Diversity and Freedom:
A Fundamental Tradeoff in Multiple Antenna Channels

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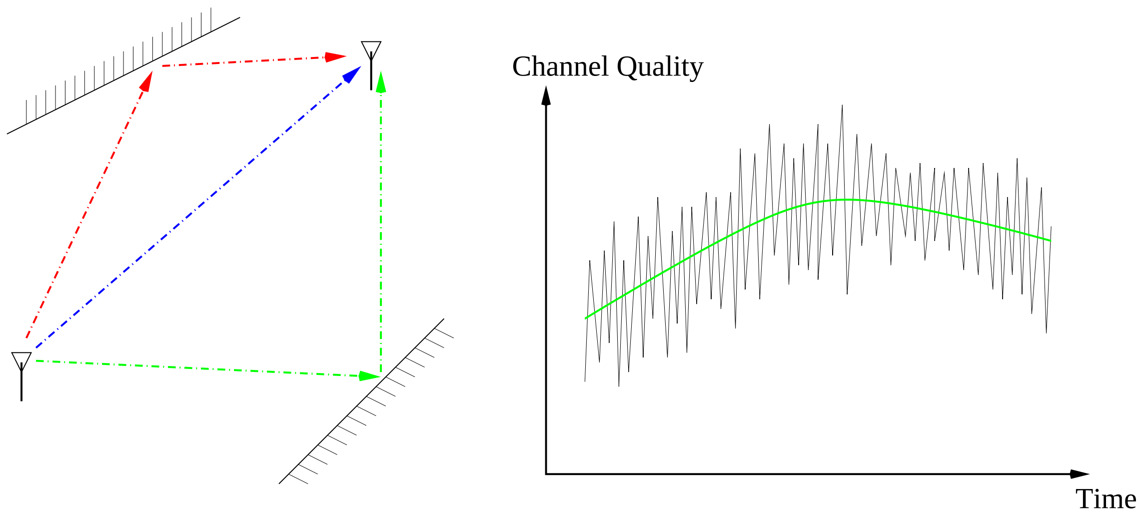
Wireless Fading Channels



Fundamental characteristic of wireless channels: multipath fading due to constructive and destructive interference.

Channel varies over time as well as frequency.

Multiple Antennas

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- Multi-antenna communication is a hot field in recent years.
- But the research community has a split personality.
- There are two very different views of how multiple antennas can be used.

Diversity

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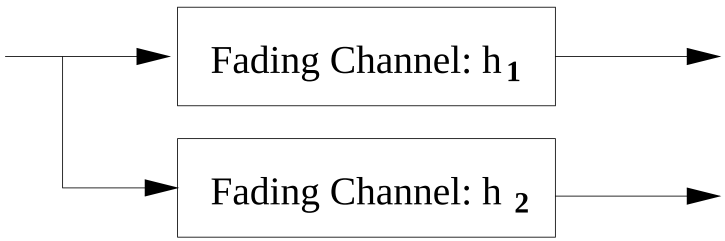
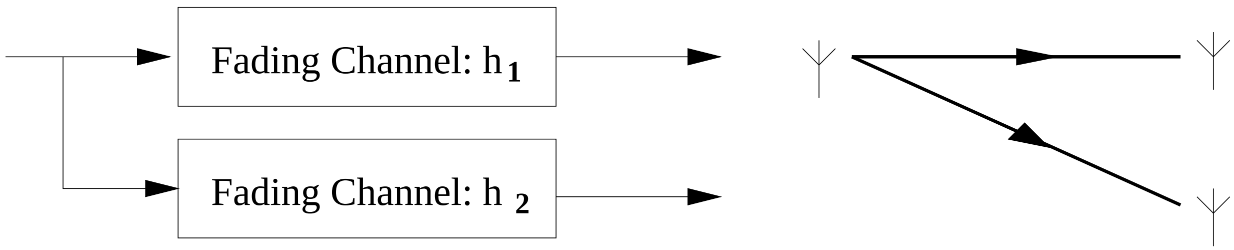


- Additional independent signal paths increase diversity.

Diversity



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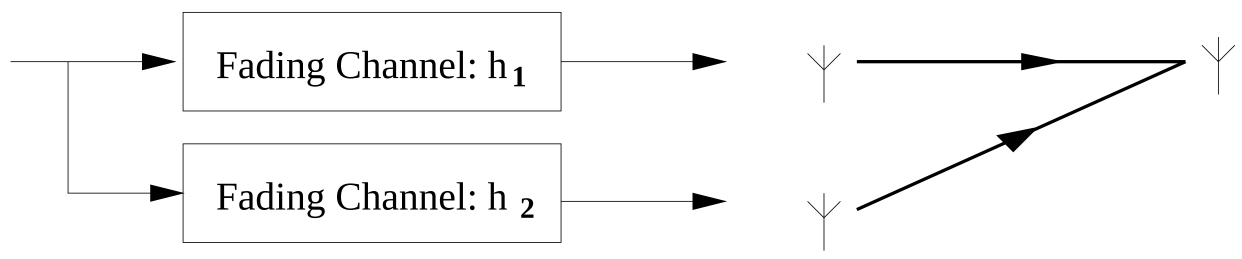
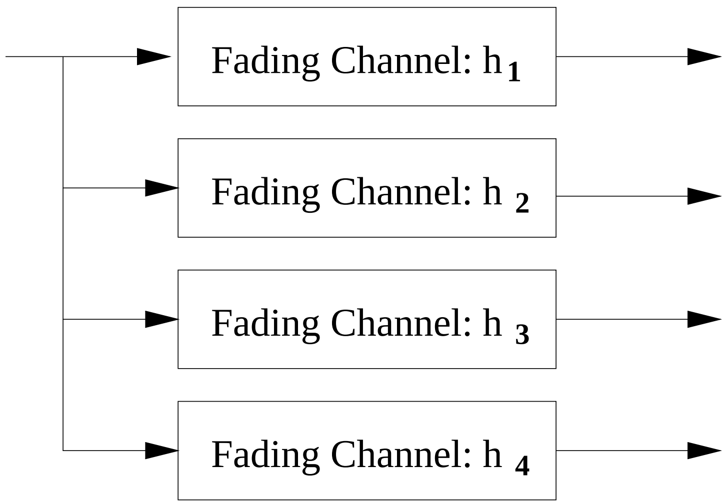
- Diversity: receive

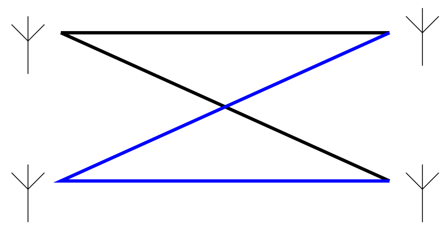
Diversity



- Additional independent signal paths increase diversity.
- Diversity: receive, transmit

Diversity





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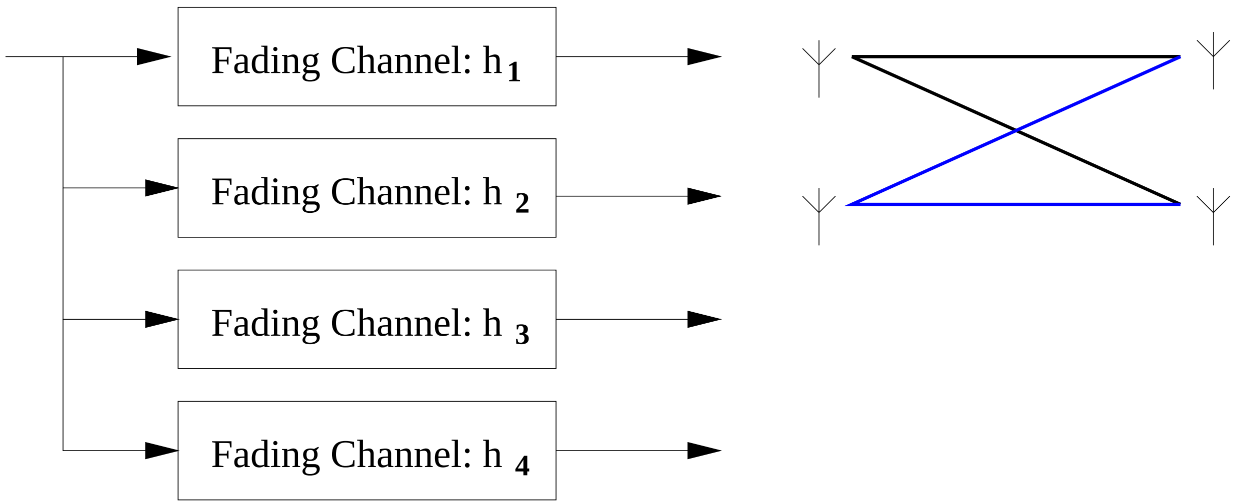
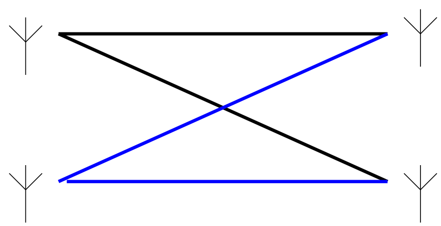


- Additional independent signal paths increase diversity.
- Diversity: receive, transmit or both.
- Compensate against channel unreliability.

Freedom



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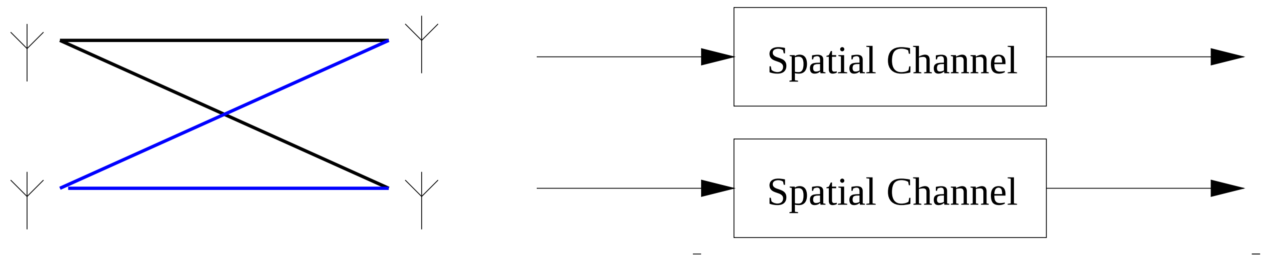
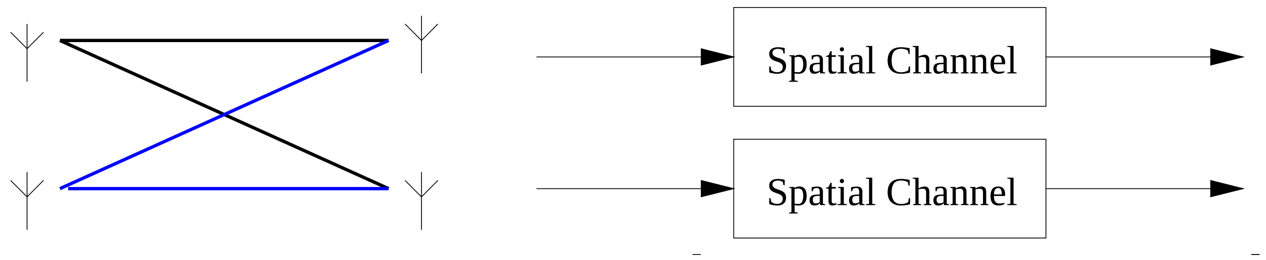


Another way to view a 2×2 system:

- Increases the degrees of freedom in the system

Freedom



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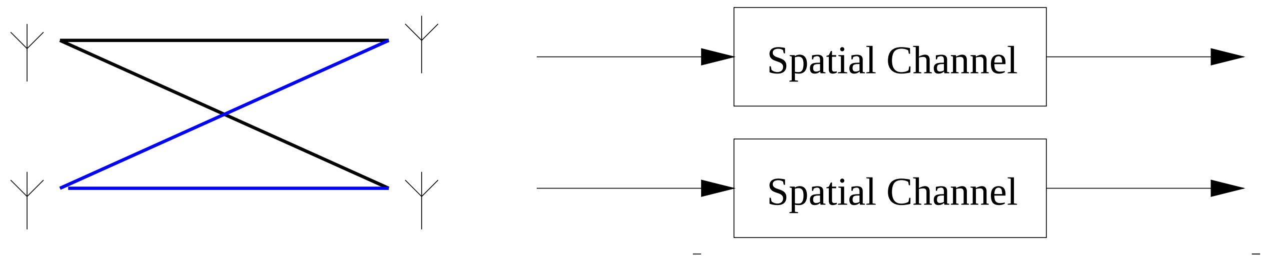

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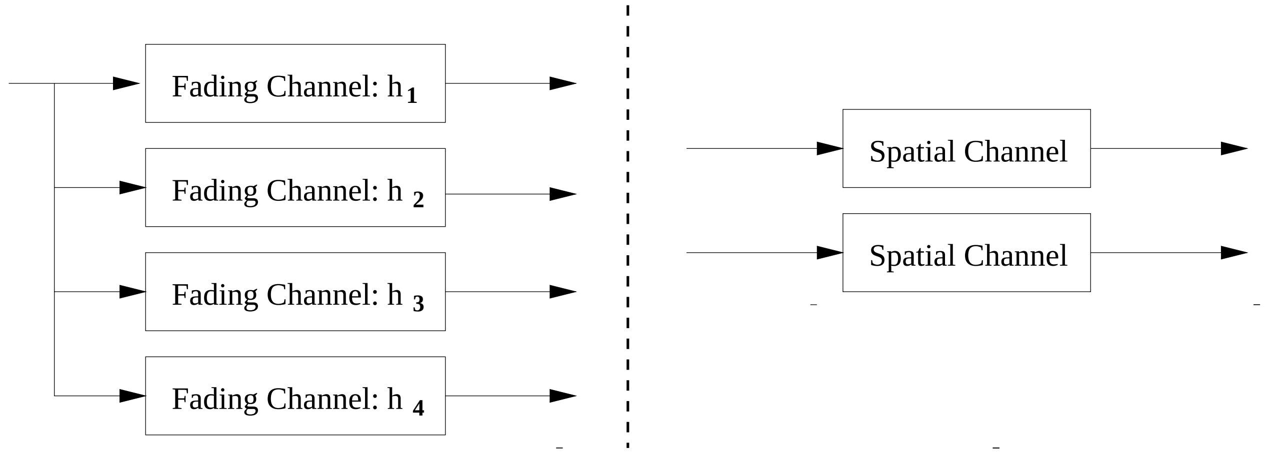
- Increases the degrees of freedom in the system
- Multiple antennas provide parallel spatial channels: spatial multiplexing
- Fading is exploited as a source of randomness.

