has made possible two new named fellowships at MSRI. The first of the Della Pietra fellowships to begin in 2017–18 was awarded this semester to Liran Rotem (also profiled here), in honor of Stephen Della Pietra. The second Della Pietra Fellowship will be announced in the Spring 2018 Emissary, including a profile of Vincent.

Stephen Della Pietra received his bachelor’s degree in physics from Princeton University in 1981, and his PhD in mathematical physics from Harvard University in 1986. From 1987–88, he was a post-doctoral fellow at The University of Texas at Austin. From 1988–89, he conducted postdoctoral research at the Institute for Advanced Study in Princeton.

From 1989–95, he was a research staff member at the IBM Thomas J. Watson Research Center in Yorktown Heights and Hawthorne, New York. As a project leader of the natural language understanding group, his primary research focused on machine translation and natural language understanding. In 1995 he joined Renaissance Technologies, where he currently co-manages the General Research Group and works on statistical methods to model the stock market.

Stephen is co-founder of the Della Pietra Lecture Series at Stony Brook University. This series brings world-renowned scientists to the Simons Center for Geometry and Physics, and is intended to bring awareness of recent and impactful discoveries in physics and mathematics to high school, undergraduate, and graduate students.

He serves on the advisory council of the Department of Astrophysical Sciences at Princeton University, as treasurer of the National Museum of Mathematics in New York, and as board member of the nonprofit organization PIVOT.

First Della Pietra Postdoc

Liran Rotem is the inaugural recipient of the new Della Pietra Fellowship this fall. Liran is a member of the Geometric Functional Analysis and Applications program.

In 2016 Liran joined the University of Minnesota as a Dunham Jackson Assistant Professor, after completing his PhD at Tel Aviv University under the supervision of Prof. Vitali Milman. In his thesis he studied functional versions of basic notions from convex geometry such as mixed volumes and convex duality, as well as functional analogs of geometric inequalities such as the Santaló inequality. An intriguing notion of geometric mean of convex bodies is introduced in his works, which is the subject of ongoing research. While pursuing his PhD, Liran was also passionate about his work as a teaching assistant, winning several awards for his excellence in this pursuit.

A Familiar MSRI Face Returns

Alaina Drake-Moore is an MSRI veteran who worked as the Assistant for Scientific Activities from 2012 until she moved to Sacramento in 2015. For the past two years she was employed as a Senior Project Coordinator for the architectural firm Lionakis, but she is thrilled to have re-joined the wonderful MSRI staff as the new Grants & Data Specialist. She works remotely from her home in Sacramento where she lives with her new husband and their two cats. In her free time, she enjoys rollerskating and attending heavy metal concerts.

MSRI: Now GuideStar Platinum

MSRI has earned a Platinum transparency rating from GuideStar, an information service specializing in reporting on US nonprofit companies and the world’s largest source of information on nonprofit organizations. The rating recognizes MSRI’s governance, fiscal management, and commitment to accountability and transparency.

You can be confident that your philanthropic gifts to MSRI are handled in accordance with the highest standards.
MSRI and the American Mathematical Society (AMS) host two Congressional briefings on mathematical topics each year in Washington, DC, to inform members of Congress and congressional staff about new developments made possible through federal support of basic science research.

On June 28, 2017, David Donoho (pictured) presented “Blackboard to Bedside: How High-Dimensional Geometry is Transforming the MRI Industry” to an attentive audience at the Russell Senate Office Building. Donoho shared the story of how U.S. investment in basic research in the mathematical sciences led to a breakthrough in technology for dynamic cardiac imaging. Recently approved by the FDA, the new technology gathers data 15 times faster than before and speeds up 3D brain imaging by a factor of eight. The speedup will allow more patients to be served at a lower cost per patient, giving US taxpayers a better return on the tens of billions of dollars in annual MRI charges. Senator Charles Schumer and Leader Nancy Pelosi both attended the event and gave remarks.

Donoho, a MacArthur Fellow and National Academy of Sciences member, is currently the Anne T. and Robert Bass Professor of Humanities and Sciences and professor of statistics at Stanford University. A video featuring Professor Donoho interviewed by MSRI Director David Eisenbud is available at msri.org/congress.

Postdoctoral Fellows in Geometric Functional Analysis

**Viterbi**

Konstantin Tikhomirov is the Viterbi Postdoctoral Fellow in this fall’s Geometric Functional Analysis and Applications program. Konstantin graduated from Samara University, Russia, where he worked under the direction of Sergey Astashkin. He obtained his PhD degree from the University of Alberta in 2016, under the supervision of Nicole Tomczak-Jaegermann and Vlad Yaskin. In 2016, Konstantin took up instructorship in mathematics at Princeton University. He is a recipient of numerous awards, with the last one being 2017 Canadian Mathematical Society Doctoral Prize. His research interests are remarkably diverse: from functional analysis to high dimensional probability, to various areas of combinatorics. His current research focuses mainly on spectral characteristics of random matrices and random graphs. Konstantin and his collaborators are studying invertibility and spectral distribution of sparse random matrices, in particular, adjacency matrices of random $d$-regular graphs. In high-dimensional convex geometry, Konstantin and his coauthors explore properties of numerous sections and projections of convex sets including some new and surprising “almost isometric” phenomena in the classical Dvoretzky’s Theorem.

The Viterbi postdoctoral scholarship is funded by a generous endowment from Dr. Andrew Viterbi, well known as the co-inventor of Code Division Multiple Access based digital cellular technology and the Viterbi decoding algorithm, used in many digital communication systems.

**Berlekamp**

Tomasz Tkocz is the Berlekamp Postdoctoral Fellow this fall in the Geometric Functional Analysis and Applications program. Tomasz finished his master’s studies at the University of Warsaw and completed his PhD at the University of Warwick. His research interests concern mainly probability theory, mathematical analysis, and their applications to convex geometry. Tomasz, with various collaborators, obtained results about geometrical properties of Gaussian measures, uniform distributions on convex bodies, random matrices, suprema of stochastic processes, entropy estimates, operators on Banach spaces, random multilinear forms, and random perpetuities. In his work he applies a wide spectrum of methods from functional and classical analysis, probability theory, and geometry. Like his fellowship’s namesake, Tomasz is fond of mathematical puzzles and joins Elwyn Berlekamp and Joe Buhler as a guest contributor to this month’s Puzzles column on page 15!

