Fall 2018

EMISSARY

Mathematical Sciences Research Institute

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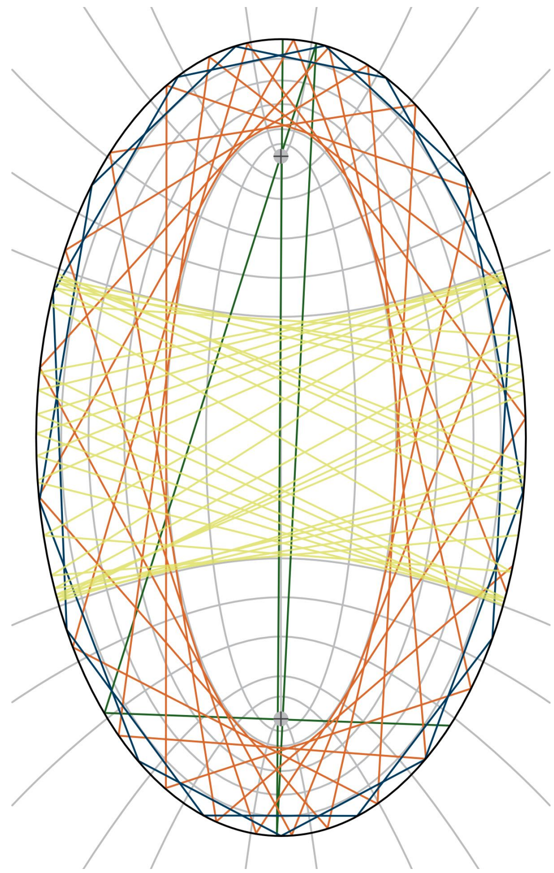
Hamiltonian Systems, from Topology to Applications through Analysis

Philip J. Morrison and Sergei Tabachnikov



Multiple trajectories for a billiard inside an ellipse: one of the popular Hamiltonian systems is the "billiard problem."

Overview

The special historic form of dynamical systems credited to Hamilton encompasses a vast array of fundamental and applied research. It is a basic form for physical law that has engendered development in many areas of mathematics, including analysis of ordinary and partial differential equations, topology, and geometry. Our jumbo program at MSRI has brought together a broad spectrum of mathematicians and scientists with research spanning emphasis on the applied to the rigorous.

It is hard to choose a starting point in the long history of Hamiltonian dynamics and its concomitant variational principles. One can go as far back as ancient Greece (Euclid, Heron), onward to the inspirational work of Fermat's principle of geometric optics (17th century), and up to the voluminous works of Lagrange (18th century). Fermat stated that the path taken by light going from one point to another in some medium is the path that minimizes (or, more accurately, extremizes) the travel time. This implies the law of optical reflection (the angle of incidence equals the angle of reflection) and Snell's law of refraction.

William Rowan Hamilton (19th century) studied the propagation of the phase in optical systems guided by Fermat's principle and realized that one could generalize it and adapt it to particle mechanics. Here is a very brief description of Hamiltonian mechanics.

(continued on page 4)

Lisa Jacobs

Come and Celebrate . . .

The **Summer Research for Women** program created and first hosted in 2017 (page 8).

Our recent **Celebration of Mind** (page 11). →

↓ Mafia, hats, and more in the **Puzzles**, as Elwyn and Joe welcome a new contributor (page 15)!

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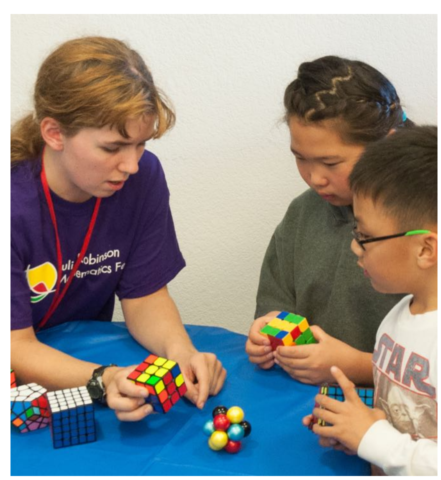


The View from MSRI

Hélène Barcelo, Acting Director

I'm excited (and I must admit, somewhat daunted!) to be taking the role of Acting Director this year while Director David Eisenbud is taking a much-deserved (and very active!) sabbatical. In the role of Assistant Director is the very capable and affable Michael Singer, Professor Emeritus at North Carolina State University, and Deputy Director at MSRI from August 2001–July 2002.

This is shaping up to be a very busy year for us, as we are preparing for the NSF recompetition for funding for the institute for the period 2020–25, formalizing our latest strategic vision, greatly expanding our fundraising efforts, and gearing up for the National Math Festival to be held in Washington, DC, this coming May. These and some other of our current projects are described below.

Survey of Postdocs

MSRI has undertaken a survey of the 204 postdoctoral fellows that we have hosted since 2009. While the analysis of the responses is not finished, so far we are encouraged by the data. Particularly striking was the 90% of respondents who rated their overall experience at MSRI as "excellent" or "very good." We will continue to share the full results once complete.

Diversity at MSRI

As we prepare for the NSF recompetition and reflect on the institute's recent programs, we are very pleased to report that the work of so many past and present members of our advisory and planning committees is coming to fruition. For example, over the last decades the percentage of women participating in our long program has increased by nearly 5%, and in the 2017–18 programs, 25% of the members, and fully 32% of the Research Professors self-identified as women. In addition, our Summer Research for Women in Mathematics program is accepting applications for the third year — Assistant Director Michael Singer discusses the development of this newer initiative on page 8.

National Math Festival

Our biggest public outreach program, the National Math Festival, will return to Washington, DC, for a third time on May 4, 2019. This year's festival includes not only presentations by mathematicians and activities for young children through adults, but also dance performances, athletic competitions, and more. We encourage any of you to join us for this celebration of math in many forms.

(continued on next page)

David Eisenbud, Director
(On sabbatical, 2018–19)

As readers will be aware, I've stepped back from many of the leadership duties at MSRI, which are being carried on very successfully this year by Acting Director Hélène Barcelo. This allows me to devote more time to some other initiatives. And it also means that in this Emissary, you get two views from MSRI: both my view (below), and Hélène's, next door in the left column!

NSF Proposal

The NSF has decided to hold an Institutes recompetition every five years. Though our "core" NSF grant is now only a little more than half our total funding, it is still the lifeblood of our research enterprise, and we take the proposal process extremely seriously. The new proposal will have input from many people, but I've taken primary responsibility for writing it. I welcome any good ideas for things MSRI should propose or change! If you have some, write them down, and send me an email (de@msri.org).

Fundraising and Strategic Vision

Annie Averitt

Annie Averitt is our new head of development, and I'm working very closely with her on other projects to support MSRI as well. Our endowment now stands at around $26 million, but needs to be much larger to ensure that MSRI can continue to serve the math community for the generations to come. Crucial for raising money is to be able to say clearly what you want to do with it, and we are thus engaged in a strategic planning exercise. Again, with much input from the trustees and from Hélène and Michael Singer, I've taken primary responsibility for drafting the vision.

Film About Maryam Mirzakhani

One project for which we've already raised funds: a film about the life and work of the wonderful mathematician Maryam Mirzakhani, who made great contributions to the study of the dynamics and geometry of Riemann surfaces. The filming will take place in the U.S., Europe, and Iran. George Csicsery is the director, and we hope for a premiere at the Joint Mathematics Meetings in January 2020.

Contents

The View *(continued from previous page)*

Mirzakhani Research Professorships

Last, but certainly not least, we wish to share some additional news about another project under development in honor of Maryam Mirzakhani: the creation of a five million dollar endowment to support Mirzakhani Research Professorships in MSRI's scientific programs, with the enthusiastic support of her husband, Jan Vondrák.

Here is an excerpt from a piece that David and I wrote for the November AMS Notices on Maryam's impact at MSRI:

> *Maryam was a key member of the MSRI Scientific Advisory Committee from 2012–2016, the committee charged with selecting the scientific programs and their members. During part of that time she was very ill with what turned out to be her terminal cancer, but she never missed a meeting — even when she couldn't travel from Stanford to Berkeley she attended by video, between harsh medical treatments — and she faithfully did the substantial committee homework. It is fair to say that she was an inspiration to the entire committee.*
>
> *She played other important roles at MSRI, too. She was one of the main organizers of the semester-long program on Teichmüller theory and Kleinian groups in 2007, and a research professor for the spring of 2015. She was even a member of the complementary program when her husband, Jan Vondrák, came to MSRI for a computational program in the spring of 2005.*
>
> *Finally, in the spring before her death, Maryam had agreed to be nominated as a trustee of MSRI — although she warned David Eisenbud, MSRI's director, in a long and deeply sad meeting, that she might well not live to take office.*

Professor Mirzakhani's scientific work will endure and remain important, and through Chairs such as the one we propose, we can preserve the memory of her humble spirit of exploration, and her indomitable quest for knowledge.