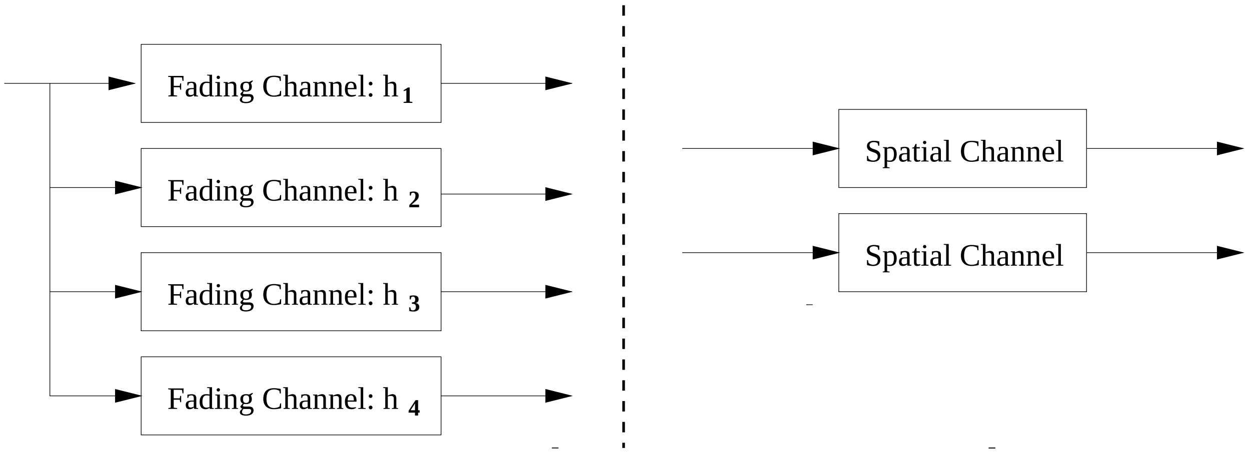
Diversity vs. Multiplexing



Multiple antenna channel provides two types of gains:

Diversity Gain vs. Spatial Multiplexing Gain

Diversity vs. Multiplexing



Multiple antenna channel provides two types of gains:

Diversity Gain vs. Spatial Multiplexing Gain

Existing schemes focus on one type of gain.

A Different Point of View

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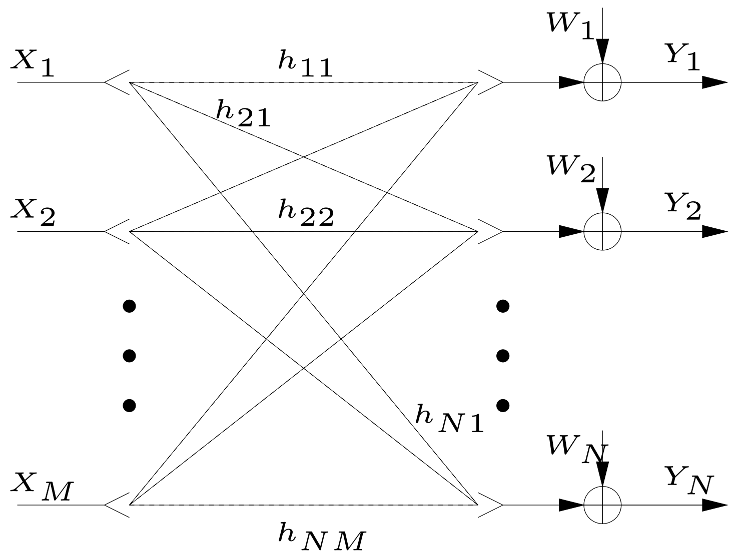
Both types of gains can be achieved simultaneously in a given multiple antenna channel, but there is a fundamental tradeoff.

We propose a unified framework which encompasses both diversity and multiplexing and study the optimal tradeoff.

Outline

- Problem formulation and main result on optimal tradeoff.
- Sketch of proof.
- Comparison of existing schemes.

Channel Model



$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}, \qquad w_i \sim \mathcal{CN}(0,1)$$

- Rayleigh fading i.i.d. across antenna pairs ($h_{ij} \sim \mathcal{CN}(0,1)$).
- Focus on codes of T symbols, where $\mathbf{H}$ remains constant (slow, flat fading)
- $\mathbf{H}$ is known at the receiver but not the transmitter.
- SNR is the average signal-to-noise ratio at each receive antenna.

How to Define Diversity Gain

Motivation: Binary Detection

$$\mathbf{y} = \mathbf{h}\mathbf{x} + \mathbf{w} \qquad\qquad P_e \approx P(\|\mathbf{h}\| \text{ is small }) \propto \mathsf{SNR}^{-1}$$

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Acutal error probability instead of pairwise error probability. (eg. Tarokh et al 98, Guey et al 99)

How to Define Spatial Multiplexing Gain

Motivation: (Telatar'95, Foschini'96)

Ergodic capacity:

$$C(\mathsf{SNR}) \approx \min\{M,N\} \log \mathsf{SNR} \quad (bps/Hz),$$

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A scheme is a sequence of codes, one at each SNR level.

Definition A scheme achieves spatial multiplexing gain r, if

$$R = r \log \mathsf{SNR} \quad (bps/Hz)$$

Increasing data rates instead of fixed data rate. (cf. Tarokh et al 98)

Fundamental Tradeoff

A scheme achieves

Spatial Multiplexing Gain r : $R = r \log \mathsf{SNR} \quad (bps/Hz)$

and

Diversity Gain d : $P_e \approx \mathsf{SNR}^{-d}$

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Spatial Multiplexing Gain r : $R = r \log \mathsf{SNR} \quad (bps/Hz)$

and

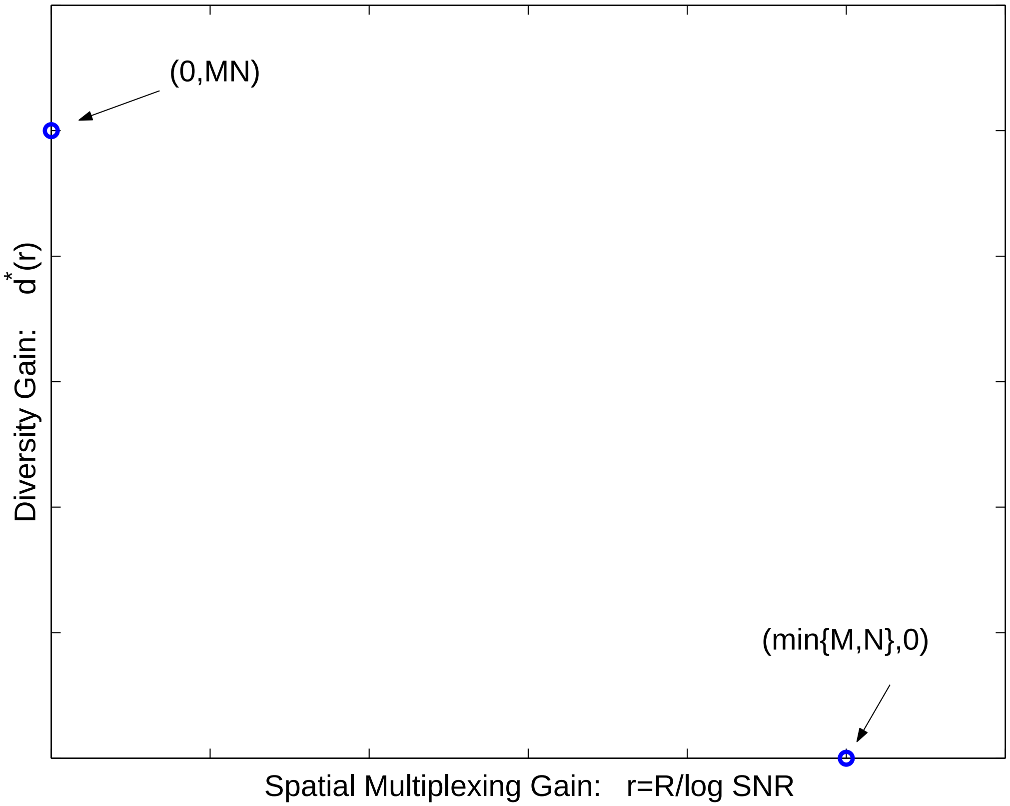
Diversity Gain d : $P_e \approx \mathsf{SNR}^{-d}$

Fundamental tradeoff: for any r, the maximum diversity gain achievable: $d^*(r)$.

$$r \longrightarrow d^*(r)$$

Main Result: Optimal Tradeoff

As long as $T \geq M + N - 1$:



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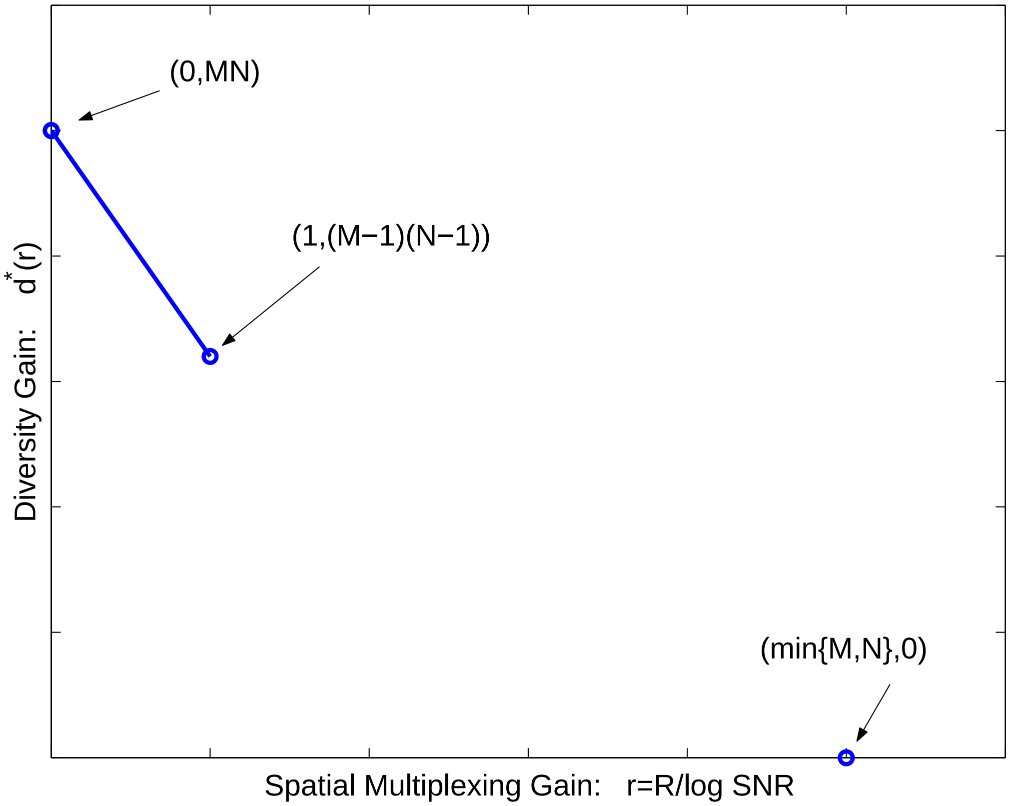

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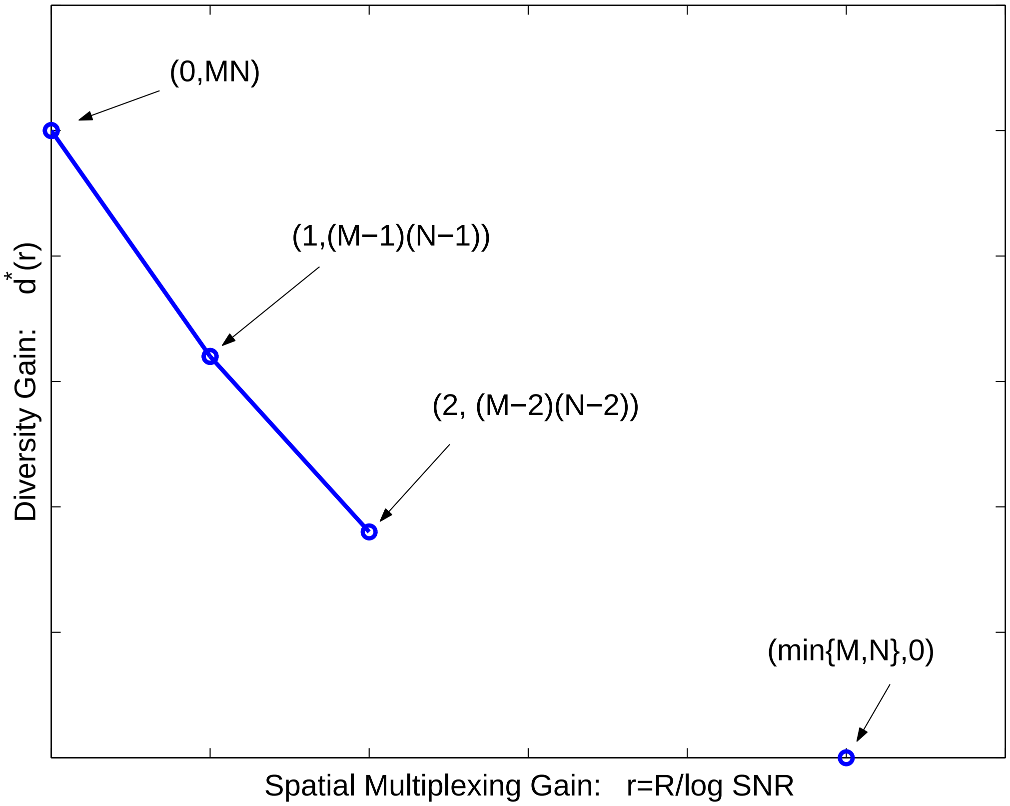