The Berlekamp Postdoctoral Fellowship was established in 2014 by a group of Elwyn Berlekamp’s friends, colleagues, and former students whose lives he has touched in many ways. He is well known for his algorithms in coding theory and has made important contributions to game theory. He is also known for his love of mathematical puzzles.

Liran Rotem, another GFA postoc, is the inaugural recipient of the new Della Pietra Fellowship and is profiled on page 3.
Convex sets in $\mathbb{R}^n$ are as diverse as normed spaces. Their incredible variety only increases with the dimension. Nevertheless, their substructures, like cross-sections or projections, exhibit a remarkable regularity. For example, Dvoretzky’s theorem says that any convex body has a high-dimensional cross-section which is nearly an ellipsoid. The theorem is one of the cornerstones of geometric functional analysis. The word “nearly” here is important: allowing a small error opens a window to finding the ideal Euclidean structure everywhere.

Given that all convex bodies have nearly ellipsoidal slices, how to find them? This is no easy task, even for specific bodies like the simplex or cross-polytope. However, by random selection it is hard to miss them. Indeed, V. Milman’ approach to Dvoretzky’s theorem shows that almost-ellipsoidal slices are more a rule than an exception. What underlies this is the concentration of measure phenomenon. Put simply, it says that a Lipschitz function on the high-dimensional sphere (in particular, the norm associated with the convex body) is nearly constant. The latter follows from the standard isoperimetric inequalities in Euclidean space or on the sphere or Gauss space. Concentration of measure is now responsible for a wealth of counterintuitive knowledge about high-dimensional phenomena in mathematics and beyond.

This semester’s program on Geometric Functional Analysis and Applications is devoted to interactions between high-dimensional convex geometry, various branches of analysis, and probability. A central theme is to understand and quantify uniform behavior that often accompanies high dimensions. The growth of the field in the last 15 years has coincided with increased connections to data science, dimension reduction, compressed sensing, and complexity of algorithms, among others. We will try to give an overview of some of the main principles by discussing a central open question, the slicing problem. This choice is just one of many fascinating and active directions, but it serves as a vehicle to explore links to probability, harmonic analysis, and isoperimetry.

### The Slicing Problem: A Recurring Obstacle

The slicing problem, posed by Bourgain, asks if all convex bodies of volume one in $\mathbb{R}^n$ admit hyperplane sections through their centroids with $(n - 1)$-volume greater than an absolute constant, independent of the body and $n$. If false, there are convex bodies of volume one such that all central slices have almost no volume when $n$ is large enough. In the 1980s it was realized, and surveyed by Milman and Pajor, that a great deal of unsolved volumetric questions in convex geometry remained open precisely because of a lack of understanding of the slicing problem. As a sample, we will discuss two.

One of the earliest problems in geometric probability, with roots in J. J. Sylvester’s famous four point problem from 1864, is to understand the expected volume of random simplices inside a convex body $K \subseteq \mathbb{R}^n$. Specifically, choose $n + 1$ random points $x_1, \ldots, x_{n+1}$ in $K$, independently according to the uniform measure: $d \mu_K(x) = \frac{1}{\text{vol}(K)} \mathbf{1}_K(x) dx$. Their convex hull is a random simplex, as in the figure at the top of the next column.

The expected proportion of volume such a simplex occupies in $K$ is the affine-invariant quantity

$$m(K) = \frac{\mathbb{E} \text{vol}(\text{conv}(x_1, \ldots, x_{n+1}))}{\text{vol}(K)},$$

which is well comparable to the determinant of the covariance matrix of $\mu_K$. By compactness, $m(K)$ admits extremizers. In $\mathbb{R}^2$, Blaschke proved that

$$m(B) \leq m(K) \leq m(\Delta),$$

where $B$ is the Euclidean disk, or, by invariance, an ellipse, and $\Delta$ is a triangle. While the lower bound extends to the Euclidean ball $B$ in all dimensions, as proved by Groemer, the natural conjecture that simplices $\Delta$ are extremizers in $\mathbb{R}^n$, $n \geq 3$, is still unsolved. In $\mathbb{R}^n$, the gap between the upper and lower bounds in (*) is insignificant: $m(B)^{1/n} \approx m(\Delta)^{1/n}$, up to an absolute constant. But sandwiching $m(K)$ between the latter quantities, even at the expense of an absolute constant, amounts to solving the slicing problem. Indeed, an equivalent form of the latter is that the quantity

$$L_K := m(K)^{1/n} \sqrt{n},$$

called the isotropic constant of $K$, is of order 1 for all convex bodies. A positive answer means that it is impossible to distinguish $K$ from the Euclidean ball by sampling and computing $m(K)$. Approximate isoperimetric inequalities are a key feature of geometric functional analysis and have played a significant role in its development, drawing tools from various sources. Quantitative bounds for $L_K$, however, are challenging as the uniform measure on $K$ has highly counterintuitive behavior when $n$ is large.

Another seminal volumetric problem about convex bodies connected to the slicing problem is the Busemann–Petty problem. The latter asks if $K$ and $L$ are convex bodies in $\mathbb{R}^n$ (say origin-symmetric) such that for each unit vector $\theta$, $\text{vol}_{n-1}(K \cap \theta^\perp) \leq \text{vol}(L \cap \theta^\perp)$, does it follow that $\text{vol}(K) \leq \text{vol}(L)$? (See the figure at the top of the next page.)
This is trivially true in dimension 2, as the assumption entails inclusion of $K$ inside $L$. However, the situation in higher dimensions goes against low-dimensional intuition. Dozens of mathematicians worked on the problem and the final answer was shown to be positive for $n \leq 4$ and negative for $n \geq 5$, by Gardner, Koldobsky, and Schlumprecht. The solution depends essentially on a Fourier-analytic approach and is a striking example of the interplay between convex geometry and harmonic analysis. However, relaxing the conclusion to $\text{vol}(K) \leq c \text{vol}(L)$ to allow an absolute constant as a correction is another equivalent form of the slicing problem.