The Mirzakhani Endowment

Monsoon0 (CC BY-SA 4.0)

To honor Maryam Mirzakhani's legacy, MSRI has proposed a five million dollar endowment to support four Mirzakhani Professors each year in programs at the institute. The photo above shows Maryam with her fellow Fields medalists from 2014 (Artur Avila at left and Martin Hairer at back), as well as with her daughter Anahita and Manjul Bhargava (far right), at the ICM 2014 in Seoul. You can learn more about Maryam's impact at MSRI in the View column on this page.

If you wish to learn more or to contribute to the endowment in Maryam's honor, you can visit msri.org/mirzakhani or contact Annie Averitt at development@msri.org or (510) 643-6056.

AMS/MSRI Congressional Briefing

In May 2018, the American Mathematical Society and MSRI hosted a joint congressional briefing on Capitol Hill featuring MacArthur Fellow and MIT computer scientist Erik Demaine. Demaine's talk on "Origami Meets Math, Science, and Engineering" shared surprising applications in manufacturing, robotics, animation, biology, medicine, nanotech, and space technology that have grown from new fundamental research in computational origami. Speakers at the Congressional briefing included Representative Jerry McNerney of California, MSRI Director David Eisenbud, AMS Associate Executive Director Karen Saxe, and AMS President Ken Ribet (UC Berkeley).

The next Congressional briefing takes place in December 2018, featuring Rodolfo H. Torres (University of Kansas), whose research includes collaborations with biologists, engineers, and economists. You can learn more and view short films about past events: msri.org/congress.

Hamiltonian Systems, from Topology to Applications through Analysis

(continued from page 1)

A mechanical system is described by its configuration space, mathematically a smooth manifold Q whose points, $q \in Q$, are understood as positions. The system is described by a Lagrangian function $L(q, \dot{q}) : TQ \to \mathbb{R}$, depending on the position q and the velocity $\dot{q}$. One considers the action functional

$$S[q(t)] = \int_{t_0}^{t_1} L(q, \dot{q})\, dt, \quad \delta q(t_0) = \delta q(t_1) = 0.$$

The time evolution of the mechanical system is described as a variational principle that the action S be extremal. This implies the Euler–Lagrange equations,

$$\frac{\partial L}{\partial q} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right).$$

Going from the tangent bundle TQ to the cotangent bundle T^*Q, one introduces momenta $p = \partial L/\partial \dot{q}$ and the Hamiltonian function $T^*Q \to \mathbb{R}$, given by the Legendre transform $H(q,p) = p\dot{q} - L(q,\dot{q})$. In the phase space T^*Q, the motion is described by Hamilton's first order differential equations,

$$\dot{p} = -\frac{\partial H}{\partial q} \quad \text{and} \quad \dot{q} = \frac{\partial H}{\partial p}.$$

The Hamiltonian form is not limited to finite-dimensional systems — indeed, action functionals are fundamental to the development of 20th century field theories in physics. A case in point is ϕ^4 field theory that has the action functional

$$S[\phi] = \int_{t_0}^{t_1} dt \int d^3x\, \mathcal{L}(\phi, \partial\phi),$$

with the Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_t \phi)^2 - \frac{1}{2}|\nabla\phi|^2 - \frac{1}{2}m^2\phi^2 - \frac{g}{4!}\phi^4.$$

The Legendre transform of this system gives the associated Hamiltonian form of the partial differential equations,

$$\partial_t \pi = -\frac{\delta H}{\delta \phi} \quad \text{and} \quad \partial_t \phi = \frac{\delta H}{\delta \pi},$$

where $\delta H/\delta\phi$ denotes the functional derivative and the Hamiltonian functional is

$$H = \int d^3x \left(\frac{1}{2}\pi^2 + \frac{1}{2}|\nabla\phi|^2 + \frac{1}{2}m^2\phi^2 + \frac{g}{4!}\phi^4\right),$$

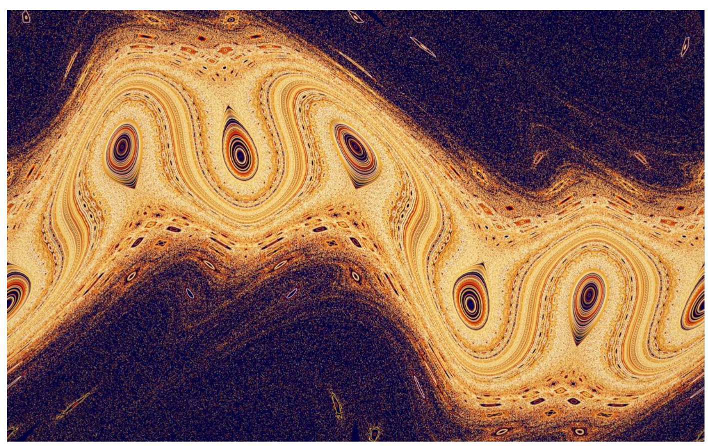
where m and g are parameters of the theory. By direct calculation one obtains a nonlinear wave equation as a Hamiltonian system.

There are many ramifications and generalizations, practical and aesthetic, of both the finite and infinite forms of Hamiltonian systems. We describe some of those addressed by our MSRI program below.

Dynamics on Symplectic Manifolds

The theoretical underpinning of Hamiltonian mechanics is symplectic geometry. (The term symplectic was adopted by H. Weyl to avoid any connotation of complex numbers that his previously-used term, "complex group," had suffered. It is simply the Greek adjective corresponding to the word "complex.") The cotangent bundle T^*Q of a smooth manifold carries a canonical symplectic structure $\omega = dp \wedge dq$, a closed non-degenerate differential 2-form. Other manifolds, not necessarily cotangent bundles, may also carry a symplectic structure. Hamilton's equations of motion describe the Hamiltonian vector field (or the symplectic gradient) X_H defined by the formula $\omega(X_H, \cdot) = -dH$.

An important example of a Hamiltonian system, and one of the research topics in this program, is the motion of a charged particle in a magnetic field. Mathematically, a magnetic field is represented by a closed differential 2-form β on a Riemannian manifold Q. One considers the twisted symplectic structure $\omega + \pi^*(\beta)$ on the cotangent bundle T^*Q, where $\pi : T^*Q \to Q$ is the projection. The motion of a charge is described as the Hamiltonian vector field of the energy function $\|p\|^2/2$ (the norm is taken using the metric on Q) with respect to the twisted symplectic structure. One of the problems of contemporary interest is the existence of periodic motions of the charge.



Iteration of the Standard Nontwist map (del-Castillo-Negrete et al., *Physica D* **91**, 1 (1996)) that shows invariant tori (continuous curves) embedded in a chaotic sea of orbits. (Figure courtesy of George Miloshevich.)

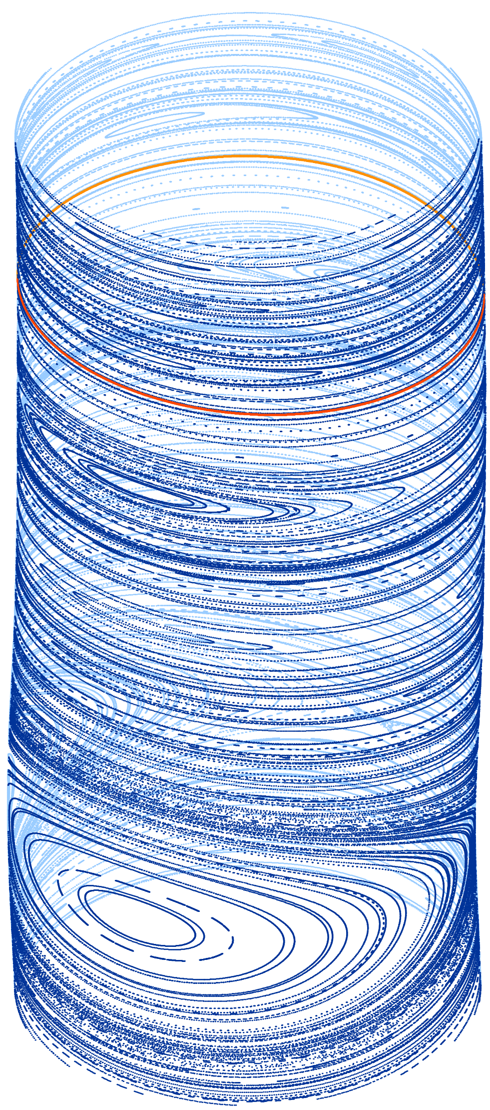
Other theoretical backgrounds of Hamiltonian dynamics include the calculus of variations, Morse theory, and Floer theory. Floer theory, in particular, is designed for the study of symplectic dynamics of Arnold's conjecture concerning fixed points of Hamiltonian diffeomorphisms (flows of time-dependent Hamiltonian vector fields). In the simplest case, this conjecture states that an area and center of mass preserving diffeomorphism of a torus has at least three, and generically at least four, distinct fixed points (proved in the 1980s by Conley and Zehnder).