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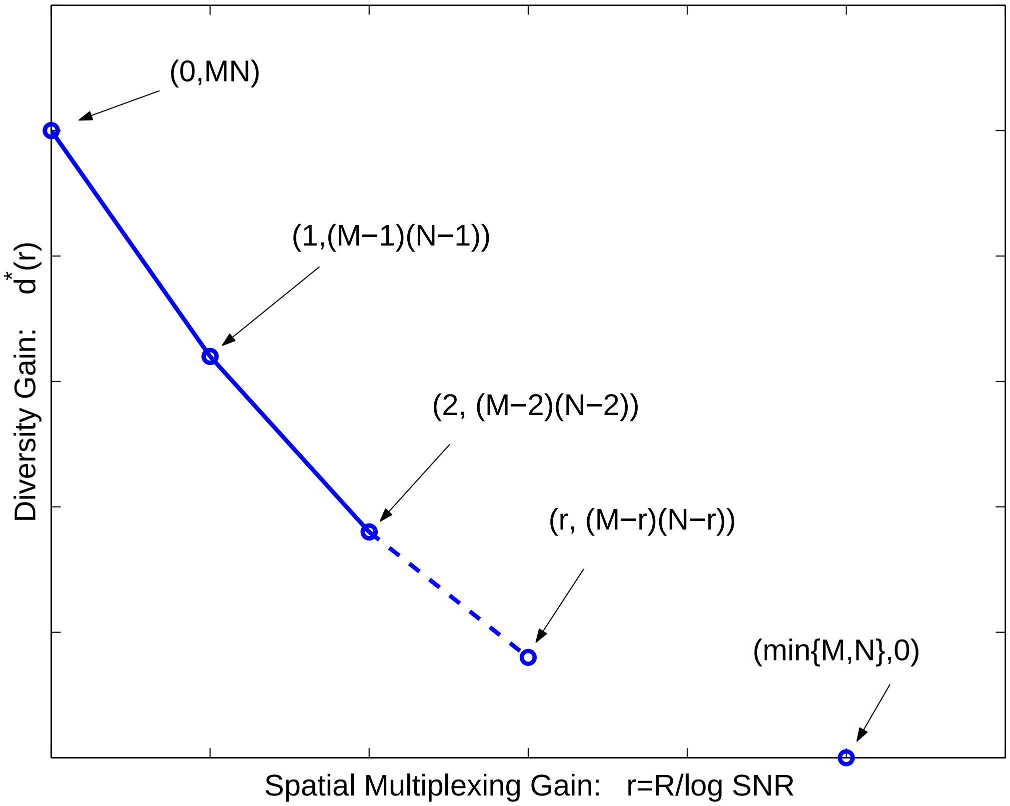

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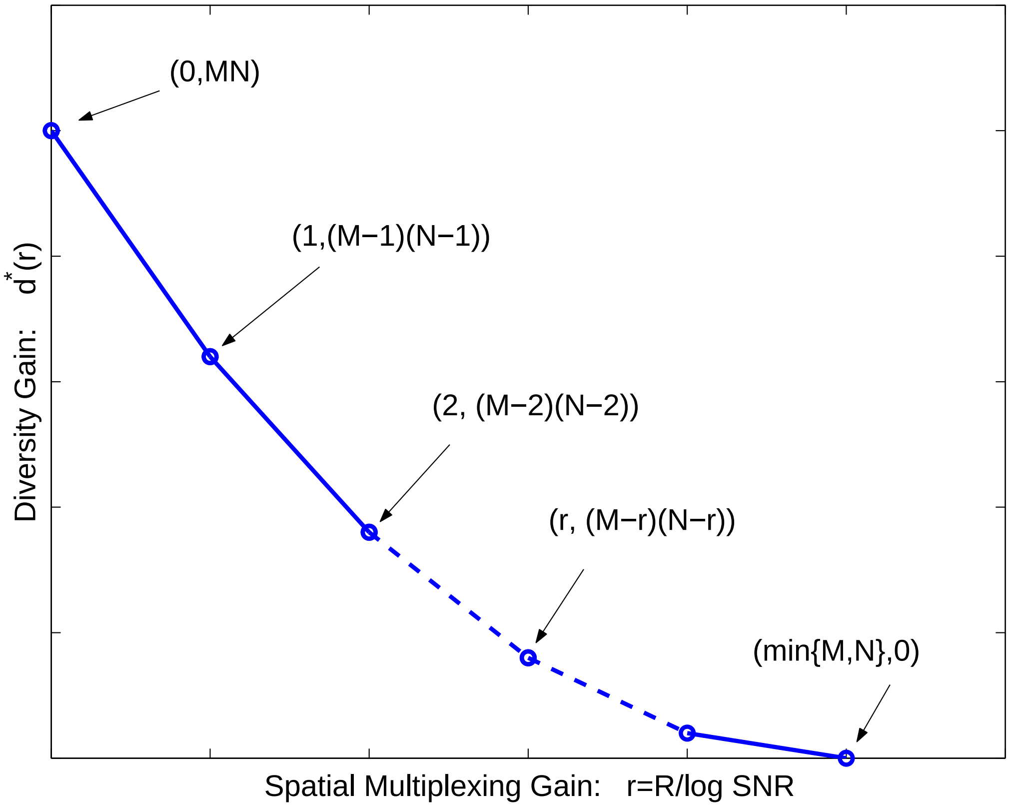

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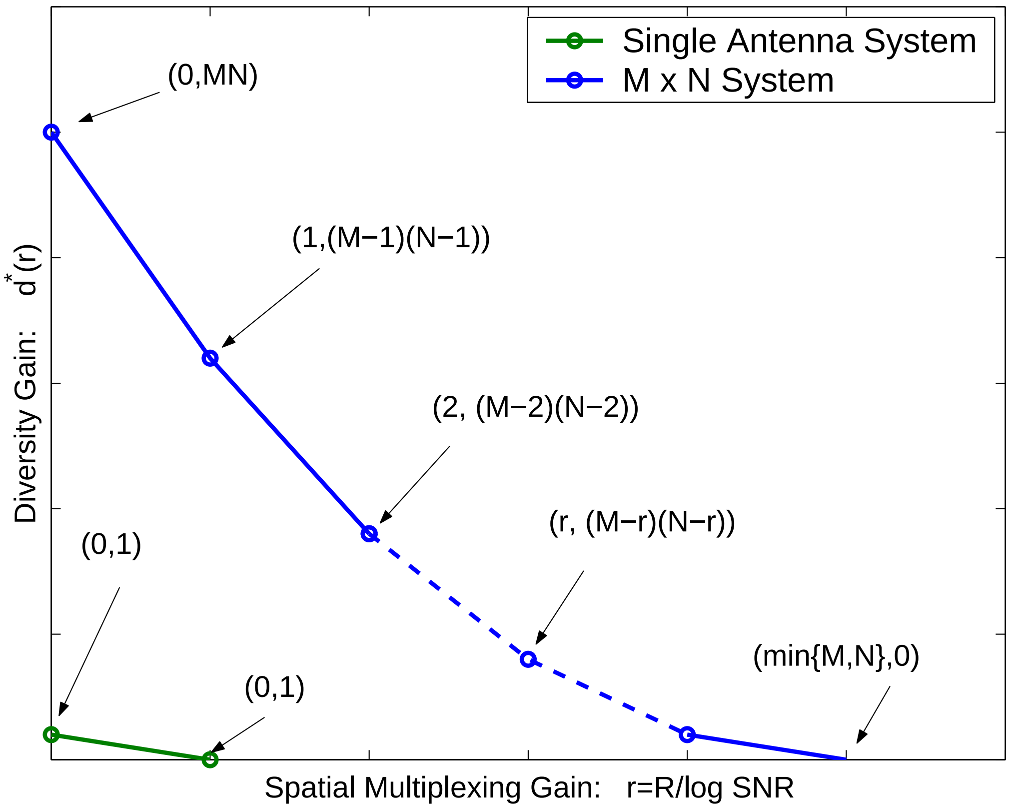

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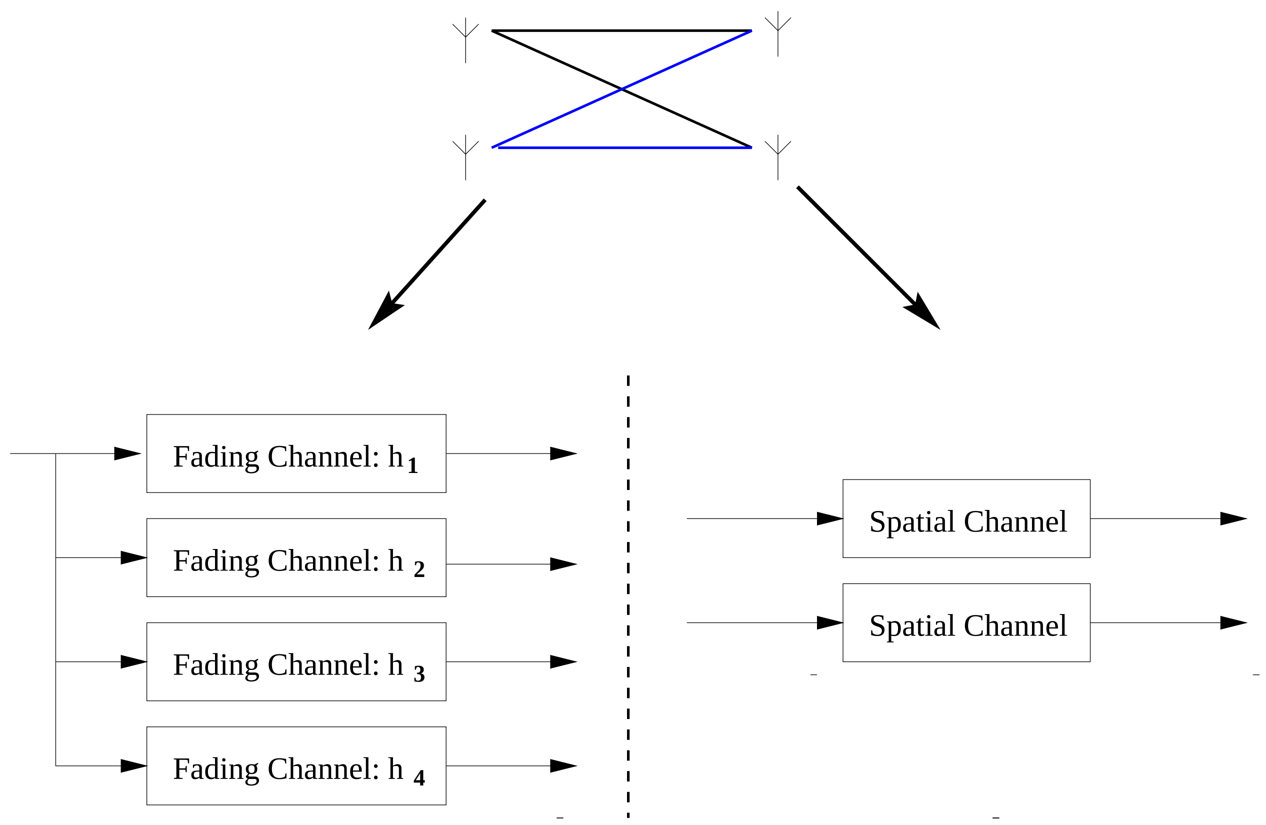
Revisit the 2×2 Example



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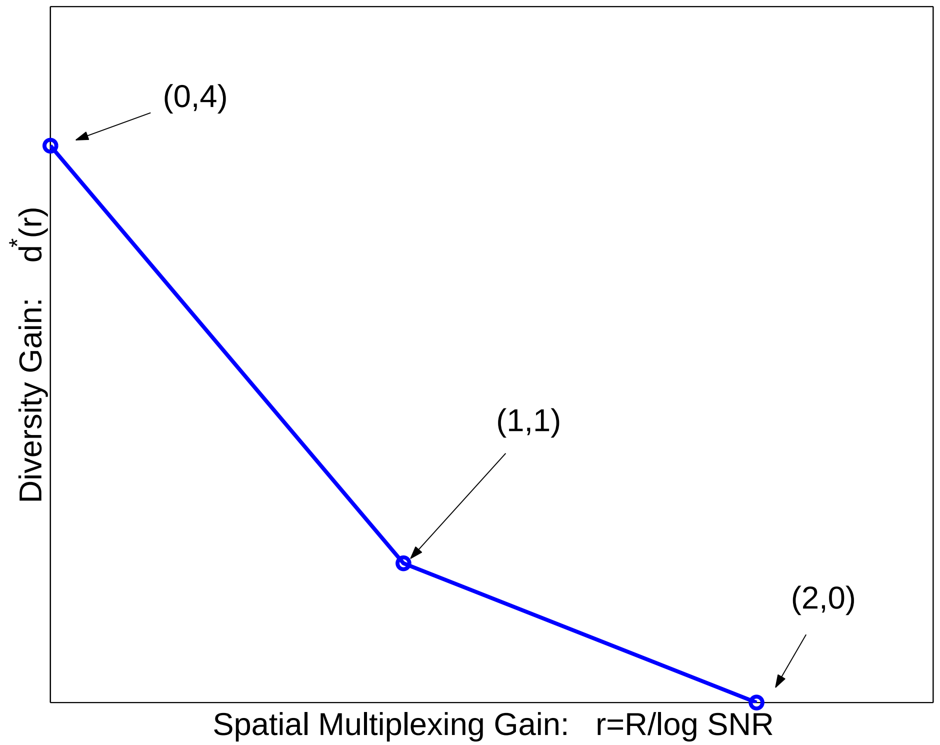