**Concentration of Volume in Convex Bodies**

Bourgain put forth a probabilistic approach to the slicing problem by studying properties of the uniform probability measure $\mu_K$ on $K$. In this setting, if $\theta$ is a unit vector, the linear functional $\langle \cdot, \theta \rangle : K \rightarrow \mathbb{R}$ is a random variable. For example, if $K = [-1,1]^n$ is the cube and $\theta$ is the diagonal direction $\frac{1}{\sqrt{n}}(1, \ldots, 1)$, the distribution of $\langle \cdot, \theta \rangle$ is the same as $\frac{x_1 + \cdots + x_n}{\sqrt{n}}$, where $x_1, \ldots, x_n$ are independent uniform random variables in $[-1, 1]$. The latter is approximately normal by the standard central limit theorem. In general, the density of $\langle \cdot, \theta \rangle$ at $t \in \mathbb{R}$ is proportional to the $(n-1)$-volume of the slice of $K$ by the hyperplane $\theta^\perp + t\theta$. If $K = B$ is the Euclidean ball, the tail of any linear functional has a Gaussian-like decay, as shown in the figure below.

Tail decay of a linear functional $\langle \cdot, \theta \rangle$ on the Euclidean ball. The shaded area has probability $\mu_B(|\langle x, \theta \rangle| \geq t) \approx e^{-t^2}$, where $E$ is the expectation of $|\langle \cdot, \theta \rangle|$.

On the other hand, not all linear functionals on convex bodies behave like Gaussian random variables. When $K$ is a cone $C$ with vertex in the direction of $\theta$, the tail of $\langle \cdot, \theta \rangle$ is more like exponential, shown below.

Tail decay of a linear functional on a centered cone $C$. The shaded area satisfies $\mu_C(|\langle x, \theta \rangle \geq t|) \approx e^{-t^2}$, where $E$ is the expectation of $|\langle \cdot, \theta \rangle|$.

Bourgain showed that for convex bodies whose linear functionals have (sub)Gaussian-like decay, the slicing problem has an affirmative answer. The link here is Talagrand’s majorizing measure theorem. For arbitrary convex bodies, the latter work led to the bound $L_K \leq C \sqrt{n} \log n$.

The probabilistic viewpoint was fruitful beyond the slicing problem and initiated a systematic study of high-dimensional measures with convexity properties. For example, Anttila, Ball, and Perissinaki proposed a central limit theorem for linear functionals, and this was established in a breakthrough result of Klartag in 2006. It says that there is a sequence $\varepsilon(n) \to 0$ such that any convex body $K \subseteq \mathbb{R}^n$ admits a direction $\theta$ for which

$$\sup_{A \subseteq \mathbb{R}} |\mu_K(A) - \gamma(A)| \leq \varepsilon(n),$$

where $\mu_K$ is the marginal distribution on $\mathbb{R}$ induced by $\theta$, that is, $\mu_K(A) = \mu_K(|x : \langle x, \theta \rangle \in A])$ and $\gamma$ is a Gaussian distribution on $\mathbb{R}$. Since (†) holds for arbitrary convex bodies $K$, where independence of coordinates may be lacking, it shows that convexity in high dimensions can be a substitute for independence. This establishes a strong resemblance between uniform measures on convex bodies (or log-concave distributions) and the Gaussian measure. However, the precise quantification of the result is still a major open question.

The fundamental reason why Klartag’s central limit theorem (†) is
true is that virtually all of the volume in a high-dimensional convex body concentrates in a “thin shell” whenever the underlying Euclidean inner-product is chosen appropriately. To be more precise, if we choose the Euclidean structure such that the covariance matrix of $\mu_K$ is the identity (assumed henceforth), then there are functions $\epsilon(\mathbb{n})$, $\phi(\mathbb{n})$, tending to $\mathcal{O}$ as $\mathbb{n} \to \infty$, such that

$$\mu_K \left( \left\{ x : 1 - \epsilon(\mathbb{n}) \leq \frac{||x||_2}{\sqrt{n}} \leq 1 + \epsilon(\mathbb{n}) \right\} \right) \geq 1 - \phi(\mathbb{n}).$$

To capture this high-dimensional phenomenon of volume concentration in a 2-dimensional picture, convex bodies are often illustrated in hyperbolic form as in the figure below.

![Hyperbolic Illustration of a Convex Body](image)

Most of the volume in a high-dimensional convex body lies in a thin shell. The decay of linear functionals also suggests the hyperbolic form in which convexity is lost but the volume decay in given directions is apparent.

As above, a high-dimensional phenomenon has been established, but the precise asymptotics for $\epsilon(\mathbb{n})$ and $\phi(\mathbb{n})$ are unknown. The behavior of $\epsilon(\mathbb{n})$ and $\phi(\mathbb{n})$ would be determined if one could solve an isoperimetric problem on $\mathbb{R}^n$ equipped with $\mu_K$, as one wants to estimate the “perimeter” of a thin shell. An isomorphic variant of the latter isoperimetric problem has been proposed by Kannan–Lovász–Simonovits. Interestingly, their motivation came from an algorithmic problem as they wanted to control mixing rate of a certain random walk in $K$ in order to efficiently estimate its volume. The latter conjecture is strongly related to geometric analysis and, by results of Cheeger and Ledoux, is equivalent to the existence of an absolute positive spectral gap for the (Neumann) Laplacian associated to the measure $\mu_K$ or equivalently to the following Poincaré inequality:

$$\text{var} f(X) \leq C\varepsilon ||\nabla f(X)||_2^2, \quad (\ddagger)$$

for all smooth functions $f$, where $X$ has distribution $\mu_K$. By a surprising and deep result of E. Milman, $(\ddagger)$ is equivalent to the weakest possible concentration statement (in the case of convex bodies) that there exist $\epsilon, t > 0$ such that for any $1$-Lipschitz function $g$,

$$\mu_K \left( \left\{ x : |g(x) - \text{Med}(g)| \geq t \right\} \right) \leq 0.5 - \epsilon,$$

where Med$(g)$ is the median of $g$ with respect to $\mu_K$. By a result of Eldan and Klartag, establishing $(\ddagger)$ with an absolute constant $C$ would solve the slicing problem. The most recent and successful attempts to attack these problems are due to Eldan, who combined methods from the theory of log-concave distributions and stochastic calculus. He showed that approximate isoperimetric minimizers for the uniform measure $\mu_K$ (up to logarithms) are ellipsoids! Based on his technique, Lee and Vempala showed the best known estimate for $C$ in $(\ddagger)$, of order $\mathbb{n}^{-1/2}$. This also provides an $\mathbb{n}^{-1/2}$ bound for the isotropic constant, matching the previously achieved bound of Klartag using a transportation of measure technique. While all of these proofs lead to the bound $L_K \leq C\sqrt{\mathbb{n}}$, the techniques in each case are very different.