Integrability and Chaos

Hamiltonian systems exhibit a wide variety of dynamical behavior, from very regular (completely integrable) to chaotic. In a completely integrable system (for example, Kepler's problem in celestial mechanics), the motion is typically confined to tori that have half the dimension of the phase space, and the motion on these invariant tori is described by constant vector fields.

While completely integrable systems are interesting, they are extremely rare. The Kolmogorov–Arnold–Moser (KAM) theory studies small perturbations of integrable systems; its fundamental result is that some invariant tori persist under small perturbations.

If the dimension of the phase space is greater than two, the invariant tori do not separate the space, and a phase trajectory may escape to infinity. This and related phenomena are known as Arnold diffusion. Arnold diffusion is one of the focal points of this topical semester, and many experts in this field are participants of the program. KAM theory and the theory of Arnold diffusion have many applications, in particular, in celestial mechanics (for example, "Is the solar system stable?")



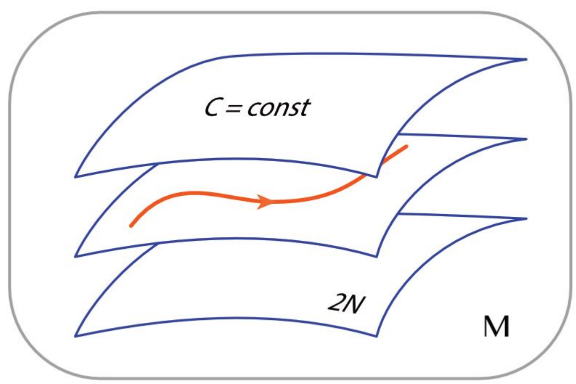
Normally hyperbolic invariant cylinder in a Froeschlé map. This type of object is relevant to the mechanism of Arnold diffusion; for example, moving around a 2-torus in a four-dimensional manifold. (Figure courtesy of Alex Haro.)

Flows on Poisson Manifolds

Poisson manifolds are a generalization of symplectic manifolds, where a manifold M has instead of a symplectic 2-form a Poisson bracket $\{,\}: C^\infty(M) \times C^\infty(M) \to C^\infty(M)$ for arbitrary functions $f, g, h \in C^\infty(M)$ satisfying

1. Bilinearity: $\{f+\lambda g, h\} = \{f,h\} + \lambda\{g,h\} \quad \lambda \in \mathbb{R}$
2. Skew symmetry: $\{f,g\} = -\{g,f\}$
3. Jacobi identity: $\{\{f,g\},h\} + \{\{g,h\},f\} + \{\{h,f\},g\} = 0$
4. Leibniz Rule: $\{fg,h\} = g\{f,h\} + f\{g,h\}$

Here, the Poisson bracket $\{f,g\} = J(df \wedge dg)$, written in terms of the Poisson bivector J, generates the vector field $X_f = \{f,\cdot\}$ whose integral curves are the trajectories of the Hamiltonian dynamics of interest. Because the bivector does not have the usual form, systems of this type are sometimes called noncanonical Hamiltonian systems. Unlike conventional Poisson brackets, the noncanonical Poisson brackets of Poisson manifolds are degenerate with special invariants know as Casimir invariants $C \in C^\infty(M)$ that satisfy $\{C,f\} = 0$ for all $f \in C^\infty(M)$. Because of this degeneracy flows are constrained to submanifolds that are in fact generically symplectic manifolds.



Cartoon of Poisson manifold with its foliation by symplectic leaves.

The study of Poisson manifolds is important both because of intrinsic mathematical interest and because infinite-dimensional versions of such flows generated by noncanonical Poisson brackets describe many physical systems. These flows are systems of partial differential equations that have a Hamiltonian form given by

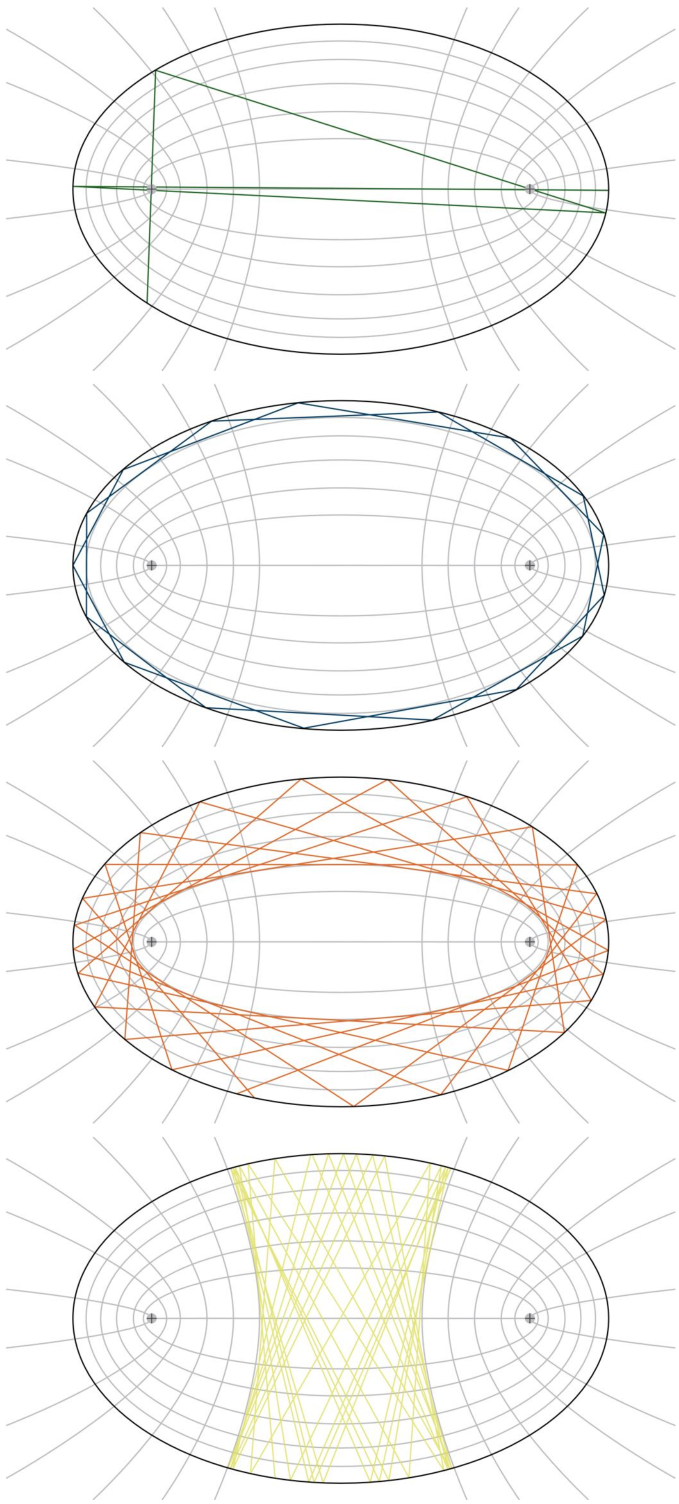
$$\partial_t \chi = \mathcal{J}(\chi)\frac{\delta H}{\delta \chi},$$

where χ denotes the set of dependent field variables and $\mathcal{J}$ is a Poisson operator that is a generalization of the Poisson tensor J of the Hamiltonian bivector for finite systems.

Mathematical Billiards

One of the popular Hamiltonian systems is a mathematical billiard that describes the motion of a mass-point in a domain, subject to specular reflections off the boundary. Many mechanical systems with elastic collisions — that is, collisions in which the energy and momentum are preserved — are described as billiard systems.