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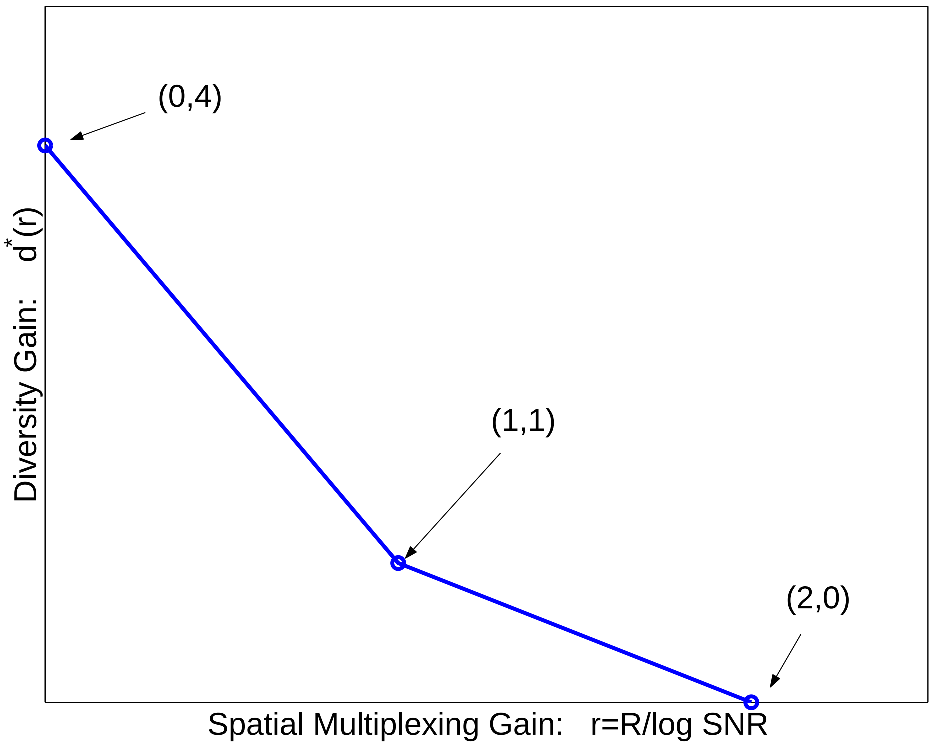
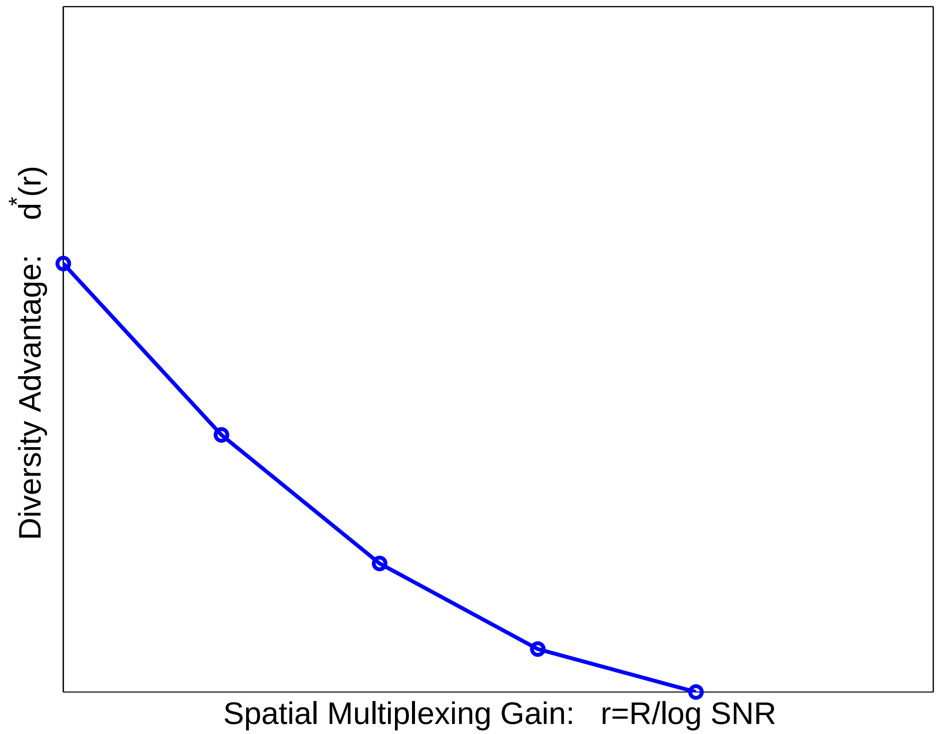


- Tradeoff bridges the gap between the two types of approaches.

Adding More Antennas



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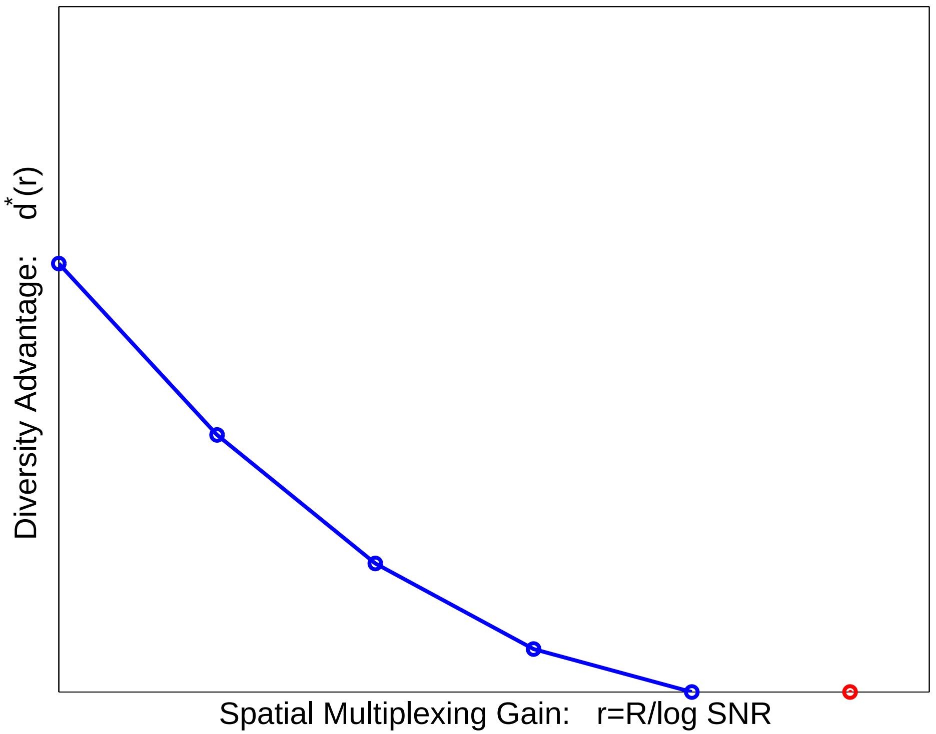

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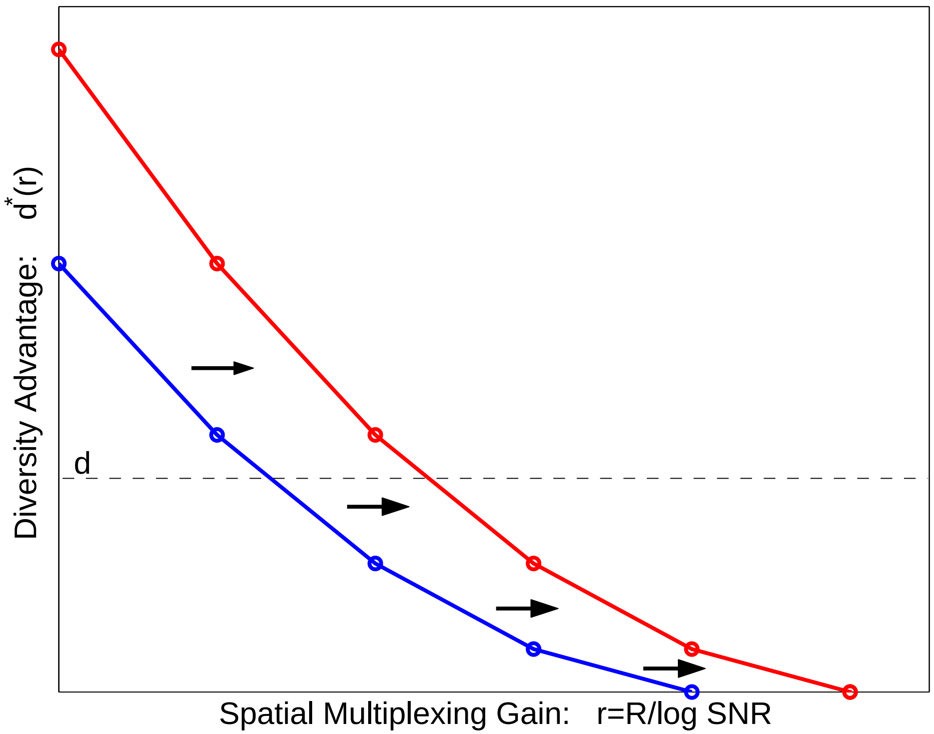
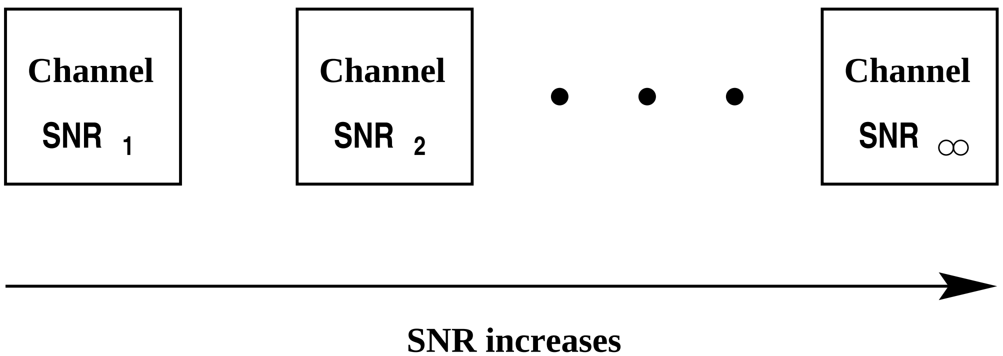


- Capacity result: increasing $\min\{M, N\}$ by 1 adds 1 more degree of freedom.
- Tradeoff curve: increasing both M and N by 1 yields multiplexing gain $+1$ for any diversity requirement d.

Increasing vs Fixed Code Rate



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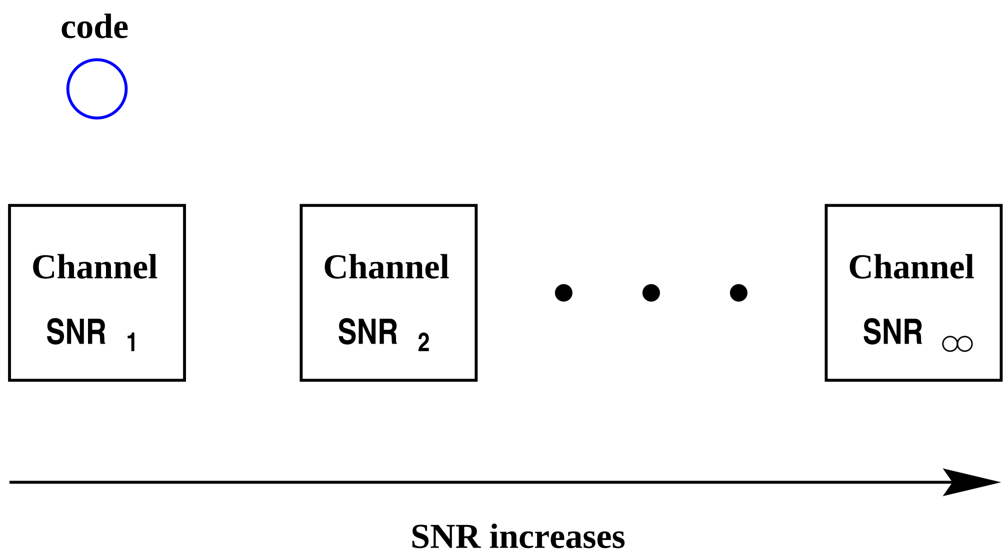

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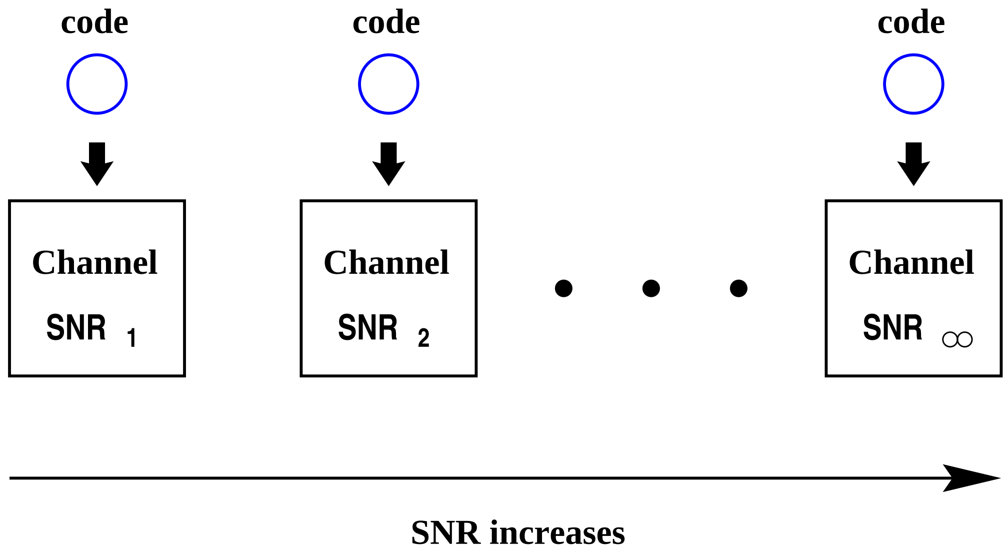