Connections and Relevance

The list of connections between the slicing problem and other fields is not limited to those discussed above but extends to information theory, transportation of measure, affine isoperimetric inequalities, and others. Convexity and isoperimetry underlie many of the phenomena that we have discussed and they are central themes in the theory. Modern geometric functional analysis serves as a conductor of ideas and techniques between analysis, probability, and convex geometry and other disciplines that share a need for precise descriptions of high-dimensional geometric structures.

Named Positions, Fall 2017

MSRI is grateful for the generous support that comes from endowments and annual gifts that support faculty and postdoc members of its programs each semester. The Clay Mathematics Institute awards its Senior Scholar awards to support established mathematicians to play a leading role in a topical program at an institute or university away from their home institution.

Chern, Eisenbud, and Simons Professors

Federico Ardila, San Francisco State University
Imre Bárány, Renyi Institute of Mathematics
Pavle Blagojević, Freie Universität Berlin
Sylvie Corteel, Université de Paris VII (Denis Diderot)
Jesús De Loera, University of California, Davis
Alexander Koldobsky, University of Missouri
Vitali Milman, Tel Aviv University
Igor Pak, Hebrew University
Grigoris Paouris, Texas A&M University
Gideon Schechtman, Weizmann Institute of Science
Francis Su, Harvey Mudd College
Nicole Tomczak-Jaegermann, University of Alberta

Named Postdoctoral Fellows

Anastasia Chavez, Univ. of California, Berkeley (Huneke)
Annie Raymond, University of Washington (Gamelin)
Liran Rotem, University of Minnesota (Della Pietra)
Konstantin Tikhomirov, Princeton University (Viterbi)
Tomasz Tkocz, Princeton University (Berlekamp)

Clay Senior Scholars

William Johnson, Texas A&M University
Francisco Santos, University of Cantabria
The National Math Festival welcomed 20,000 visitors of all ages to explore the unexpected sides of mathematics on April 22 in Washington, DC. The National Math Festival runs every two years, and the next festival is planned for Washington, DC, in spring 2019. The dates and venue will be announced in the spring of 2018. Below are just a few of the many, many events of the day, to share a flavor of the experience!

Presentations
The 2017 festival included presentations on nearly two dozen topics, including the mathematics of “tipping points”—dramatic moments when a system suddenly shifts from one state to another—and their effect on the earth’s climate with Mary Lou Zeeman (Bowdoin College), how Google searches work (Emille Davie Lawrence, University of San Francisco), and how to inspire a love of math through unconventional activities such as baking with Eugenia Cheng (School of the Art Institute of Chicago). The recent blockbuster film Hidden Figures was screened in tandem with related talks on the math behind the movie (Talitha Washington, Howard University) and of space travel (Andrea Razzaghi, Deputy Director of Astrophysics at NASA). Mariel Vazquez (University of California, Davis) shared a window into her research on the human body’s inner workings using links between math, physics, and biology.

From the United Kingdom, Marcus du Sautoy (University of Oxford) opened eyes with the ways that Einstein’s theory of relativity and other laws of the universe break down at the Big Bang and inside black holes.

Activities
Interactive and hands-on activities also abounded. Here is a small sampling of the 45 happenings at the festival.

MoMath invited attendees to slice shapes with lasers, explore giant mazes, and design roller coasters. The Julia Robinson Mathematics Festival hosted a Celebration of Mind, with dozens of mathy puzzles, board games, and magic tricks from participating organizations including ThinkFun Games, Gathering 4 Gardner Foundation, and the Elwyn and Jennifer Berlekamp Foundation. The Association for Women in Mathematics introduced a new generation to the classic Conway’s Game of Life, while puzzle master Scott Kim shared how to make your own Sudoku. Members of Science Cheerleader got kids on their feet with a Math Cheer Clinic, and hands-on sessions for all ages were held by magician Mark Mitton and educator James Gardner, Alissa Crans (Loyola Marymount University), and Natural Math.

Students and faculty from the math departments at George Washington University and
Ithaca College delighted attendees of all ages with their geometric balloon twisting. The Young People’s Project hosted demonstrations of the Flagway Game, an athletic competition based on the Möbius function, as well as a Flagway tournament for teams of fifth and sixth graders from around the U.S. The Bridges Organization brought a gallery of mathematical art, and in honor of the Mathematical Book Prize (see page 14), MSRI invited several winning children’s book authors to share readings and hands-on activities with the festival, including Christopher Danielson, Joan Holub, and Laura Overdeck.

The Alfred P. Sloan Foundation presented an all-day film room, screening live action and animated math films including MSRI’s newest documentary, Navajo Math Circles. In addition to the festivities in Washington, an additional 27 science museums around the U.S. held Giant SOMA Cube Workshops and other events to celebrate Mathematics and Statistics Awareness Month.

Learn More and Stay in Touch Online

Most of the talks are available in video on the National Math Festival website, and there are many more photos to enjoy as well: nationalmathfestival.org/photos-videos. Additionally, a curated collection of mathy games, logic puzzles, videos, books, and other resources is updated each month at the More Math! section of the website at nationalmathfestival.org/more-math. If you would like to suggest resources to share, please email mathfestival@msri.org. To receive future National Math Festival news and updates, subscribe to the e-newsletter at www.tinyurl.com/nmfnews.

Special Thanks

MSRI is grateful for the support of our fellow mathematics organizations who joined us in many ways in participating in and supporting the 2017 festival. While space prevents us from an exhaustive list of their specific contributions, we want to acknowledge the American Mathematical Society, the Association for Women in Mathematics, the Mathematical Association of America, the National Association of Mathematicians, the National Council of Teachers of Mathematics, the Society for Advancement of Chicanos/Hispanics and Native Americans in Science, and the Society for Industrial and Applied Mathematics.