On a basic level, billiards can be viewed as a study of non-smooth Hamiltonian vector fields, one motivated by a physically relevant setup. The study of billiards was put forward by Birkhoff, who observed (in 1927) that "... in this problem the formal side, usually so formidable in dynamics, almost completely disappears, and only the interesting qualitative questions need to be considered."



Trajectories for a billiard inside an ellipse, broken out from the cover image. (Figure courtesy of Vadim Kaloshin.)

The billiard system can be considered as a continuous-time Hamiltonian system with discontinuities corresponding to the reflections. It can be also considered as a discrete-time system, that is, a transformation acting on the oriented lines, thought of as segments of a billiard trajectory. The space of oriented lines (rays of light) carries a symplectic structure, and the billiard ball map is a symplectic transformation.

Just like general Hamiltonian systems, planar billiards exhibit a full spectrum of dynamical behaviors, from completely integrable to chaotic. A completely integrable example is the billiard inside an ellipse: the interior of the billiard table is foliated by caustics which are confocal conics.

A notoriously hard problem in this area is known as Birkhoff's conjecture: *If a neighborhood of the boundary of a billiard table is foliated by caustics, then it is an ellipse*. There was a very substantial progress made toward the proof of this conjecture and its variation (for example, algebraic integrability, where the conserved quantity is a polynomial in momentum). A number of the main players in this field participate in the program.

Applications

Given the history of its development, it comes as no surprise that Hamiltonian dynamics plays an important role in modern theoretical physics and an array of applications, a few of which we note.

The Hamiltonian dynamics of bodies under the influence of gravity, celestial mechanics, is of course of basic importance for understanding the dynamics of the solar system and beyond (planets and stars). However, it also is needed for practical satellite navigation, for communication and space exploration, and the tracking of possibly deleterious astroids.

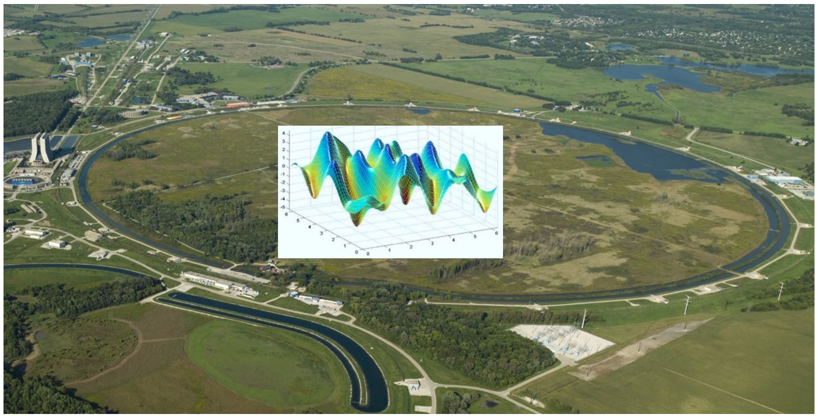
The Hamiltonian dynamics of gravitating bodies is a special case of so-called natural Hamiltonian systems where $H(q,p) = T(q,p) + V(q)$ with $T(q,p)$ the kinetic energy (mathematically, a Riemannian metric on the fiber of T^*Q) and $V(q)$ the potential energy. Examples of natural Hamiltonian systems are spring systems, pendula, particles in potential wells, and the N-body problem with $q_i \in \mathbb{R}^3$, $i = 1, \ldots, N$, where

$$H(q,p) = \sum_{i=1}^{N} \frac{\|p_i\|^2}{2m_i} + \sum_{i,j=1}^{N} \frac{c_{ij}}{|q_i - q_j|}$$

and c_{ij} represents the interaction. In the context of celestial mechanics, the interaction represents gravitational attraction, but the N-body setup can also describe the electrostatic interaction of repelling electrons, electrons and ions (protons) that attract each other, and the collection of both that occurs in plasmas.

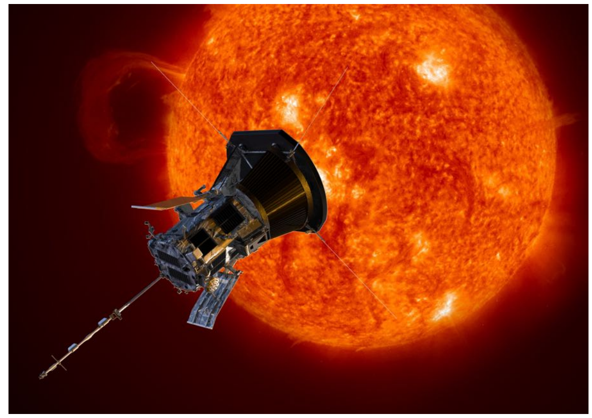
If charged particles experience the full electromagnetic interaction then the Hamiltonian is given by

$$H(q,p) = \frac{\|p - eA(q,t)\|^2}{2m} + e\phi(q,t),$$



Fermilab

The Tevatron — a large, 1 km-radius particle accelerator at Fermilab near Chicago — accelerated particles to near the speed of light and stored them for many hours. Understanding the particle orbits in such a system is a complicated Hamiltonian dynamics problem of estimating minute deviations from integrability. (Inset courtesy of Martin Berz.)



NASA/Johns Hopkins APL/Steve Gribben

The Parker Solar Probe, already in orbit around the sun, will characterize plasma near the sun (within 10 solar radii) and investigate magnetic fields, energetic particles, and the creation of the solar wind that impacts the Earth's environment and affects space weather.

where the magnetic field $B = \nabla \times A$ and the electric field $E = -\nabla\phi - \partial_t A$. The dynamics under (relativistic versions of) such Hamiltonians is of great importance for the design of particle accelerators such as Fermilab where one must corral and accelerate particles with sufficient luminosity to probe the nature of elementary particles. In addition accelerator technology is essential for the medical physics of radiotherapy, radiology, nuclear medicine, and oncology.

The Hamiltonian dynamics of charged particles is of fundamental importance for understanding naturally occurring plasmas such as that near the sun that will be explored with the Parker Solar probe, which will investigate the origin of the solar wind that impacts the earth's magnetosphere. In addition the Hamiltonian dynamics of charged particles is essential for understanding the laboratory plasmas created in large machines such as the ITER Tokamak that is being built in France. Particle dynamics in magnetic fields is particularly important for the design of such devices that use a strong magnetic field to confine plasma within a toroidal region with the goal of producing controlled thermonuclear fusion to generate power.

In addition to the finite-dimensional systems described above, there are infinite-dimensional systems — field theories — that describe collective motion. An important example is the Vlasov–Maxwell system, a kinetic theory that takes into account the fact that charges, in addition to producing electric fields, have motions, currents, that produce magnetic fields and electromagnetic wavelike motions. A reduced form of this system describes average features of large scale stellar dynamics.

Other field theories include Euler's equations of fluid mechanics; shallow water theory and the quasi-geostrophic equations, important equations of geophysical fluid dynamics that describe aspects of atmospheric and ocean dynamics; magnetohydrodynamics and two-fluid theory, fluid theories of plasma physics that include the magnetic and electric fields as dynamical variables; and sundry additional equations that describe various aspects of media treated as a continuum.

All of these infinite-dimensional systems are Hamiltonian systems, and they all possess the form of flows on infinite-dimensional Poisson manifolds as described above. Indeed, the discovery of their Hamiltonian form provided a major impetus for the study of Poisson manifolds begun in the 1980s.

Structure-Preserving Computation

Given that Hamiltonian systems are defined in terms of a differential 2-form, it comes as no surprise that various structures are preserved by the dynamics. In fact, the 2-form itself is preserved in the sense $\pounds_{X_H}\omega = 0$, where $\pounds_{X_H}$ is the Lie derivative with the Hamiltonian vector field X_H. One understands that Hamiltonian dynamics is a one-parameter temporal map of the phase space manifold to itself by a canonical transformation (symplectomorphism) that preserves symplectic area defined by the 2-form.

Numerical algorithms that preserve this structure are known as symplectic integrators. In addition to the exact conservation of a symplectic area, which can be wedged together to make a notion of volume preservation, symplectic integrators prevent the Hamiltonian (energy) from deviating significantly from its theoretically constant value. Thus they provide improved performance for long-time computation.

Recently, the variational form of Hamiltonian dynamics has been exploited to obtain variational integrators based on discretizing the variational principle, giving rise to desirable preservation of geometric structure. Poisson integrators are numerical algorithms that have exact conservation of the symplectic leaves of a Poisson manifold, as well as being symplectic on symplectic leaves.