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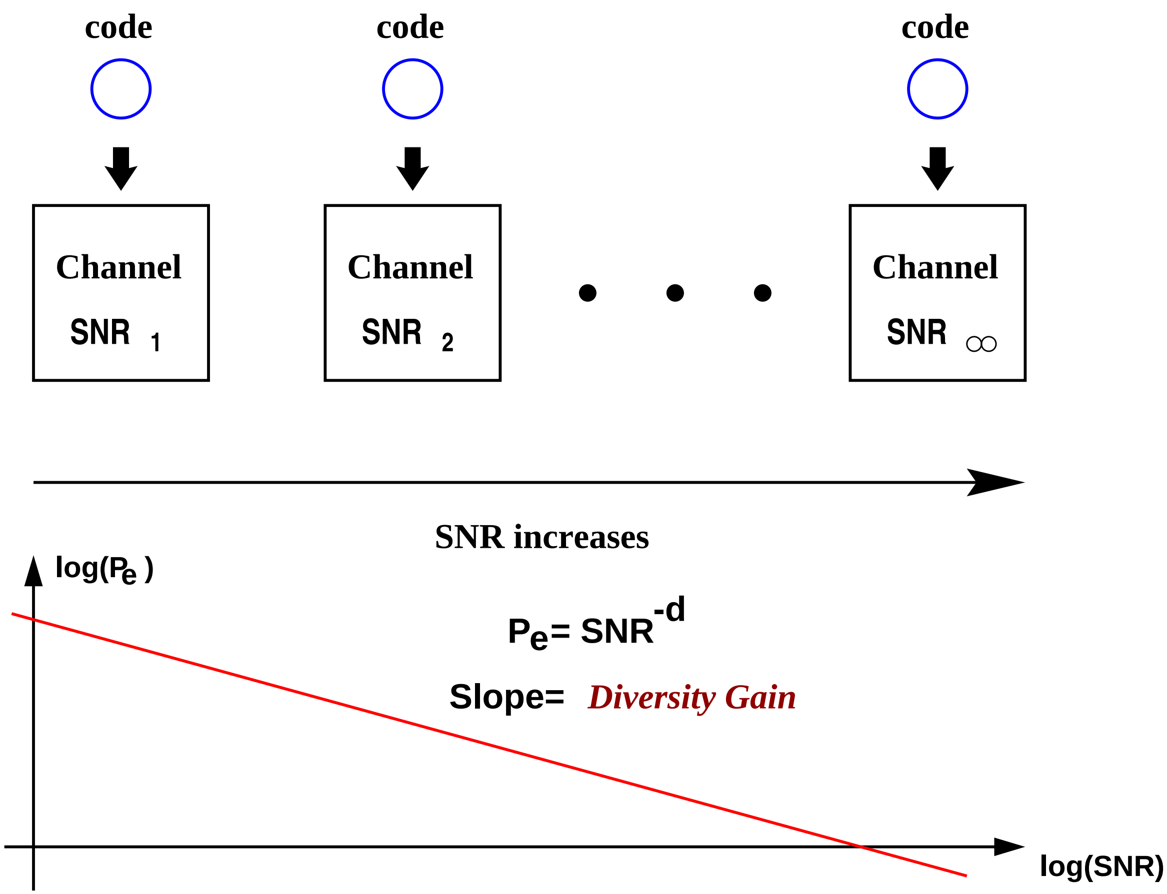

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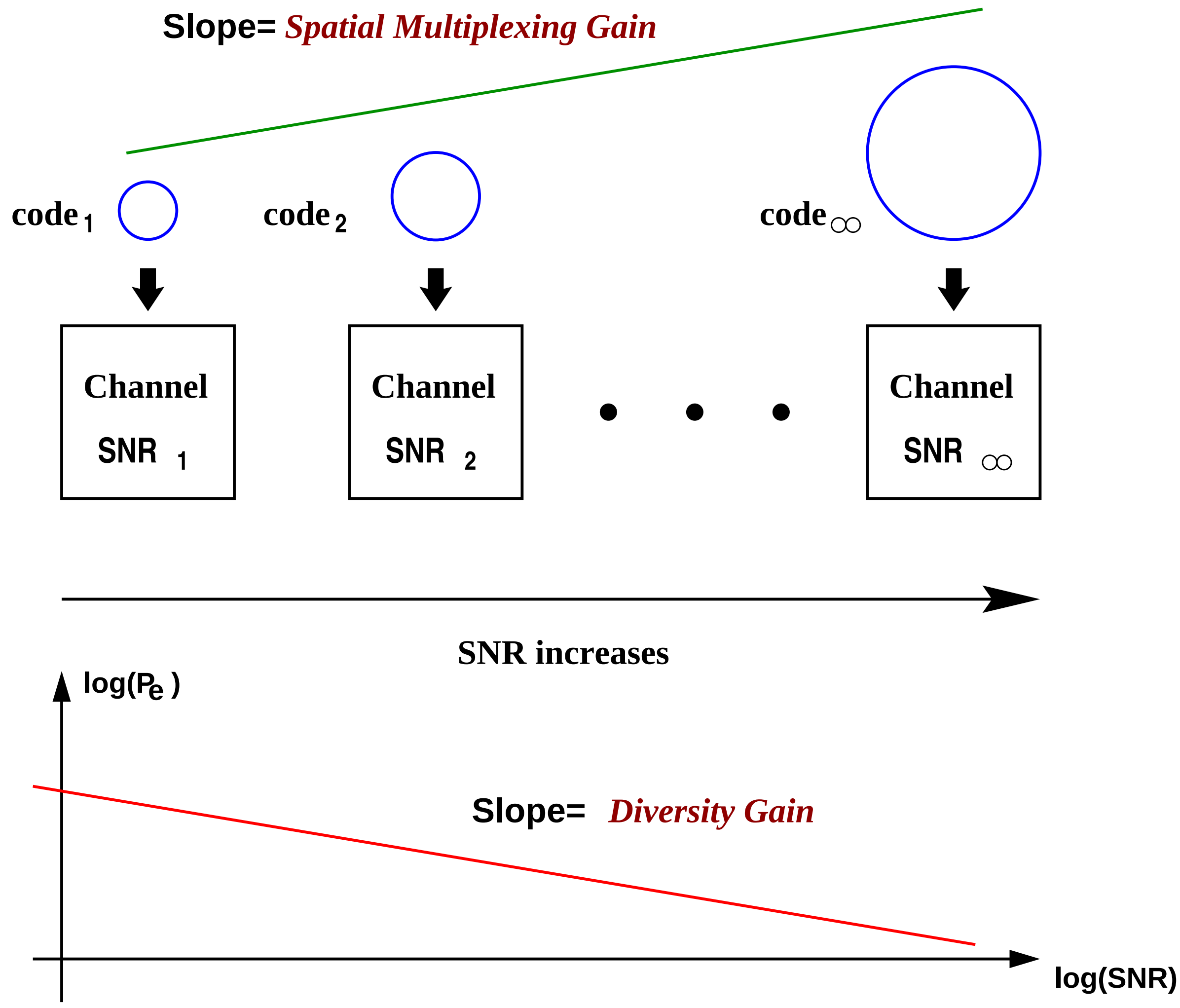

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Converse: Outage Bound

- Outage formulation for quasi-static scenarios. (Ozarow et al 94, Telatar 95)
- Look at the mutual information per symbol $I(Q, \mathbf{H})$ as a function of the input distribution and channel realization.

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$$\inf_{Q} P_{\mathbf{H}} \left[I(Q, \mathbf{H}) < R \right].$$

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- Error probability for finite block length T is asymptotically lower bounded by the outage probability:

$$\inf_{Q} P_{\mathbf{H}} \left[I(Q, \mathbf{H}) < R \right].$$

- At high SNR, i.i.d. Gaussian input Q^* is asymptotically optimal, and

$$I(Q^*, \mathbf{H}) = \log \det \left[I + \mathsf{SNR} \mathbf{H} \mathbf{H}^* \right].$$

Outage Analysis

- **Scalar 1×1 channel:**

  For target rate $R = r \log \mathsf{SNR}$, $r < 1$,

$$
\begin{aligned}
& P\{\log(1 + \mathsf{SNR}\|\mathbf{h}\|^2) < r \log \mathsf{SNR}\} \\
\sim \quad & P\left\{\|\mathbf{h}\|^2 < \mathsf{SNR}^{-(1-r)}\right\} \\
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- Outage occurs when the channel gain $\|\mathbf{h}\|^2$ is small.
- More generally, outage occurs for the multi-antenna channel when some or all of the singular values of $\mathbf{H}$ are small.
- But unlike the scalar channel, there are many ways for this to happen in a vector channel.

Typical Outage Behavior

$\mathbf{v}$ = vector of singular values of $\mathbf{H}$.

Laplace Principle:

$$p_{out} = \min_{\mathbf{v} \in \mathsf{Out}} \mathsf{SNR}^{-f(\mathbf{v})}$$

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Result:

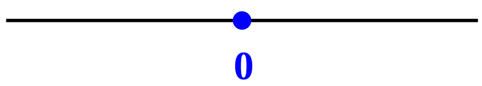
At target rate $R = r \log \text{SNR}$, outage typically occurs when $\mathbf{H}$ is near a rank $\lfloor r \rfloor$ matrix, i.e. out of the $\min\{M, N\}$ non-zero squared singular values:

- $\lfloor r \rfloor$ of them are order 1;
- $\min\{M, N\} - \lfloor r \rfloor + 1$ of them are are order SNR^{-1};
- 1 of them is order $\text{SNR}^{-(r - \lfloor r \rfloor)}$ (just small enough to cause outage)

When r is integer, exactly r squared singular values are order 1 and $\min\{M, N\}$ are order SNR^{-1}.

Geometric Picture (integer r)

Scalar Channel



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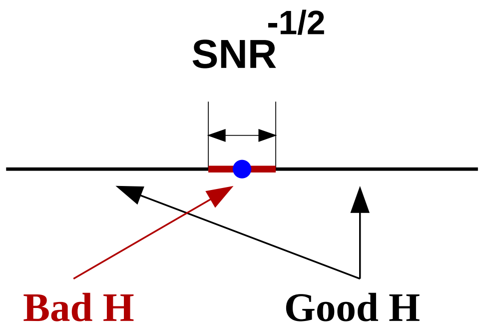

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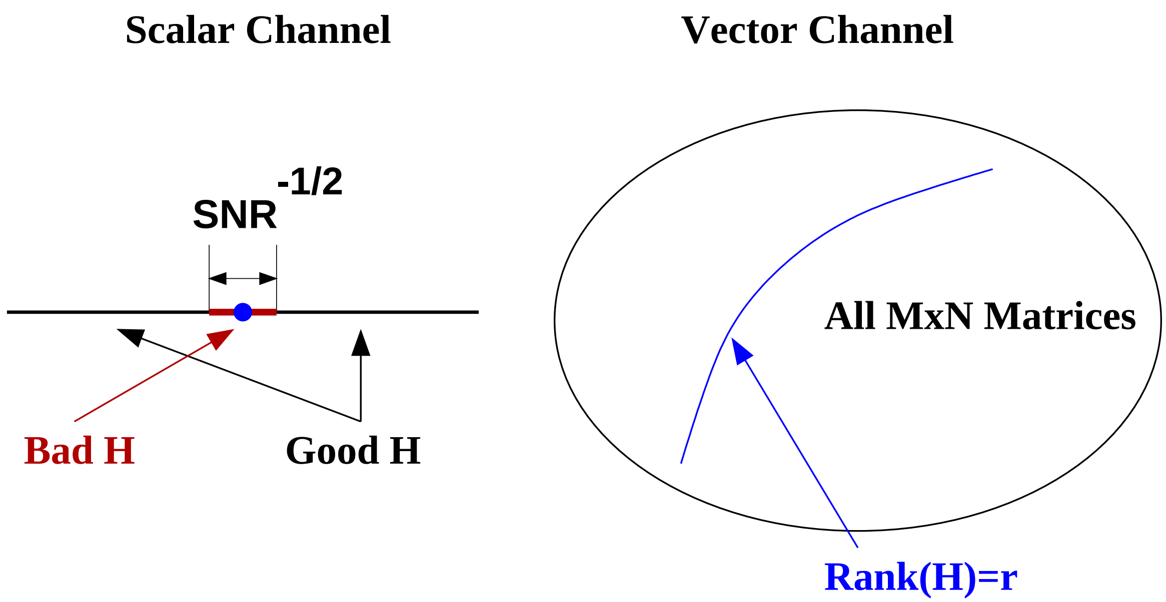

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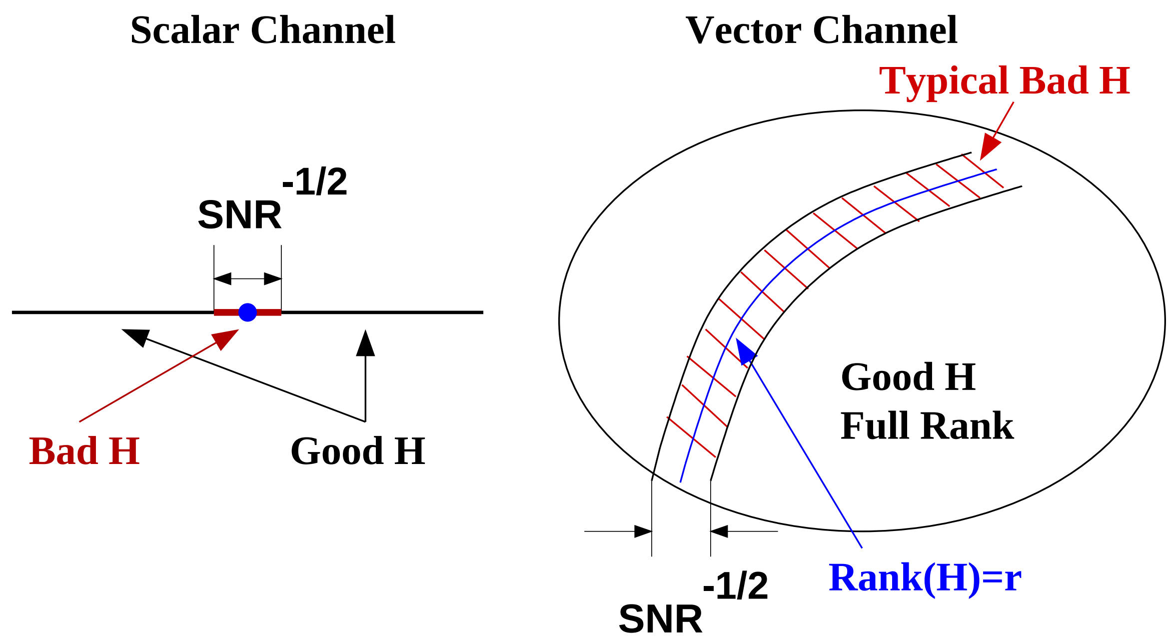