The 2017 National Math Festival was organized by MSRI in cooperation with the Institute for Advanced Study and the National Museum of Mathematics, with the generous support of the Simons Foundation; the Alfred P. Sloan Foundation; the Carnegie Corporation of New York; the Charles and Lisa Simonyi Fund for Arts and Sciences; Google.org; Eric and Wendy Schmidt; Renaissance Technologies; the Kavli Foundation; the Gordon and Betty Moore Foundation; the National Science Foundation; the Qualcomm Foundation; Research Corporation for Science Advancement; Amazon; Northrop Grumman; the John Templeton Foundation; and VISA.
Postdoctoral Fellows, Geometric & Topological Combinatorics

Gamelin

Annie Raymond is the Gamelin Postdoctoral Fellow in the Geometric and Topological Combinatorics program. Annie received her PhD in 2014 from the Technical University in Berlin under the supervision of Martin Grötschel. Her thesis work was centered around problems in polyhedral combinatorics and combinatorial optimization, spanning both theory and applications. From 2014–17, Annie was an Acting Assistant Professor at the University of Washington in Seattle. In this period she and her collaborators established an unexpected direct connection between the celebrated “flag algebra” methods of Alexander Razborov and the classical theory of sums of squares polynomials in optimization and real algebraic geometry. This work opens the door for a systematic approach to an array of problems in computer science, combinatorics, and optimization, and has already received considerable attention from these communities. Annie gives herself tirelessly to math outreach and education. For the past two years she has been teaching a weekly class at the Monroe Correctional Complex, the second largest prison in Washington, and she will continue teaching inmates in the Bay Area. In January 2018, she will join the University of Massachusetts at Amherst as an Assistant Professor.

The Gamelin postdoctoral fellowship was created in 2014 by Dr. Ted Gamelin, Emeritus Professor of the UCLA Department of Mathematics. The Gamelin Fellowship emphasizes the important role that research mathematicians play in the discourse of K-12 education.

Huneke

Anastasia Chavez is the Fall 2017 Huneke Endowed Postdoctoral Fellow as part of the Geometric and Topological Combinatorics program. Anastasia is a first-generation college student trained entirely in the California public school system. She obtained her AS degree from Santa Rosa Junior College, her BS and MA degrees from San Francisco State University, and her PhD from the University of California, Berkeley, in 2017 under the direction of Federico Ardila and Lauren Williams, while raising her daughters Ayla and Asha. She earned numerous awards along the way; most recently, she received the University of California President’s Postdoctoral Fellowship. Anastasia’s research on posets, polytopes, and positroids has uncovered beautiful combinatorial structures that are hidden behind very classical questions. Her joint work with Nicole Yamzon shed new light on the Dehn–Sommerville equations: they described which d/2-subsets determine the f-vector of a simplicial d-polytope; these subsets are enumerated by the Catalan numbers. In joint work with Felix Gotti, she gave an elegant matroid-theoretic characterization of semiorders. In addition, Anastasia is a leader in various efforts toward a more diverse and equitable mathematical community.

The Huneke postdoctoral fellowship is funded by a generous endowment from Professor Craig Huneke, who is internationally recognized for his work in commutative algebra and algebraic geometry.

National Association of Math Circles — News

Mentorship and Partnership Program Grows into Year Three

NAMC’s Math Circles–Mentorship and Partnership Program is entering its third year. We have added new mentors, and 18 groups across the country are participating.

Joint Meetings: Come to our Minicourse and Events!

NAMC Director Diana White and Associate Director Jane Long are partnering with Brianna Donaldson of AIM and the Math Teachers’ Circle Network and Gabriella Pinter of the University of Wisconsin–Milwaukee to deliver a minicourse, “How to Run Successful Math Circles for Students and Teachers,” at the Joint Mathematics Meetings in San Diego, January 10–13, 2018. We enjoyed connecting with the Math Circles community at our booth at MathFest in July, and encourage attendance at the many Math Circles events at the Joint Meetings!
Extensions to higher dimensions were conjectured by Birch and proven by Tverberg almost exactly fifty years ago:

**Theorem** (Tverberg, 1966). Given integers \( r \geq 2 \) and \( d \geq 1 \) and a set \( X \subset \mathbb{R}^d \) of at least \( (r-1)(d+1)+1 \) points, there is a partition \( X_1 \sqcup \cdots \sqcup X_r \) of \( X \) such that the convex hulls of the \( X_i \) all share a common point, that is, \( \bigcap_i \text{conv}X_i \neq \emptyset \).

This innocent looking statement is a generalization of Radon’s theorem which deals with the case \( r = 2 \) and for which there is an easy proof. While several proofs of Tverberg’s theorem are known, they are quite a bit more involved than the \( r = 2 \) case. Tverberg’s theorem derives its importance from its numerous applications ranging from discrete and computational geometry and abstract convexity to quantum error correcting codes. We outline a few of those applications.

**Initial Consequences**

A purely combinatorial consequence — originally a theorem of Lindström — was found by Tverberg himself: If \( A_1, \ldots, A_{(r-1)n+1} \) is a sequence of nonempty subsets of an \( n \)-element set, then there are nonempty and disjoint subsets \( J_1, \ldots, J_r \) of \( \{1, \ldots, (r-1)n+1\} \) such that \( \bigcup_{l \in J_l} A_l = \cdots = \bigcup_{l \in J_1} A_l \). In particular, purely combinatorial partition problems can be solved by a transfer to convex geometry.

In combination with the colorful Carathéodory theorem (if \( X_0, \ldots, X_d \subset \mathbb{R}^d \) have convex hulls capturing the origin, then there is a choice of one point \( x_i \) in each \( X_i \) with \( 0 \in \text{conv}\{x_0, \ldots, x_d\} \)), Tverberg’s theorem yields the selection lemma: Given a finite set \( X \subset \mathbb{R}^d \) of points in general position, there is a point common to the convex hull of a positive fraction of the \( (d+1) \)-tuples of \( X \).