Currently, one major challenge is to extract from Hamiltonian partial differential equations finite-dimensional (semi-discrete) systems of ordinary differential equations that inherit their parent Hamiltonian form and then to implement the resulting system with a symplectic or Poisson integrator. This is particularly difficult for infinite-dimensional systems that have noncanonical Poisson brackets.

MSRI Creates Summer Research for Women in Mathematics Program

Michael Singer, Assistant Director

> *Our week at MSRI enabled us to develop a small project into something substantial. We are working slowly for exactly the same reasons that [the time at MSRI] helped us: we are overflowing with commitments at home (eight children across three authors!), and the isolation permitted the focus needed to advance.*
>
> — *SRWM participant, Summer 2017*

The Challenge

Research in mathematics requires concentration and time. It is often greatly enhanced by the opportunity to collaborate in a small group: motivation is increased by the social interaction, ideas bounce between the collaborators, and team spirit plays a very positive role.

Women have not typically been encouraged to go into mathematical research. Even after women enter the profession, a number of societal forces tend to put them in less research-active environments. This, possibly together with other factors, has led to a waste of talent. A number of conferences for women in particular areas (Women in Topology, Women in Combinatorics, Women in Numerical Methods for PDEs, . . .) have sprung up to ameliorate this situation.

In a typical conference for women, for example at the Banff International Research Station (BIRS), a group of women meet for a week and divide into small research teams, mixing established researchers and younger PhDs, to focus on particular research problems. However, a week is too short: projects remain unfinished when the teams return to their professional and personal responsibilities and the interaction is hard to maintain.

To improve the odds of completion and enhance the quality of the products of such collaborations, as well as others begun in different ways, MSRI initiated the Summer Research for Women in Mathematics (SRWM) program. Groups of two to six women with established projects can apply for funding to spend a minimum of two weeks together at MSRI, living and working together and making use of the institute's excellent resources for collaborative work in a period when the weather is cool — this is the Bay Area, after all — and MSRI has spare office capacity.

A Great Idea, Quickly Acted On

The idea for such a program originated in May 2017 when BIRS hosted a week-long workshop, Algebraic Combinatorixx 2, which brought together 42 women who worked, in small groups, on eight different projects in algebraic combinatorics. The workshop was an astounding success: some of the participants had not even known each other prior to the workshop, but they formed cohesive groups and made serious progress toward solving the problems that were set forth by the team leaders. The energy and excitement of being among peers was palpable throughout the week.

As the end of the week approached, Georgia Benkart (Trustee of MSRI) and Hélène Barcelo (MSRI's Deputy Director and current Acting Director), who were among the participants, realized that to preserve the energy built and the work accomplished throughout the week, these groups must be given the opportunity to meet

Summer 2018 participants Candice Price, Suzanne Sindi, Shelby Wilson, Nina Fefferman, and Nakeya Williams. *On the cover:* Summer 2018 participant Zajj Daugherty.

again. MSRI acted quickly and opened its doors to the participants during the following summer to so that they could pursue (and, when possible, complete) the projects started at BIRS.

Immediate Success

Four groups (a total of 13 researchers) applied and spent a week in Berkeley during the summer of 2017. One group has had a paper published, one has submitted a paper, and the remaining two groups have nearly completed manuscripts and hope to submit papers in the next few months.

It is remarkable that so many of the participants who attended both the BIRS workshop and the week at MSRI were able to produce a publication from their work. Several mentioned that knowing they would reunite at MSRI helped them maintain contact following the BIRS workshop. This added to the energy to push their projects along, and ensured that they would make the most out of the added time together.

Keeping the Program Going

The SRWM program continued in the summer of 2018 with support from the National Security Agency (NSA), supplemented by MSRI funds. MSRI was able to expand its reach beyond women from the BIRS workshops, and we received applications from 22 groups (totaling 81 individual applicants). MSRI was able to accept 21 researchers forming six groups of two to five researchers each, covering topics in Lie algebras, Riemannian geometry, Hecke algebras, PDEs, and evolutionary biology. All six groups continue to work together, and many expect to be able to submit papers for publication within the next few months.

An important factor in the success of this project was MSRI's ability to offer family support funds to the many women with children. Because of this, they were able to either bring their children to Berkeley accompanied by a family member or leave them at home with trusted childcare. Being away from family during the summer is not easy, and without adequate financial support, it would not have been possible for many of them to participate in these working groups at MSRI.

In exit surveys, participants noted the benefits of distraction-free, in-person collaboration; the value of candid discussions among peers regarding family and work/life balance; and their unexpected sense of relief and comfort in sharing partial or incomplete ideas and asking questions in a women-only setting.

The program will continue in Summer 2019 with an increased number of participants, thanks to the generous support of the Lyda Hill Foundation, the NSA, Microsoft Research, and the National Science Foundation. To learn more, visit msri.org/srw2019.

Save the Date! — National Math Festival: May 4, 2019

Welcome to the Third NMF!

We welcome you to join us at the National Math Festival this coming May 4th in Washington, DC! This free, all-ages event takes place at the Washington Convention Center and features a day of lectures, demos, art, films, performances, puzzles, games, children's book readings, and more to bring out the unexpected in mathematics for everyone. Over 20,000 visitors joined the 2015 and 2017 Festivals. The 2019 National Math Festival is organized by MSRI in cooperation with the Institute for Advanced Study (IAS) and the National Museum of Mathematics (MoMath).

Highlights

The **Alfred P. Sloan Foundation Film Room** will be home to many short, creative math films throughout the Festival, as well as opportunities for Q&A with several 2019 Festival presenters and winning filmmakers from the National Science Foundation-sponsored "We Are Mathematics" contest.

The **Make or Take Spiral** hosts dozens of national and local math organizations offering hands-on activities and take-home resources for math educators and visitors of all ages.

Explore **Mathical Book Prize** award-winning titles with live author readings for ages 2-18!

You can find more information at nationalmathfestival.org, including resources to share the invitation with your family, friends, and community. A full schedule of all 2019 Festival events is online at nationalmathfestival.org/events.

Featured Presenters

BARKIN/SELISSEN PROJECT • Holly Krieger (University of Cambridge) • Mark Mitton • Lillian Pierce (Duke University) • Annie Raymond (University of Massachusetts Amherst) • Emily Riehl (Johns Hopkins University) • Marcus du Sautoy (University of Oxford) • Nancy Scherich (University of California, Santa Barbara) • Francis Su (Harvey Mudd College) • James Tanton (Mathematical Association of America) • Amelia Taylor (Zymergen) • Joseph Teran (University of California, Los Angeles) • John Urschel (Massachusetts Institute of Technology) • Suzanne L. Weekes (Worcester Polytechnic Institute) • Avi Wigderson (IAS) • Mary Lou Zeeman (Bowdoin College)

Major Activity Presenters

National Museum of Mathematics (MoMath) • Julia Robinson Mathematics Festival • MIND Research Institute • Natural Math and Math On-A-Stick • The Bridges Organization • The Young People's Project

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*We thank our **2019 Festival Sponsors**, including the Simons Foundation, the Alfred P. Sloan Foundation, Eric and Wendy Schmidt, the National Science Foundation, the Gordon and Betty Moore Foundation, the Kavli Foundation, the American Mathematical Society, the Charles and Lisa Simonyi Fund for Arts and Sciences, and Northrop Grumman.*

Focus on the Scientist: Vered Rom-Kedar

Vered Rom-Kedar is a Chern Professor in the MSRI program Hamiltonian Systems, from Topology to Applications through Analysis. She has made outstanding contributions in the field of dynamical systems and their applications.

Vered Rom-Kedar

Vered was born in Haifa, Israel. Her mother, Yael Rom, was the first female pilot trained and certified by the Israeli Air Force.

By the age of 19, Vered had already obtained her first university degree from the Technion–Israel Institute of Technology located in Haifa. Then, after completing two years of military service in the IDF, Vered moved to Cal Tech where she obtained a Ph.D. in applied mathematics. After a postdoctoral fellowship at the University of Chicago, she settled at the Weizmann Institute of Science.

Since 2009 she has been the Estrin Family Chair of Computer Science and Applied Mathematics at the Weizmann Institute. She served as the head of the Computer Science and Applied Mathematics Department from 2014–16.