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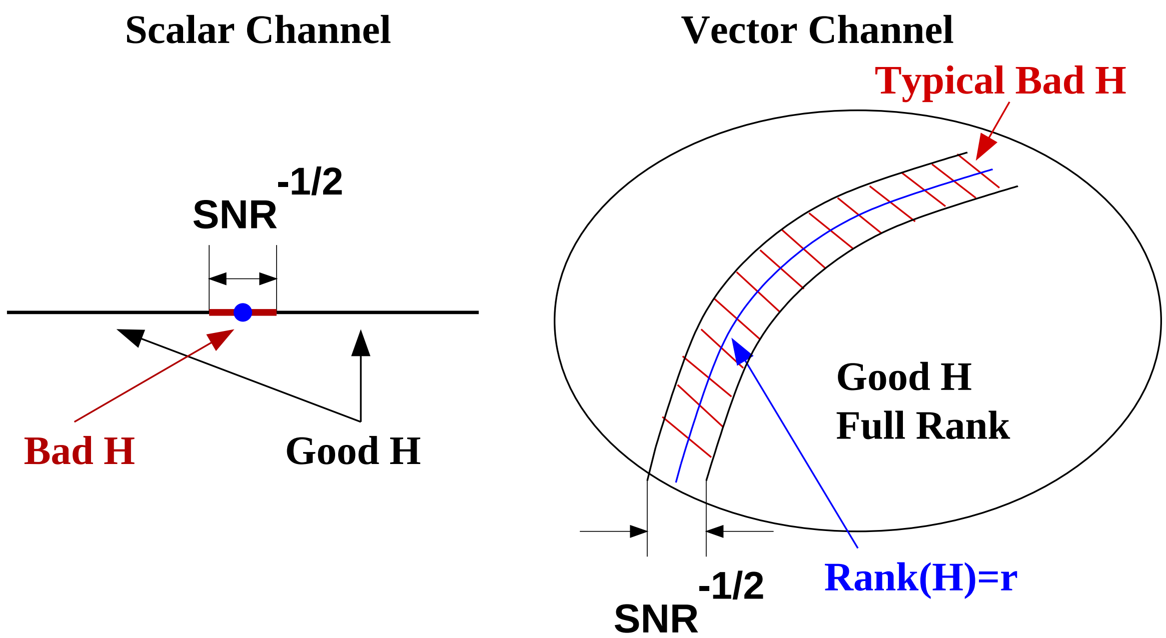
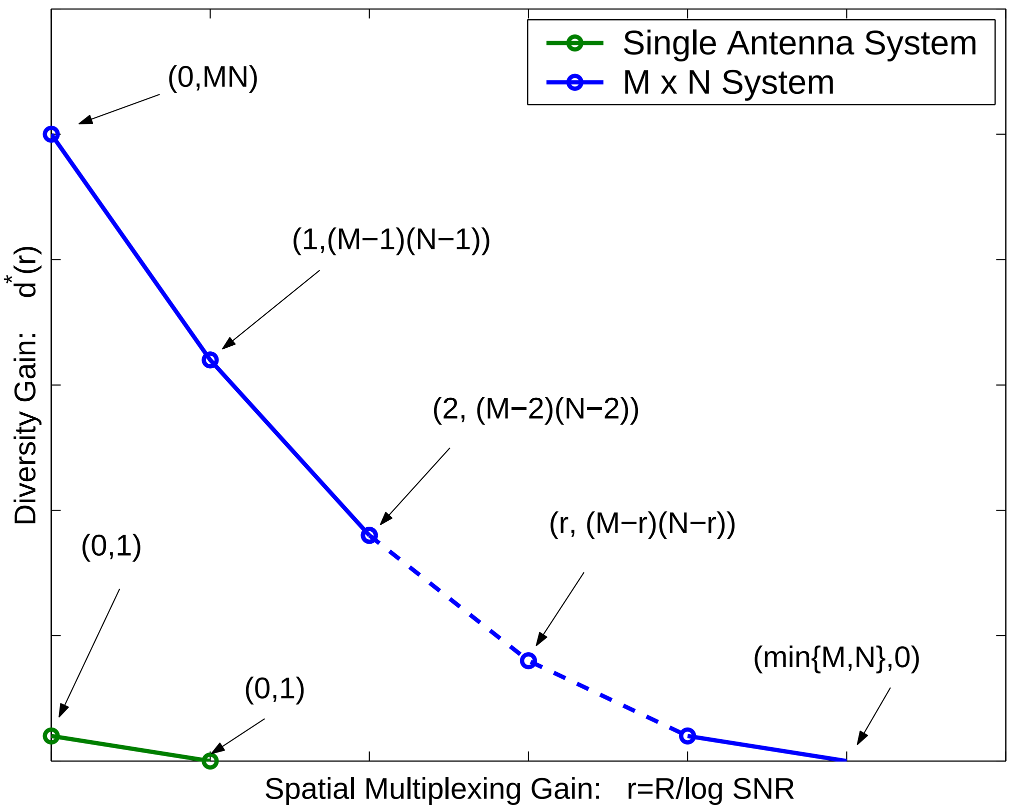


$$p_{out} \sim \mathsf{SNR}^{-(M-r)(N-r)},$$

$(M-r)(N-r)$ is the dimension of the normal space to the sub-manifold of rank r matrices within the set of all $M \times N$ matrices.

Piecewise Linearity of Tradeoff Curve



Scalar channel: qualitatively same outage behavior for all r.

Vector channel: qualitatively different outage behavior for different r.

Achievability: Random Codes

- Outage performance achievable as codeword length $T \to \infty$.
- But what about for finite T?
- Look at the performance of i.i.d Gaussian random codes.
- Can the outage behavior be achieved?

Analysis of Random Codes

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Outage analysis only needs to focus on the first event, but for finite T all three effects come into play.

Multiplicative Fading vs Additive Noise

Look at two codewords at Euclidean distance x.

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Look at two codewords at Euclidean distance x.

- Error Event A: $\mathbf{H}$ typical, AWGN large

$$P(A) \sim \exp(-x).$$

- Error Event B: $\mathbf{H}$ near singular, AWGN typical

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At high SNR, $x \to \infty$

$$\Longrightarrow P(B) \gg P(B).$$

Multiplicative Fading vs Code Randomness

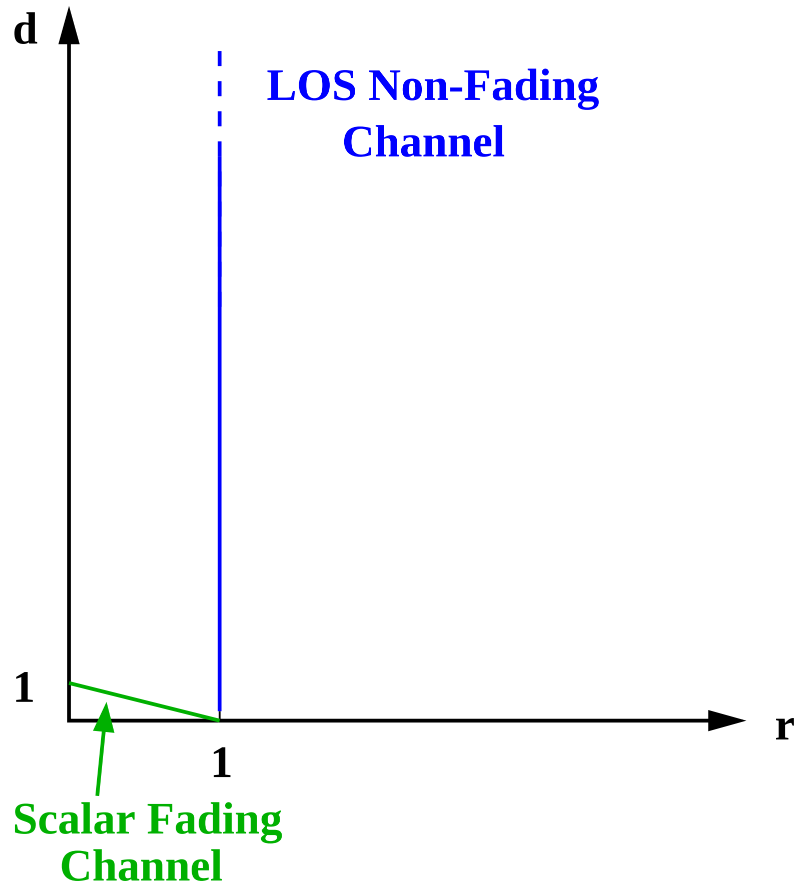
- Distance between random codewords may deviate from typical distance x.
- Error Event C: codewords atypically close

$$P(C) \sim x^{-\beta},$$

  Also polynomial in x, just like the effect due to channel fading.
- As long as $T \geq M + N - 1$, the typical error event is due to bad channel rather than bad codewords.
- For $T < M + N - 1$, random codes are not good enough. (ISIT 02)

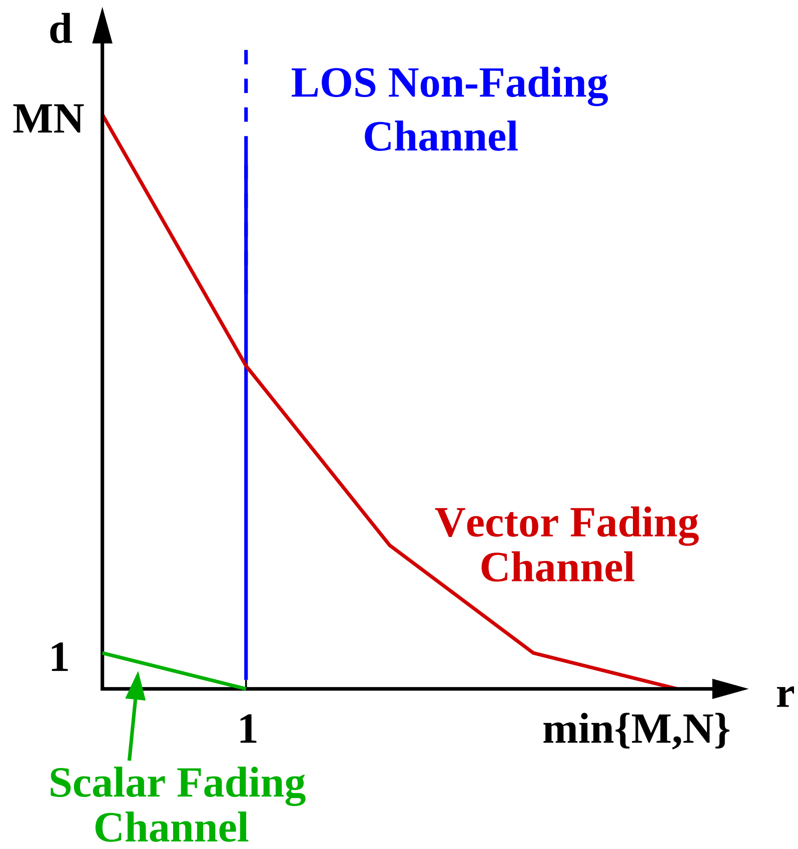
To Fade or Not to Fade?

Line-of-Sight vs Fading Channel



- In a scalar 1×1 system, line-of-sight AWGN is better.

Line-of-Sight vs Fading Channel



- In a scalar 1×1 system, line-of-sight AWGN is better.
- In a vector $M \times N$ system, it depends.

Outline

- Problem formulation and main result on optimal tradeoff.
- Sketch of proof.
- Comparison of existing schemes.

Using the Optimal Tradeoff Curve

Provide a unified framework to compare different schemes.

Use the Optimal Tradeoff Curve

Provide a unified framework to compare different schemes.

For a given scheme, compute

$$r \longrightarrow d(r)$$

Compare with $d^*(r)$

Two Diversity-Based Schemes

Focus on two transmit antennas.

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{W}$$

Repetition Scheme:

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}_1 & 0 \\ 0 & \mathbf{x}_1 \end{bmatrix}$$

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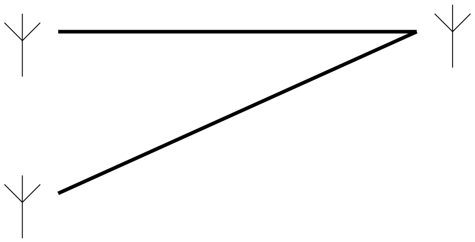
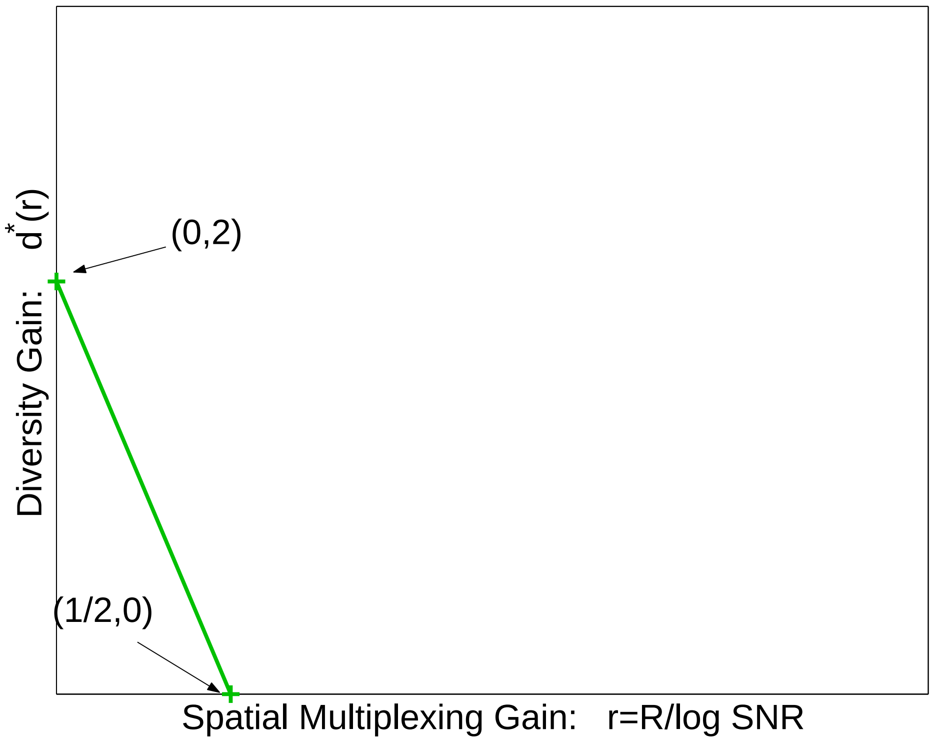
Comparison: 2×1 System