A halving plane of a finite set \( X \subset \mathbb{R}^3 \) of points in general position is a plane spanned by three points of \( X \) that leaves equally many points of \( X \) on either side of it. While the first author was working with Füredi and Lovász on establishing upper bounds on the number of halving planes, they encountered the following question: Given a set \( X \subset \mathbb{R}^2 \) of \( n \) points in general position, a crossing is the intersection of the lines spanned by \( x, y \) and by \( u, v \) where \( x, u, v \) are distinct points from \( X \). It is evident that there are \( \frac{1}{2} \binom{n}{2} \binom{n-2}{2} \) \( \approx n^3 \) crossings. How many of them are contained in a typical triangle spanned by points in \( X \)? A direct application of Tverberg’s theorem shows that the number of crossings is again of order \( n^3 \). This was a key step in establishing an \( O(n^{3-\varepsilon}) \) bound on the number of halving planes. It also led to what is now called the colorful Tverberg theorem. The moral is that when working on a question in combinatorial convexity it is always good to check what Tverberg’s theorem says in the given situation.

**Topological Radon Theorem**

A second possible fascination with Tverberg’s theorem beyond its applicability to a broad range of combinatorial and geometric problems stems from the following question of the first author: Does Tverberg’s theorem admit a continuous relaxation, in the sense that we may continuously deform convex hulls (provided that these deformations match up for common subsets) and still expect an \( r \)-fold intersection? On the one hand such a less rigid version of Tverberg’s theorem would allow us to apply it to an even broader range of questions. On the other hand this question has sparked deep and interesting connections to equivariant algebraic topology.

Let us first give a precise statement and focus on the case \( r = 2 \). Radon’s theorem, which equivalently can be phrased as any affine map \( f: \Delta_{d+1} \to \mathbb{R}^d \) from the \( (d+1) \)-simplex to \( \mathbb{R}^d \) identifies two points from disjoint faces. Does this remain true for merely continuous \( f \)? In fact it does, as can be shown using the Borsuk–Ulam theorem:

**Theorem** (Bajmóczy and Bárány, 1978). For any continuous map \( f: \Delta_{d+1} \to \mathbb{R}^d \) there are two disjoint faces \( \sigma_1 \) and \( \sigma_2 \) of \( \Delta_{d+1} \) such that \( f(\sigma_1) \cap f(\sigma_2) \neq \emptyset \).

Actually, more generally, any continuous map from the \( d \)-skeleton of a \( (d+1) \)-polytope to \( \mathbb{R}^d \) identifies points from two parallel, disjoint faces. As a further application of this more general continuous version, Lovász and Schrijver used a slight extension of this topological Radon theorem to show that linklessly embeddable graphs have Colin de Verdière number at most four.

**Topological Tverberg Theorem**

Tverberg’s theorem can be reformulated in the same way as for the \( r = 2 \) case: For \( n = (r-1)(d+1) \), any affine map \( f: \Delta_n \to \mathbb{R}^d \) identifies \( r \) points from \( r \) pairwise disjoint faces of the \( n \)-simplex \( \Delta_n \). Can one avoid this \( r \)-fold intersection among the pairwise disjoint faces of \( \Delta_n \) if one is allowed to deform them continuously? In 1976 the first author conjectured that Tverberg’s theorem is in fact topological, that is, that it remains true for continuous maps \( f \). The answer turned out to depend on whether the intersection multiplicity \( r \) is a prime power or not. On the positive side, topological machinery more involved than the Borsuk–Ulam theorem can be used to show:
**Theorem** (Bárány, Shlosman, Szücs, 1981; Özaydin, 1987). Given $r > 2$ a power of a prime, an integer $d > 1$, and a continuous map $f: \Delta_n \to \mathbb{R}^d$, where $n = (r - 1)(d + 1)$, there are $r$ pairwise disjoint faces $\sigma_1, \ldots, \sigma_r$ of $\Delta_n$ such that $f(\sigma_1) \cap \cdots \cap f(\sigma_r) \neq \emptyset$.

Here the case $r$ a prime was proven via an extension of the Borsuk–Ulam theorem to free group actions known as Dold’s theorem, while the general case of prime powers was solved by Özaydin with a generalization to fixed-point-free actions in an influential unpublished manuscript.

The condition that $r$ be a power of a prime was long believed to be an artifact of the proof method, which employs techniques from equivariant topology to show that certain continuous maps that commute with given symmetries of the symmetric group do not exist. Mabillard and Wagner developed an $r$-fold Whitney trick to show that the failure of the proof methods outside of the prime power case actually gives rise to continuous maps from certain simplicial complexes without $r$-fold points of coincidence among their pairwise disjoint faces. Their method, however, has technical restrictions such as the simplicial complex being of dimension at most $d - 3$ that prevents a direct application to the topological Tverberg conjecture. Indeed, notice that the dimension of the simplex $\Delta_n$ in Tverberg’s theorem is much larger than $d$.

It was the insight of the second author that the constraint method developed by Blagojević, the second author, and Ziegler (which was earlier observed in special cases by Sarkaria, de Longueville, and Gromov) may be used to circumvent the codimension restriction in the Mabillard–Wagner theorem, thus yielding counterexamples to the topological Tverberg conjecture for every $r$ that is not a power of a prime. The smallest counterexample known today is a continuous map $\Delta_{65} \to \mathbb{R}^{12}$ that does not identify points from six pairwise disjoint faces; it was constructed, along with counterexamples in dimension $2r$ for every $r$ that is not a power of a prime, by Avvakumov, Mabillard, Skopenkov, and Wagner. Tverberg’s theorem remains a constant source of new and exciting questions and results, even fifty years after its discovery.

### Forthcoming Workshops

**Nov 13–17, 2017:** Geometric Functional Analysis and Applications

**Nov 29–Dec 1, 2017:** Women in Topology

**Jan 18–19, 2018:** Connections for Women: Enumerative Geometry Beyond Numbers

**Jan 22–26, 2018:** Introductory Workshop: Enumerative Geometry Beyond Numbers

**Feb 1–2, 2018:** Connections for Women: Group Representation Theory and Applications

**Feb 5–9, 2018:** Introductory Workshop: Group Representation Theory and Applications

**Feb 21–23, 2018:** CIME 2018: Access to Mathematics by Opening Doors for Students Currently Excluded from Mathematics

**Mar 8–10, 2018:** LaTeX in the Mathematical Sciences Conference 2018

**Mar 12–16, 2018:** Hot Topics: The Homological Conjectures

**Mar 19–23, 2018:** Structures in Enumerative Geometry

**Apr 9–13, 2018:** Representations of Finite and Algebraic Groups

**Apr 14–15, 2018:** The 2018 Infinite Possibilities Conference

**Jun 16–Jul 29, 2018:** MSRI-UP 2018: The Mathematics of Data Science

**Summer Graduate Schools**

**Jun 11–22, 2018:** The δ-Problem in the Twenty-first Century

**Jun 11–22, 2018:** Séminaire de Mathématiques Supérieures 2018: Derived Geometry and Higher Categorical Structures in Geometry and Physics