In spite of the Weizmann Institute being principally a research establishment, Vered is actively involved in the development of mathematical education at various levels. In addition to supervising M.Sc. and Ph.D. students, she served as the head of the Moross Research School of Mathematics and Computer Science from 2010–13. Since 2013 she has been a committee member for the development of national high school curricula in mathematics.

She is also a member of the editorial board for the SIAM Journal on Applied Dynamical Systems and a member of the editorial advisory board of Chaos: An Interdisciplinary Journal of Nonlinear Science.

Vered is well known for her important contributions to the problems of transport in chaotic systems, creation of islands of stability, and mathematical features of Fermi acceleration. In addition, she has applied her knowledge of dynamical systems to the modeling of the immune response in humans.

Her current research interests are centered around Hamiltonian systems, including both the theoretical aspects, such as near-integrable and near-billiard dynamics in multi-dimensional systems, and applications of the theory to fluid mixing, non-linear optics and resonant waves. She also works on mathematical modeling of the innate immune system with medical implications.

On her department website, Vered writes about herself: "In life, like in science, [I like] to do (too?) many different things (mothering three kids, painting, reading, water sports, biking, skiing, scuba-diving, and others), and thus suffer from the very good problem of lack of time to take a breath (which is yet another thing I like to do from time to time)." In addition, she is an excellent painter and a person highly respected and appreciated by her colleagues and friends.

— Vasily Gelfreich

Forthcoming Workshops

Jan 28–30, 2019: *Connections for Women: Derived Algebraic Geometry, Birational Geometry, and Moduli Spaces*

Jan 31–Feb 8, 2019: *Introductory Workshop: Derived Algebraic Geometry and Birational Geometry and Moduli Spaces*

Mar 6–8, 2019: *CIME 2019: Mathematical Modeling in K-16: Community and Cultural Context*

Mar 25–29, 2019: *Derived Algebraic Geometry and its Applications*

Apr 8–12, 2019: *Hot Topics: Recent Progress in Langlands Program*

May 6–10, 2019: *Recent Progress in Moduli Theory*

Jun 10–12, 2019: *Improving the Preparation of Graduate Students to Teach Undergraduate Mathematics*

Jun 15–Jul 28, 2019: *MSRI-UP 2019: Combinatorics and Discrete Mathematics*

Aug 15–16, 2019: *Connections for Women: Holomorphic Differentials in Mathematics and Physics*

Aug 19–23, 2019: *Introductory Workshop: Holomorphic Differentials in Mathematics and Physics*

Aug 29–30, 2019: *Connections for Women: Microlocal Analysis*

Sep 3–6, 2019: *Introductory Workshop: Microlocal Analysis*

Summer Graduate Schools

Jun 3–14, 2019: *Commutative Algebra and its Interaction with Algebraic Geometry*

Jun 10–21, 2019: *Random and Arithmetic Structures in Topology*

Jun 24–Jul 5, 2019: *Representation Stability*

Jul 1–13, 2019: *Séminaire de Mathématiques Supérieures 2019: Current Trends in Symplectic Topology*

Jul 1–12, 2019: *Geometric Group Theory*

Jul 8–19, 2019: *Polynomial Method*

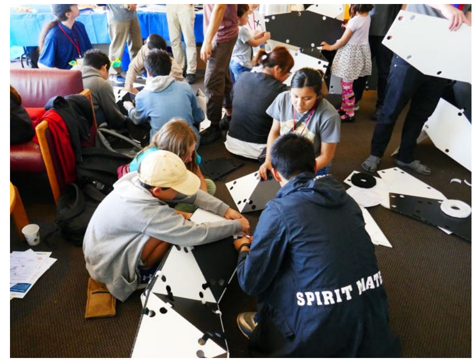
Jul 22–Aug 2, 2019: *Recent Topics on Well-Posedness and Stability of Incompressible Fluid and Related Topics*

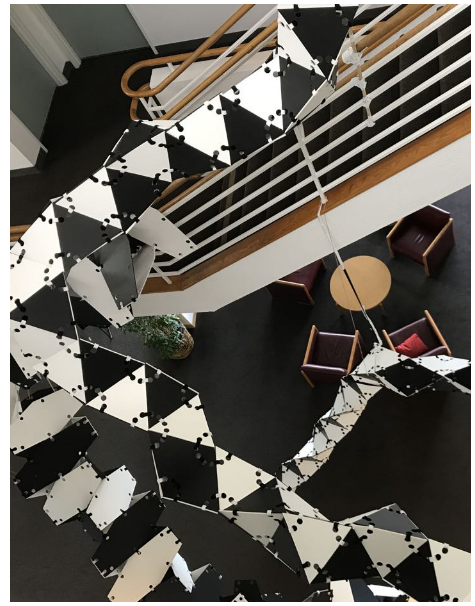
Jul 29–Aug 9, 2019: *Toric Varieties*

Jul 29–Aug 9, 2019: *Mathematics of Machine Learning (Microsoft)*

Jul 29–Aug 9, 2019: H-*Principle*

To find more information about any of these workshops or summer schools, as well as a full list of all upcoming workshops and programs, please visit msri.org/scientific.





Celebration of Mind 2018: Math, Magic, Games, Puzzles, & More

On October 21, 2018, MSRI hosted over 300 visitors at the institute's annual Celebration of Mind festival. Developed by the Gathering 4 Gardner Foundation, each year events are held around the world on or around Martin Gardner's birthday (October 21) so that people can meet and share in the legacy of this polymath.

This year's event featured stage presentations and activities by Carlo Séquin (UC Berkeley), Henry Segerman (Oklahoma State University), Brady Haran (Numberphile), magician Mark Mitton, and speedcuber Sydney Weaver. There was also hands-on fun with the California Math Festival, the Julia Robinson Mathematics Festival, and the Elwyn and Jennifer Berlekamp Foundation. In one activity, dozens of attendees worked together to build a three-story-tall dual helix sculpture with MoMath co-founder Glen Whitney. MSRI staff also hosted a Zometool geometric bubble blowing station and Mathical Book Prize reading nook. Mark your calendar to join us next October!

Left column: Gretchen Muller (at right) of the California Math Festival shares activities with families, while three young participants take a break with Mathical Book Prize winning titles in the Hearst Library. *Center:* Considering a move at the Julia Robinson Mathematics Festival in our commons room. *Right column:* UC Berkeley math students and festival visitors assemble a three-story dual helix with MoMath co-founder Glen Whitney, and the sculpture hanging in the atrium.

Named Positions, Fall 2018

MSRI is grateful for the generous support that comes from endowments and annual gifts that support faculty and postdoc members of its programs each semester.

Chern, Eisenbud, and Simons Professors

Gonzalo Contreras, Centro de Investigación en Matemáticas
Wilfrid Gangbo, University of California, Los Angeles
Jeffrey Lagarias, University of Michigan
James Meiss, University of Colorado Boulder
Richard Montgomery, University of California, Santa Cruz
Vered Rom-Kedar, The Weizmann Institute
Antonio Siconolfi, Sapienza – Università di Roma
Zensho Yoshida, University of Tokyo

Named Postdoctoral Fellows

Huneke: Rosa Vargas, Universidad Nacional Autónoma de México
Strauch: Gabriel de Oliveira Martins, University of California, Santa Cruz
Uhlenbeck: Thomas McConville, MIT
Viterbi: Joshua Burby, Courant Institute of Mathematical Sciences

More News from MSRI

Connections for Women, Fall 2018

Lisa Jacobs

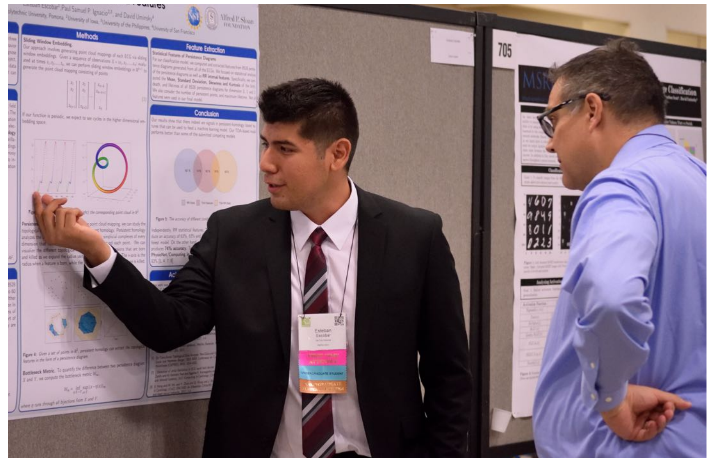
Connections for Women (CfW) is an ongoing workshop series that serves as an introduction to each semester's programmatic topics. In August, **Nancy Hingston** (The College of New Jersey) gave a CfW workshop on loop products and self-intersections in the Simons Auditorium for this semester's Hamiltonian Systems program.