Repetition: $\mathbf{r}_1 = \|\mathbf{H}\|\mathbf{x}_1 + \mathbf{w}$

Alamouti: $[\mathbf{r}_1 \mathbf{r}_2] = \|\mathbf{H}\|[\mathbf{x}_1 \mathbf{x}_2] + [\mathbf{w}_1 \mathbf{w}_2]$



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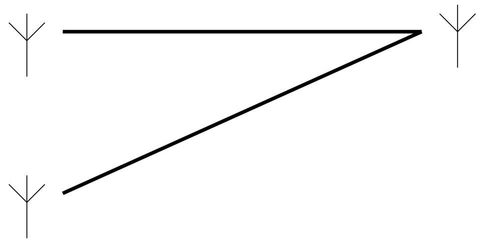

Repetition: $\mathbf{r}_1 = \|\mathbf{H}\|\mathbf{x}_1 + \mathbf{w}$

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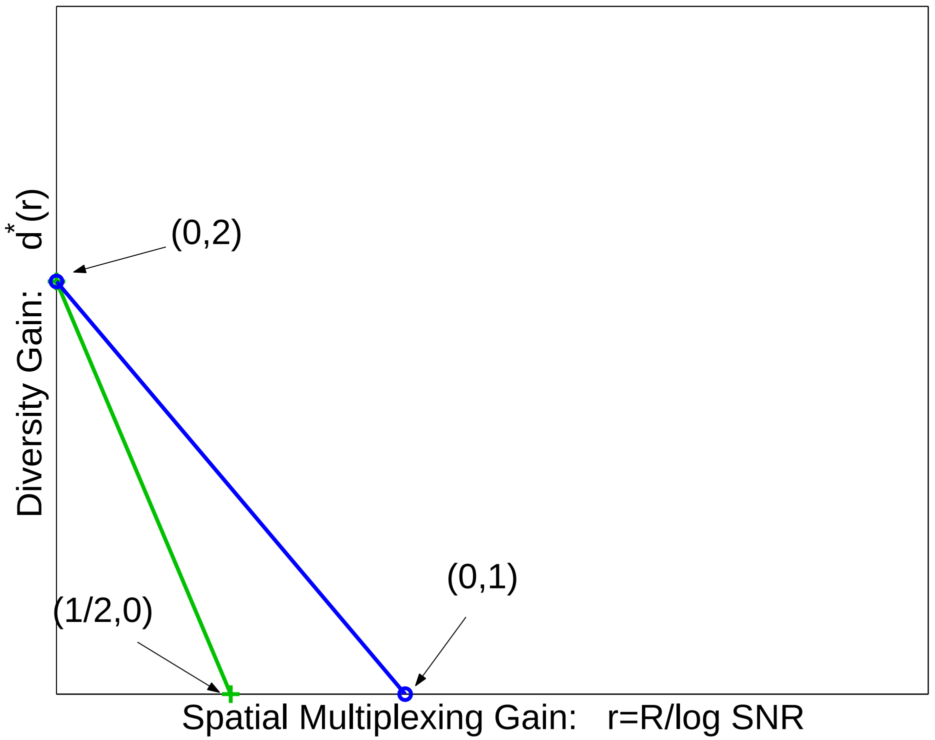

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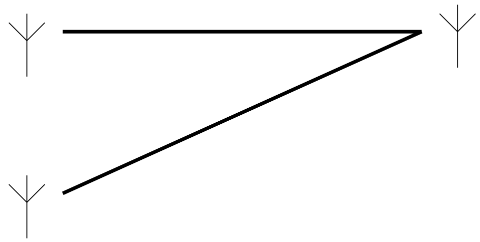

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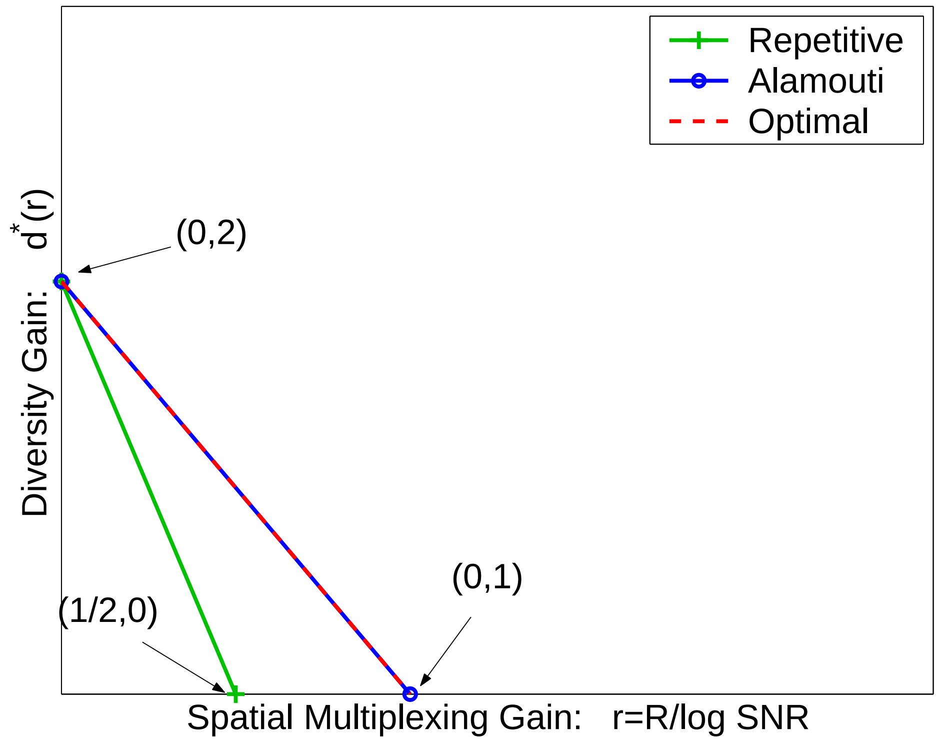
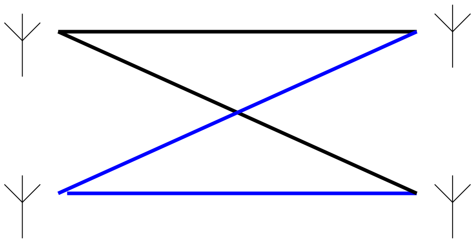
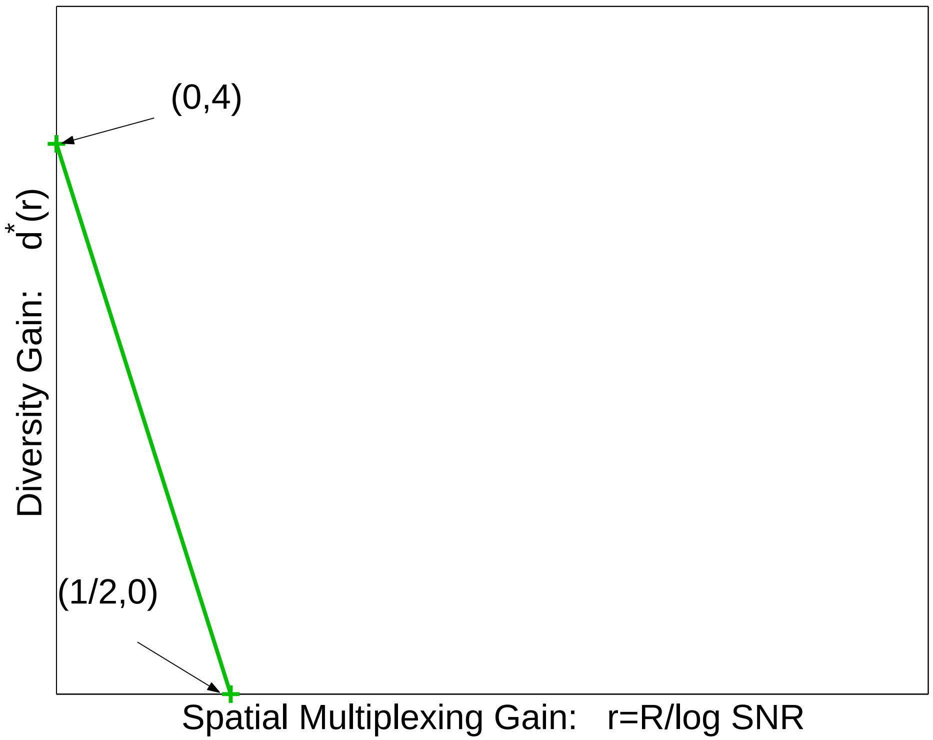


Comparison: 2×2 System

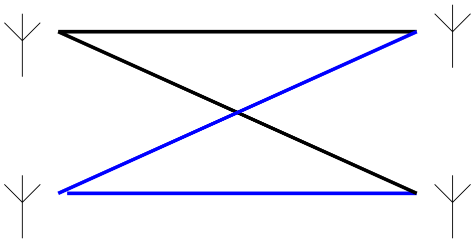


Repetition: $\mathbf{r}_1 = \|\mathbf{H}\|\mathbf{x}_1 + \mathbf{w}$

Alamouti: $[\mathbf{r}_1\mathbf{r}_2] = \|\mathbf{H}\|[\mathbf{x}_1\mathbf{x}_2] + [\mathbf{w}_1\mathbf{w}_2]$



Comparison: 2×2 System



Repetition: $\mathbf{r}_1 = \|\mathbf{H}\|\mathbf{x}_1 + \mathbf{w}$

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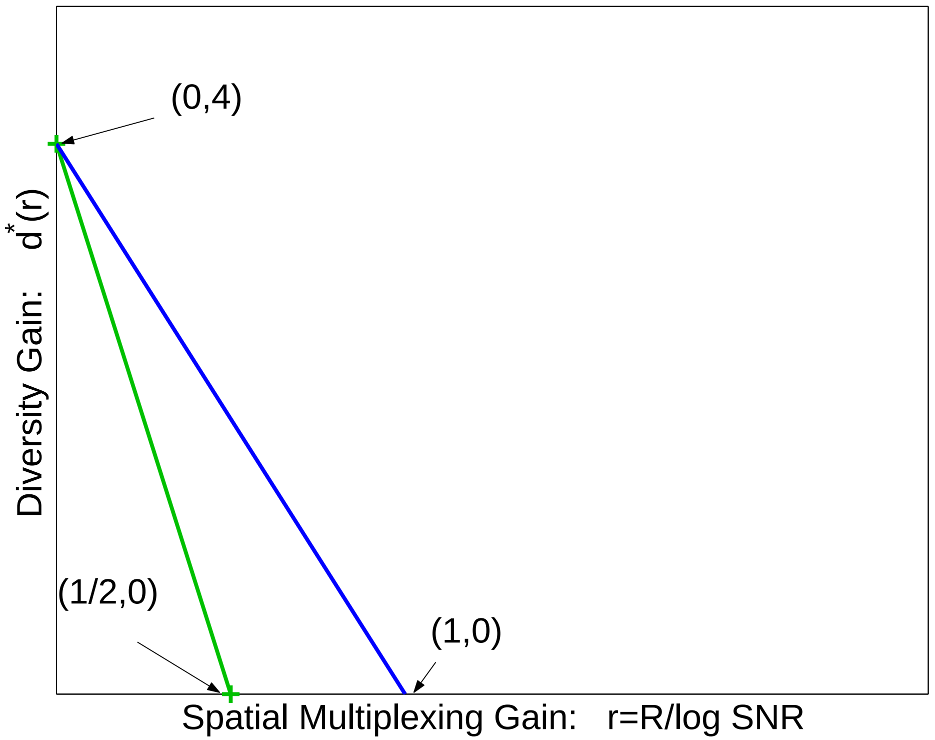

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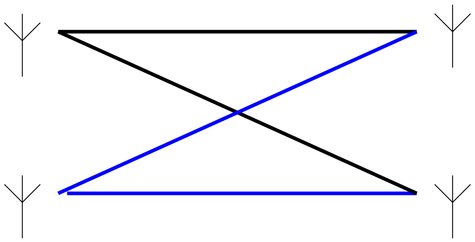

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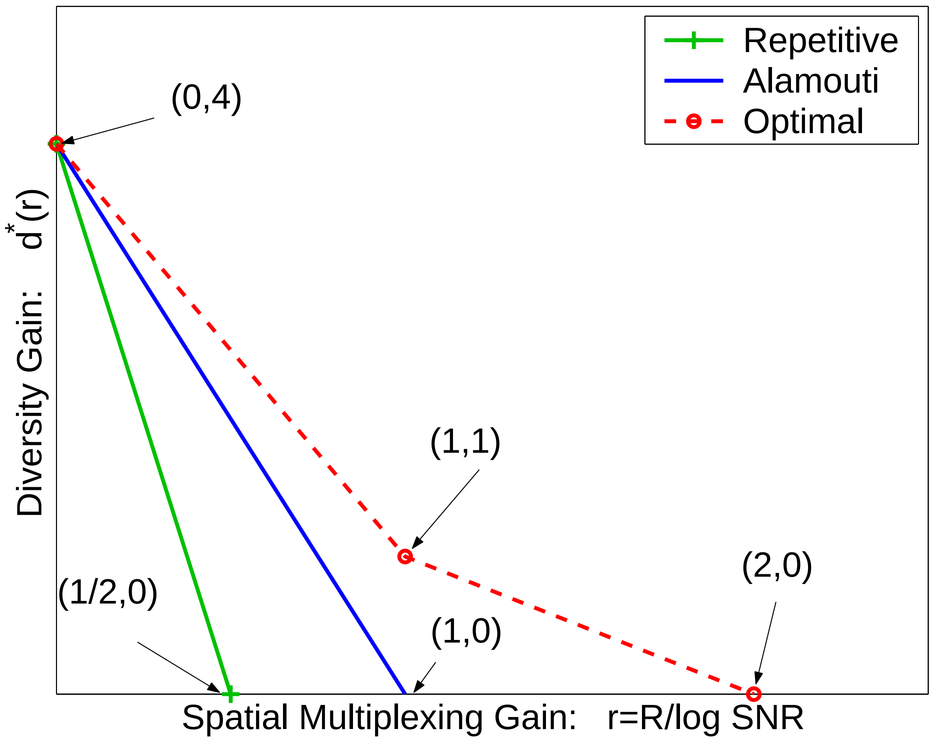
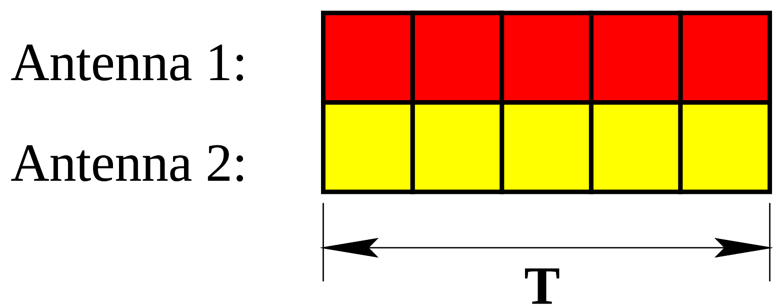
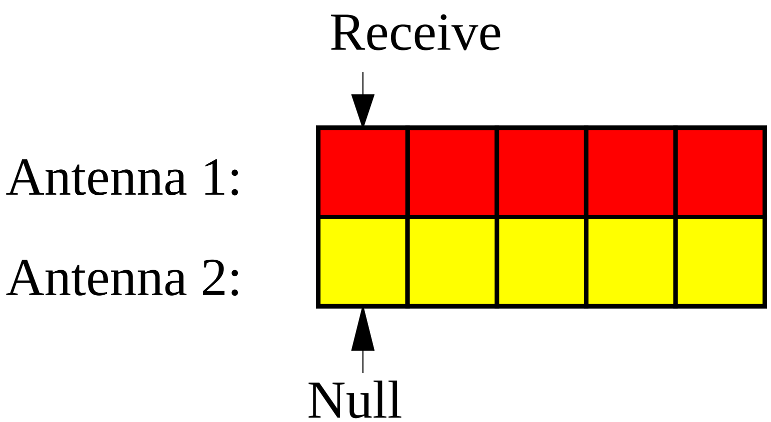