**June 17–30, 2018:** Mathematical Analysis of Behavior

**Jun 25–Jul 6, 2018:** Derived Categories

**Jun 25–Jul 6, 2018:** H-principle

**Jul 1–21, 2018:** IAS/PCMI 2018: Harmonic Analysis

**Jul 9–20, 2018:** Representations of High Dimensional Data

**Jul 23–Aug 3, 2018:** From Symplectic Geometry to Chaos

To find more information about any of these workshops or summer schools, as well as a full list of all upcoming workshops and programs, please visit [msri.org/scientific](http://msri.org/scientific).
Focus on the Scientist: Imre Bárány

Imre Bárány is a leading figure in combinatorial geometry and convex optimization. He is one of the distinguished research professors in the MSRI program on Geometric and Topological Combinatorics.

Imre started his career in 1978 by providing a short and elegant proof of Kneser’s conjecture, which had just been solved by Lovász. Imre’s solution uses the Gale transform and the theory of neighborly polytopes. A little bit later he proved the colorful version of Carathéodory’s theorem, the topological version of Tverberg’s theorem (together with Shlosman and Szűcs), and he initiated the study of quantitative versions of Helly’s theorem (together with Katchalski and Pach).

All these beautiful and fundamental theorems from his first five years of research make Imre one of the pioneers in bringing algebraic topology to be an everyday tool in convexity and discrete geometry. They are not only classical “textbook” results in convexity and combinatorics, but remain the subject of intensive research. He came back to these types of questions often, giving a colored version of Tverberg’s theorem in 1992 (together with Larman) and a fractional version of Helly’s theorem for lattice points in 2003 (together with Matousek).

Another early breakthrough of a more geometric and algorithmic nature was his proof, together with Füredi in 1987, that “computing the volume is hard”: no polynomial time algorithm can approximate the volume of convex bodies to a factor better than essentially $d^d$. This improved on an earlier estimate of Elekes, showed that Lovász algorithm based on the ellipsoid method was not far from optimal, and it partially motivated Dyer, Frieze, and Kannan to design their famous randomized algorithm.

Several of Imre’s results have helped us understand the family of lattice polytopes. With Vershik (1992) he found the right order of magnitude of the number of lattice polytopes of a given dimension and volume, thus answering a question of Arnold motivated by work on singularities of complex manifolds. Together with Howe and Scarf (1994), he proved that the complex of lattice-free copies of a given simplex is homeomorphic to the Euclidean space. He also described the typical shape of a lattice polygon contained in a given convex domain $K$.

Two landmark results were his answer to a celebrated question of Sylvester’s on the probability that a given number of points taken at random from a convex body in the plane are in a convex position (1999) and the six-circle conjecture of Fejes Tóth (with Füredi and Pach, 1983).

Imre’s oeuvre and career have been awarded numerous prizes and honors, such as the Mathematical Prize (now Erdős Prize) of the Hungarian Academy of Sciences (1996), the Széchenyi Prize (2016), an invitation to speak at ICM (2002), and an ERC grant. He is a member of the Hungarian Academy of Sciences and a Fellow of the American Mathematical Society.

Besides his versatile, deep, and prolific research, Imre is an avid sportsman and traveler, having positions both at University College London and at the Rényi Institute, Budapest. He is also an inspiration and a friend for many of us. His smile constantly sends the message, “Don’t worry, there is a solution.”

— Károly Böröczky and Francisco Santos

Call for Proposals

All proposals can be submitted to the Director or Deputy Director or any member of the Scientific Advisory Committee with a copy to proposals@msri.org. For detailed information, please see the website msri.org.

Thematic Programs

The Scientific Advisory Committee (SAC) of the Institute meets in January, May, and November each year to consider letters of intent, pre-proposals, and proposals for programs. The deadlines to submit proposals of any kind for review by the SAC are March 15, October 15, and December 15. Successful proposals are usually developed from the pre-proposal in a collaborative process between the proposers, the Directorate, and the SAC, and may be considered at more than one meeting of the SAC before selection. For complete details, see tinyurl.com/msri-progprop.

Hot Topics Workshops

Each year MSRI runs a week-long workshop on some area of intense mathematical activity chosen the previous fall. Proposals should be received by March 15, October 15, or December 15 for review at the upcoming SAC meeting. See tinyurl.com/msri-htw.

Summer Graduate Schools

Every summer MSRI organizes several two-week long summer graduate workshops, most of which are held at MSRI. Proposals must be submitted by March 15, October 15 or December 15 for review at the upcoming SAC meeting. See tinyurl.com/msri-sgs.
Focus on the Scientist: Vitali Milman

Vitali Milman is a member of this fall’s Geometric Functional Analysis and Applications program. He has been highly influential in the development of the field over the last decades, not only through his many mathematical results and ideas, but also thanks to his vision, vigor, numerous students, and academic activities—including serving as one of the organizers of the 1996 MSRI program on Convex Geometry and Geometric Analysis.

Michel Talagrand writes: “The idea of concentration of measure (that was discovered by V. Milman) is arguably one of the great ideas of Analysis in our times.” Surprisingly, typical functions on a high-dimensional space behave in many cases as if they were approximately constant functions. This is related to a geometric property which follows from Paul Levy’s isoperimetric inequality, that in the high-dimensional Euclidean sphere most of the mass is close to the equator, for any equator. Vitali found that this concentration phenomenon is rather general and goes well beyond the specific case of the sphere, and that it is a powerful effect with consequences in various branches of mathematics and science. For example, the concept of an expander graph from computer science is an interpretation of concentration of measure in graph theory.