MSRI Connections Abound in Recent Fields Medalists and AWM Fellows

In August, the International Mathematical Union announced this year's **Fields Medalists** at the International Congress of Mathematicians in Rio de Janeiro, Brazil. MSRI extends our sincere congratulations to the winners, all of whom have been connected to the Institute: three of the four — **Alessio Figalli** (ETH Zurich), **Peter Scholze** (Universität Bonn), and **Akshay Venkatesh** (Institute for Advanced Study) — have been in residence for recent programs at MSRI, while the work of the fourth winner, **Caucher Birkar** (University of Cambridge), was featured in a Hot Topics workshop on canonical models of algebraic varieties.

The Association for Women in Mathematics (AWM) has named their second class of **AWM Fellows**, whose commitment to supporting and growing women across the mathematical sciences is praised by their students and colleagues. Amongst the 2019 Fellows are MSRI Acting Director **Hélène Barcelo**, MSRI Trustees **Maria Klawe** (Harvey Mudd College) and **Dusa McDuff** (Barnard College), and Committee of Academic Sponsors Chair **Judy Walker** (University of Nebraska). Fourteen of this year's 23 fellows (and 22 of 32 fellows in the 2018 inaugural class) have been involved in governance and/or scientific programs at MSRI. For a complete list, visit tinyurl.com/AWMfellows.

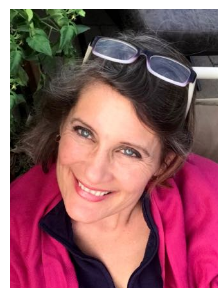
MSRI-UP at SACNAS

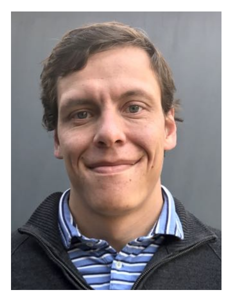


Paul Samuel Ignacio

MSRI-UP participants **Esteban Escobar** (Cal Poly Pomona; pictured) and **Heyley Gatewood** (Stetson University) won awards for poster presentations at this year's SACNAS national meeting. The posters presented results from their group research projects in the 2018 "Mathematics of Data Science" Research Experience for Undergraduates, funded by the National Science Foundation and the Alfred P. Sloan Foundation.

New Faces at MSRI

MSRI welcomed the following new staff members during the summer and fall of 2018. Pictured at top (left to right): **Bertram Ladner**, Facilities and Food & Beverage Coordinator; **Kimberly Manning**, Executive Assistant to the Deputy Director; **Nate Orage**, Technical Support. At bottom (left to right): **Demaur Owens**, Systems Administrator; and **Benjamin Parker**, Receptionist. **Annie Averitt**, Director of Advancement and External Relations, is introduced in the View from MSRI on page 2.

Call for Membership

MSRI invites membership applications for the 2019–2020 academic year in these positions:

Research Members	by December 1, 2018
Postdoctoral Fellows	by December 1, 2018

In the academic year 2019–2020, the research programs are:

Holomorphic Differentials in Mathematics and Physics
Aug 12 – Dec 13, 2019
Organized by Jayadev Athreya, Steven Bradlow, Sergei Gukov, Andrew Neitzke, Anna Wienhard, Anton Zorich

Microlocal Analysis
Aug 12 – Dec 13, 2019
Organized by Pierre Albin, Nalini Anantharaman, Kiril Datchev, Raluca Felea, Colin Guillarmou, Andras Vasy

Quantum Symmetries
Jan 21 – May 29, 2020
Organized by Vaughan Jones, Scott Morrison, Victor Ostrik, Emily Peters, Eric Rowell, Noah Snyder, Chelsea Walton

Higher Categories and Categorification
Jan 21 – May 29, 2020
Organized by David Ayala, Clark Barwick, David Nadler, Emily Riehl, Marcy Robertson, Peter Teichner, Dominic Verity

MSRI uses **MathJobs** to process applications for its positions. Interested candidates must apply online at mathjobs.org after August 1, 2018. For more information about any of the programs, please see msri.org/scientific/programs.

Call for Proposals

All proposals can be submitted to the Director or Deputy Director or any member of the Scientific Advisory Committee with a copy to proposals@msri.org. For detailed information, please see the website msri.org.

Thematic Programs

The Scientific Advisory Committee (SAC) of the Institute meets in January, May, and November each year to consider letters of intent, pre-proposals, and proposals for programs. The deadlines to submit proposals of any kind for review by the SAC are **March 1**, **October 1**, and **December 1**. Successful proposals are usually developed from the pre-proposal in a collaborative process between the proposers, the Directorate, and the SAC, and may be considered at more than one meeting of the SAC before selection. For complete details, see tinyurl.com/msri-progprop.

Hot Topics Workshops

Each year MSRI runs a week-long workshop on some area of intense mathematical activity chosen the previous fall. Proposals should be received by **March 1**, **October 1**, or **December 1** for review at the upcoming SAC meeting. See tinyurl.com/msri-htw.

Summer Graduate Schools

Every summer MSRI organizes several two-week-long summer graduate workshops, most of which are held at MSRI. Proposals must be submitted by **March 1**, **October 1**, or **December 1** for review at the upcoming SAC meeting. See tinyurl.com/msri-sgs.

Focus on the Scientist: Jacques Féjoz

Jacques Féjoz is a mathematician of depth and impressive power. He has made several important contributions to stability and instability in celestial mechanics and Hamiltonian dynamics. Let me start with his remarkable result on the stability of a planetary system.

Jacques Féjoz

Stability versus instability in the solar system is one of the most important problems in dynamics. It is natural to study dynamics of the solar system using Newtonian equations. The most important result about stability of the solar system says that for a planetary system with small enough masses of planets, there is a positive measure of initial conditions leading to quasiperiodic motions, densely filling invariant tori in the phase space. The first important step toward proving this result was made by V. Arnold in the 1960s. In the 1990s, M. Herman made another important step, and finally in the early 2000s, Jacques completed the proof.

Another one of Jacques' fascinating results is proof of the existence of reparametrized quasiperiodic solutions accumulating to a double collision. This is fairly surprising since, heuristically, collisions lead to sensitive dependence on initial conditions.

Analysis of instabilities in the solar system is full of puzzles and open questions. One place in the solar system where instabilities are quite illuminating is the asteroid belt. It turns out that between the orbits of Jupiter and Mars, there are over one million astroids of diameter greater than 1 km. When one looks at the distributions of asteroids with respect to orbital period, there are gaps in the distribution. These so-called Kirkwood gaps correspond to rational relations between the periods of the asteroids and Jupiter. The most notorious ones are 1:3, 2:5, and 3:7.

One possible explanation of the 1:3 Kirkwood gap was proposed by J. Wisdom and, independently, by A. Neishtadt. Jacques, jointly with Guadia, Kaloshin, and Roldan, proposed another explanation and proved the existence of unstable motions in the regime of small Jupiter eccentricity.

Jacques is versatile. He does mathematics, plays oboe in the Université PSL orchestra, and also plays water polo, all at a high level. Jacques explains that doing mathematics and playing oboe have at least one thing in common: people generally don't know how hard it is.

— Vadim Kaloshin

Named Postdocs — Fall 2018

Huneke

Rosa Vargas

Rosa Maria Vargas Magaña is the Huneke Postdoctoral Fellow in the Hamiltonian Systems, from Topology to Applications through Analysis program. In 2017 she received her Ph.D. in Mathematical Sciences from the Universidad Nacional Autónoma de México working on surface water waves under the supervision of Panayotis Panayotaros. Her principal research interests are the derivation of simplified models for the study of physical phenomena that lead to nonlinear partial differential equations with rich mathematical structure. In particular, she introduced a Whitham–Boussinesq-type model that describes the effects caused by large order depth variations on surface water waves when the full linear dispersion is included in a weakly nonlinear approximation and exhibits a combination of nonlinear and variable depth effects involving non-local operators.

The Huneke postdoctoral fellowship is funded by a generous endowment from Professor Craig Huneke, who is internationally recognized for his work in commutative algebra and algebraic geometry.

Strauch

Gabe Martins

Gabriel Martins is this semester's Strauch fellow in the Hamiltonian Systems, from Topology to Applications through Analysis program. He grew up in São Paolo and Rio de Janeiro, Brazil. He got his bachelor's and master's degrees in the Universidade Federal do Rio de Janeiro where he wrote a master's thesis on the basics of Morse Theory and Symplectic Topology advised by Umberto Hryniewicz. He received his Ph.D. from UC Santa Cruz in the spring of 2018, where he worked with Richard Montgomery. Gabriel studies the dynamics of a charged particle under the influence of a strong magnetic field, specifically in a container whose field tends to infinity at the boundary. He proved that if the field blows up fast enough, then the charged particle is trapped in the interior for all time. Gabriel's methods also show promise for providing quantitative leakage results for containers with bounded magnetic fields — such as have been proposed as a way to achieve sustained nuclear fusion. Gabriel's current work also involves exploring the quantum/classical discrepancy between his work for classical particles and a quantum trapping theorem proven by Colin de Verdière and Truc, as well as investigating trapping when using the non-Abelian generalization of magnetic fields, known as Yang–Mills fields.

The Strauch postdoctoral fellowship is funded by a generous annual gift from Roger Strauch, Chairman of The Roda Group. He is a member of the Engineering Dean's College Advisory Boards of UC Berkeley and Cornell University, and is also currently the chair of MSRI's Board of Trustees.

Viterbi

Joshua Burby

Joshua Burby is the Viterbi Postdoctoral Fellow in this semester's Hamiltonian Systems, from Topology to Applications through Analysis program. After receiving his Ph.D. in 2016 from Princeton University's Plasma Physics Laboratory, under the supervision of Hong Qin, he obtained a US DOE Fusion Energy Sciences Postdoctoral Research Fellowship with residence at the Courant Institute of Mathematical Sciences in New York. After the Viterbi Fellowship, he will take up residence at the Los Alamos National Laboratory as a Feynman Postdoctoral Fellow. He was also awarded, but declined, a Humboldt Research Fellowship. Josh has broad interest in the geometric structure of physical law, but particularly the Hamiltonian and variational structure of dynamical systems that describe plasmas. Given the multi-scale complexity of these systems, he brings geometrical techniques of fast-slow dynamical systems theory to obtain faithful reduced models that maintain structure, both in removing fast scales to obtain reduced partial differential equations, and further to obtain structure-preserving discretizations for numerics.

The Viterbi postdoctoral fellowship is funded by a generous endowment from Dr. Andrew Viterbi, well known as the co-inventor of Code Division Multiple Access (CDMA) based digital cellular technology and the Viterbi decoding algorithm, used in many digital communication systems.

Correction!

In the Spring issue, the brief bio of Karen Uhlenbeck that appeared with her fellowship stated that she was the first woman mathematician named to the National Academy of Sciences in 1986. This is not the case: Julia Robinson, in 1976, became the first woman to be elected to the mathematical section of the National Academy of Sciences. We apologize for the error.

Puzzles Column

Elwyn Berlekamp, Joe P. Buhler, and Tanya Khovanova

*This puzzle column has been running for almost exactly 18 years. There have been guest editors along the way, but Elwyn and Joe, and MSRI, would like to take this opportunity to welcome **Tanya Khovanova** as a new long-term composer and editor. Tanya is at MIT, where she is Head Mentor for RSI (the Research Science Institute), and for MIT PRIMES. Tanya is well known in the recreational mathematics community for her blog, her online videos, and her clever problems. Two of her papers were chosen for the* Best Writing on Mathematics *volumes in 2014 and 2016. Tanya received her Ph.D. in 1988 from the Moscow State University (under Israel Gelfand) and worked as a systems engineer, applied mathematician, and lecturer before coming to MIT. Her early research was in representation theory and related topics, and she is now also interested in combinatorics and recreational mathematics.*

1. Amy and Bob play a game. Initially, there are 20 cookies on a red plate and 14 on a blue plate. A legal move consists either of eating two cookies on one plate, or moving one cookie from the red plate to the blue plate. The last player to move wins, and of course both Amy and Bob want to win. Amy makes the first move, and then they alternate moves. Who will win?

Comment: This problem appeared on the 2014 Bay Area Mathematical Olympiad.

2. Five people, *A*, *B*, *C*, *D*, *E*, sit around a table playing "Mafia." The players are secretly assigned roles: two are in the mafia, one is a detective, and other two are innocent (that is, neither a detective nor in the mafia). The mafiosi always lie, and the other players always tell the truth. The mafiosi know each other, and the detective knows who they are. The mafia members do not know who the detective is, and the two innocents know nothing about the others' roles. All players know the rules, and the number of players given each role. The following four statements are made in order.

A: I know who *B* is.

B: I know who the detective is.

C: I know who *B* is.

D: I know who *E* is.

Who is who?

Comment: The game is well known (for example, it has a Wikipedia page), and this puzzle can be found on the QWERTY YouTube channel. It also appeared on Tanya's blog.

3. Let $A = \{1, 2, 3, 4, 5\}$ and let P be the set of all nonempty subsets of A. A function f from P to A is a "selector" function if $f(B)$ is in B, and $f(B \cup C)$ is either equal to $f(B)$ or $f(C)$. How many selector functions are there?

Comment: We saw this on Stan Wagon's Problem of the Week blog (stanwagon.com/potw/), and it apparently originally appeared in a Turkish national competition.

© Lucian Alexandru Motoc | Dreamstime.com (Modified for use)

4. Let C be a circle of radius $r \geqslant 1$ in the plane. You are allowed to augment the set C by repeatedly performing the following operation: Take two points in C that are a distance 1 apart, and add the line segment between them to C. Show that it is possible to perform finitely many such operations so that the center of the circle is in the resulting set.

Comment: This problem is due to Gregory Galperin, who also asks the much harder question of what a reasonable upper bound is, as a function of r, on the number of augmentations that are required.

5. Assume that a correctly set clock has hour, minute, and second hands of the same length, and that the motion of those hands is continuous. Prove that it is impossible for the tips of those hands to be at the vertices of an equilateral triangle. Suppose that a clock builder chooses a wrong gear and the result is a clock which has correct hour and second hands, but the minute hand makes only 11 revolutions in a 12-hour period. Show that, despite the clock's flaws, it does have the virtue that the hand tips do occasionally lie at the vertices of an equilateral triangle. (Assume that at noon and midnight the hands all point straight up, as on a normal clock.)

6. Although hats problems have been something of a staple in our puzzle columns, a quite novel one appeared in an article by Tanya in the *Mathematical Intelligencer* in 2014. We follow this with a variant that we heard from Larry Carter and Stan Wagon; Wagon, Rob Pratt, Michael Wiener, and Piotr Zielinski have a paper on the arXiv that discusses further generalizations of these questions.

(a) There are n prisoners and $n+1$ hats. Each hat has its own distinctive color. The prisoners are put into a line by their friendly warden, who randomly places hats on each prisoner (note that one hat is left over). The prisoners "face forward" in line which means that each prisoner can see all of the hats in front of them. In particular, the prisoner in the back of the line sees all but two of the hats: the one on her own head, and the leftover hat. The prisoners (who know the rules, all of the hat colors, and have been allowed a strategy session beforehand) must guess their own hat color, in order starting from the back of the line. Guesses are heard by all prisoners. If all guesses are correct, the prisoners are freed. What strategy should the prisoners agree on in their strategy session?

(b) Suppose that there are three prisoners and *five* hats — that is, the warden will place three hats, and there will be two extra hats. The rules are as above. Find an optimal strategy!

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Make a Donation to MSRI

Autumnal Greetings from MSRI!

. . . and from the postdocs in this fall's jumbo program on Hamiltonian Systems, who enjoyed carving some jumbo pumpkins at a Halloween social in October.

Join us at the 2019 JMM in Baltimore

Mathematical Institutes Open House

Wednesday, January 16 5:30 – 8:00 pm	Marriott Inner Harbor Grand Ballroom East West ABC 1st Floor

MSRI Reception for Current and Future Donors

Thursday, January 17 6:30 – 8:00pm	Hilton Baltimore Carroll A 3rd Floor

For more information, contact development@msri.org.

MSRI Directorate

Add *@msri.org* to email addresses. All area codes are (510) unless otherwise noted.

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Hélène Barcelo, Acting Director, 643-6040, *director*
Michael Singer, Assistant Director, 2018-19, 643-6070, *msinger*
Annie Averitt, Director of Advancement and External Relations, 643-6056, *aaveritt*