Another fundamental concept promoted by Vitali, beginning with his work on distortion and spectrum in Banach spaces, is the idea that certain high-dimensional structures necessarily contain sub-structures that are very regular. Ramsey’s theorem and Dvoretzky’s theorem are two typical examples. His 1980s construction of the “Milman ellipsoid” is central to high-dimensional convex geometry, and is intimately connected with his quotient-of-subspace theorem, the reverse Brunn–Minkowski inequality, and the Bourgain–Milman theorem.

Vitali was born in the Soviet Union in 1939 and completed his PhD at the University of Kharkov in 1965. In 1973 he immigrated to Israel and joined Tel Aviv University, where he is still a Professor. Vitali is a recipient of the Landau Prize (2002) and the Emet prize (2007), delivered lectures at the International Congress of Mathematicians (ICM) in Berkeley (1986) and Berlin (1998), and a plenary lecture at the European Congress of Mathematics in Budapest (1996). Throughout his career, Vitali has been active in aiding the absorption of immigrant scientists in Israel, and in 1991 received an award from the Israeli government for his exceptional contribution.

— Bo’az Klartag

Mathical Book Prize Winners Announced for 2017

MSRI, in partnership with the National Council of Teachers of Mathematics and the National Council of Teachers of English, announced the 2017 winners of the Mathical: Books for Kids from Tots to Teens youth literature prize in April 2017.

The Mathical Book Prize cultivates a love of mathematics in the everyday world. Each year’s winners and honor books join a selective and ever-growing list of new and previously published fiction and nonfiction titles for kids of all ages. The 2017 Mathical Prize winners (published in 2016) are:

Pre-K  ONE Very Big Bear, by Alice Brière-Haquet (author) and Olivier Philipponneau and Raphaële Enjary (illustrators)

Grades K–2 Absolutely One Thing: Featuring Charlie and Lola, by Lauren Child

Grades 3–5 Which One Doesn’t Belong? A Shapes Book, by Christopher Danielson

Grades 6–8 Mind-Boggling Numbers, by Michael J. Rosen (author) and Julia Patton (illustrator)

Grades 9–12 Genius: The Game, by Leopoldo Gout

For a full list of current and past winners, you can visit mathicalbooks.org.
Puzzles Column

Elwyn Berlekamp, Joe P. Buhler, and Tomasz Tkocz

Tomasz Tkocz is a postdoc at Princeton who is visiting MSRI as part of the Geometric Functional Analysis and Applications program this semester. From 2011–2015, Tomasz was in charge of a problem solving corner in the Polish popular monthly Delta, and Elwyn and Joe are delighted to include him as a guest columnist this fall. Problems 3–6 are due to Tomasz, and appeared earlier in Delta.

1. An ordinary six-sided die is rolled repeatedly. What is the expected number of rolls before a 6 appears? (The number of rolls includes the final roll.) What is the expected number of rolls before a 6 appears, conditioned on the event that no prior rolls were odd?

Comment: This puzzle has gone viral, and can be found in numerous places on the internet. (Perhaps you’ve already been exposed to it?) It seems that Gil Kalai’s marvelous blog is one of the first places where this virus appeared. The problem is apparently due to Elchanan Mossel.

2. Define a function \( f \) on the closed interval \([0,1]\) by letting \( f(x) \) be the real number whose base 10 expansion is the base 2 expansion of \( x \). For instance, \( f(3/4) = f(0.11_2) = 0.11_{10} = 11/100 \). Find the integral of \( f \) on \([0,1]\).

Comment: The problem appeared on the 2014 undergraduate problem competition at the University of Illinois, and we first saw it on Stan Wagon’s weekly problem blog. The function is ill-defined at dyadic rational numbers, but continuous elsewhere and therefore integrable.

3. If points \( A, B, C, D \) in 3-space are such that all four angles \( \angle ABC, \angle BCD, \angle CDA, \) and \( \angle DAB \)
are right angles, then prove that those four points are coplanar.

Comment: Tomasz heard about this problem from Keith Ball.

4. A closed curve on the sphere with radius 1 has length less than \( 2\pi \). Show that the curve is contained in some hemisphere.

5. Each square of the usual \( 8 \times 8 \) chessboard is initially either white or black. At each step we choose a subboard of size \( 3 \times 3 \) or \( 4 \times 4 \) and reverse the colour of each square of the subboard. Is it true that no matter what the initial colouring of the chessboard is, there exists a sequence of steps such that the chessboard becomes all white?

Comment: Tomasz heard about this problem from Keith Ball.

6. Given an \( m \) by \( n \) matrix with 0, 1 entries, containing an even number of 1’s, is it always possible to find a submatrix containing exactly a half of the 1’s? Here, a submatrix is a matrix obtained by crossing out specific rows and columns of the original matrix, not necessarily consecutive.

Comment: Tomasz heard about this problem from Keith Ball.

Call for Membership

MSRI invites membership applications for the 2018–2019 academic year in these positions:

- **Research Members** by December 1, 2017
- **Postdoctoral Fellows** by December 1, 2017

In the academic year 2018–2019, the research programs are:

**Hamiltonian Systems, from Topology to Applications through Analysis**

*Aug 13–Dec 14, 2018*

Organized by Rafael de la Llave, Albert Fathi (Lead), Vadim Kaloshin, Robert Littlejohn, Philip Morrison, Tere M. Seara, Serge Tabachnikov, Amie Wilkinson

**Derived Algebraic Geometry**

*Jan 22–May 24, 2019*

Organized by Julie Bergner, Bhargav Bhatt (Lead), Dennis Gaitsgory, David Nadler, Nikita Rozenblyum, Peter Scholze, Gabriele Vezzosi

**Birational Geometry and Moduli Spaces**

*Jan 22–May 24, 2019*

Organized by Antonella Grassi, Christopher Hacon (Lead), Sándor Kovács, Mircea Mustaţă, Martin Olsson

MSRI uses MathJobs to process applications for its positions. Interested candidates must apply online at mathjobs.org. For more information about any of the programs, please see msri.org/scientific/programs.
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— Join us! —

For two events at the 2018 JMM in San Diego

Mathematical Institutes Open House
Wednesday, January 10
5:30–8:00 pm
San Diego Ballroom B
Lobby Level, North Tower

MSRI Reception for Current and Future Donors
Thursday, January 11
6:30–8:00 pm
Presidio Room 1
Lobby Level, North Tower
(Both events in the Marriott Marquis San Diego Marina)