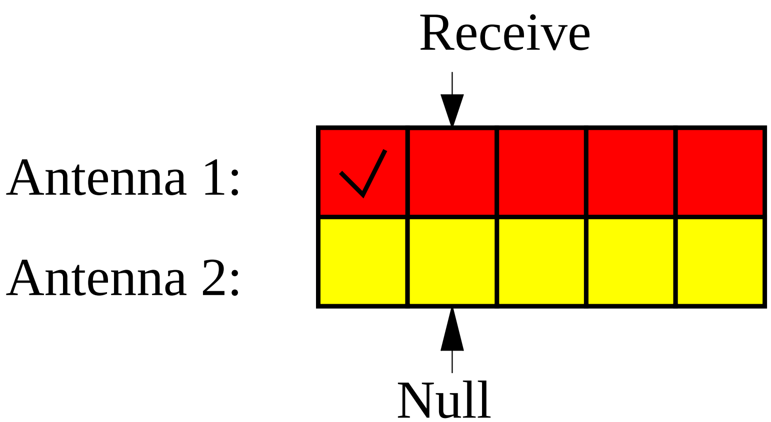
V-BLAST



V-BLAST



V-BLAST



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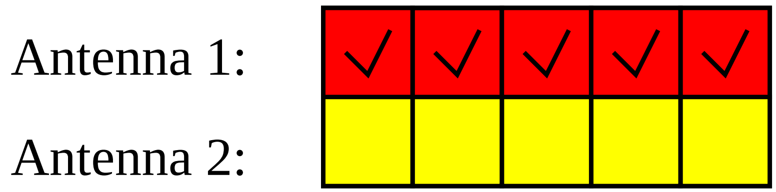
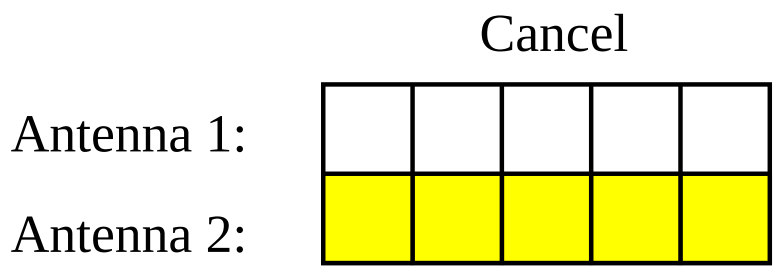
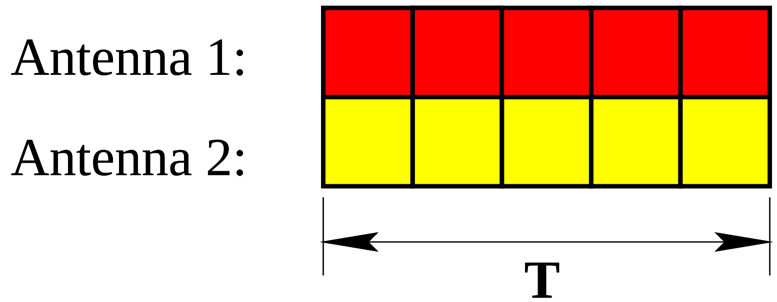


V-BLAST



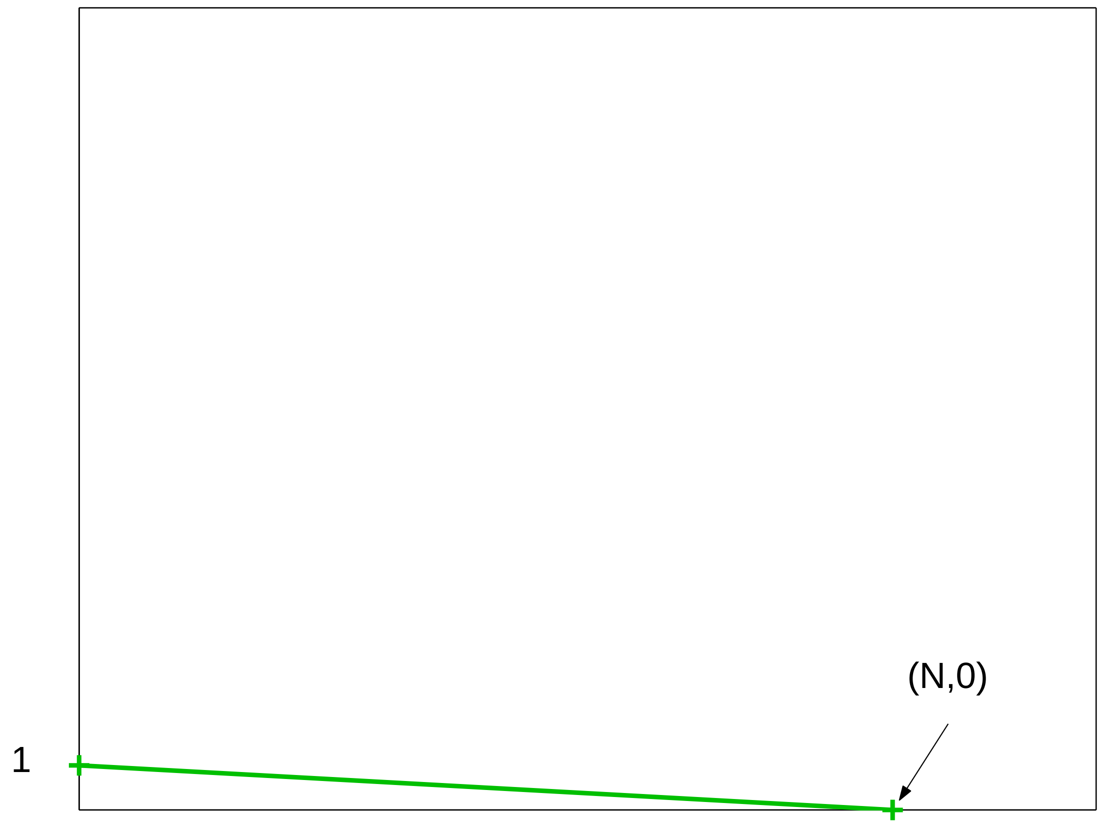
V-BLAST



- Nulling and Canceling
- Independent data streams transmitted over antennas

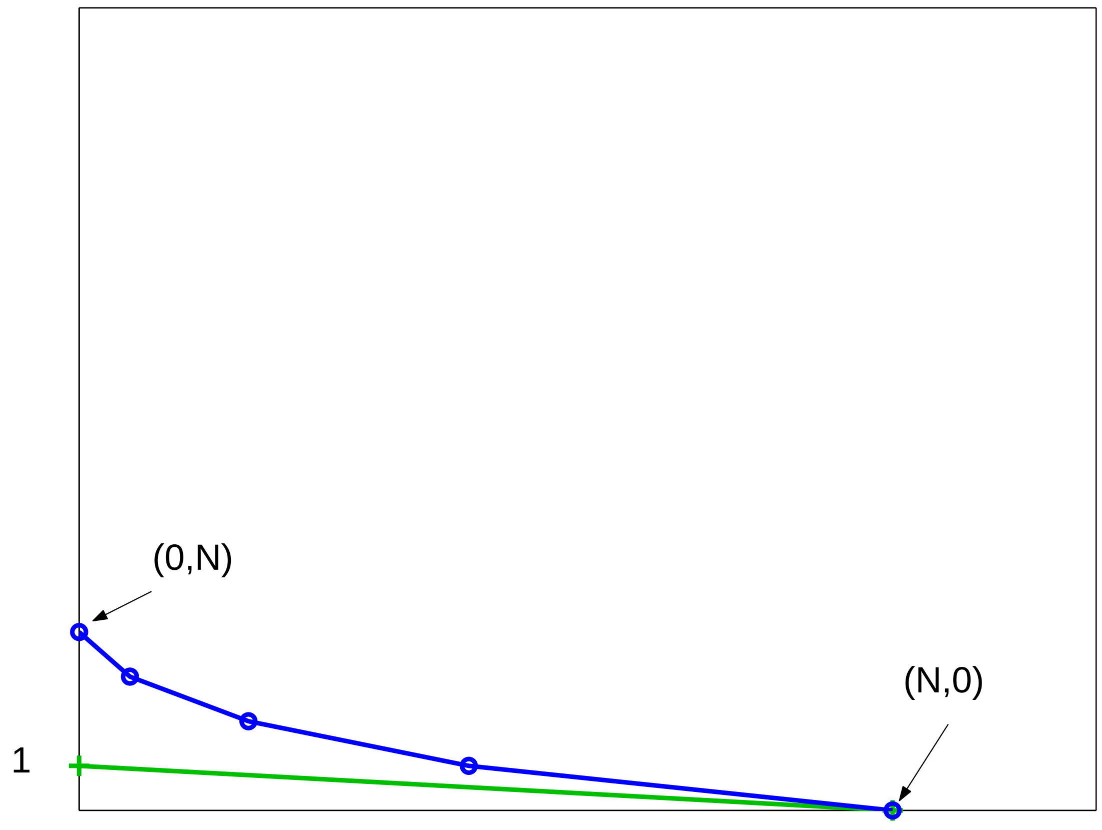
Tradeoff Performance of V-BLAST ($N \times N$)

Original V-BLAST



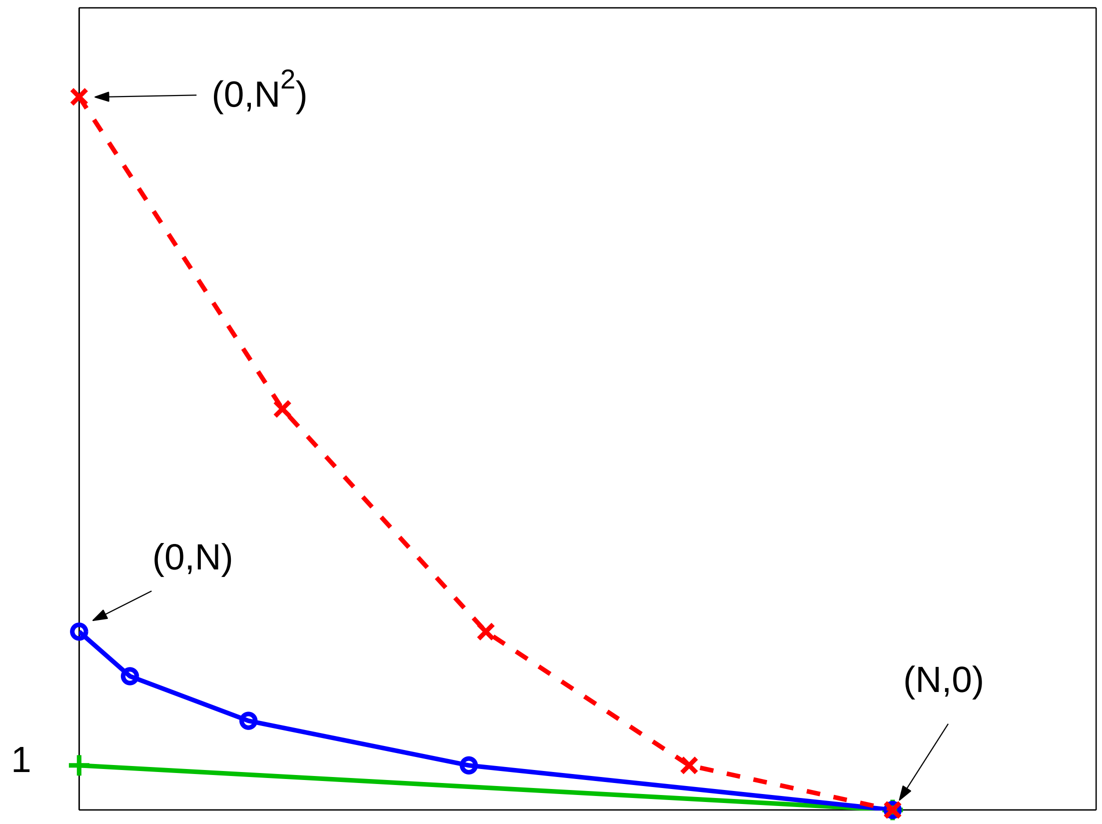
Tradeoff Performance of V-BLAST ($N \times N$)

V-BLAST with optimal rate allocation



Tradeoff Performance of V-BLAST ($N \times N$)

Compare to the optimal tradeoff



Tradeoff Performance of V-BLAST ($N \times N$)

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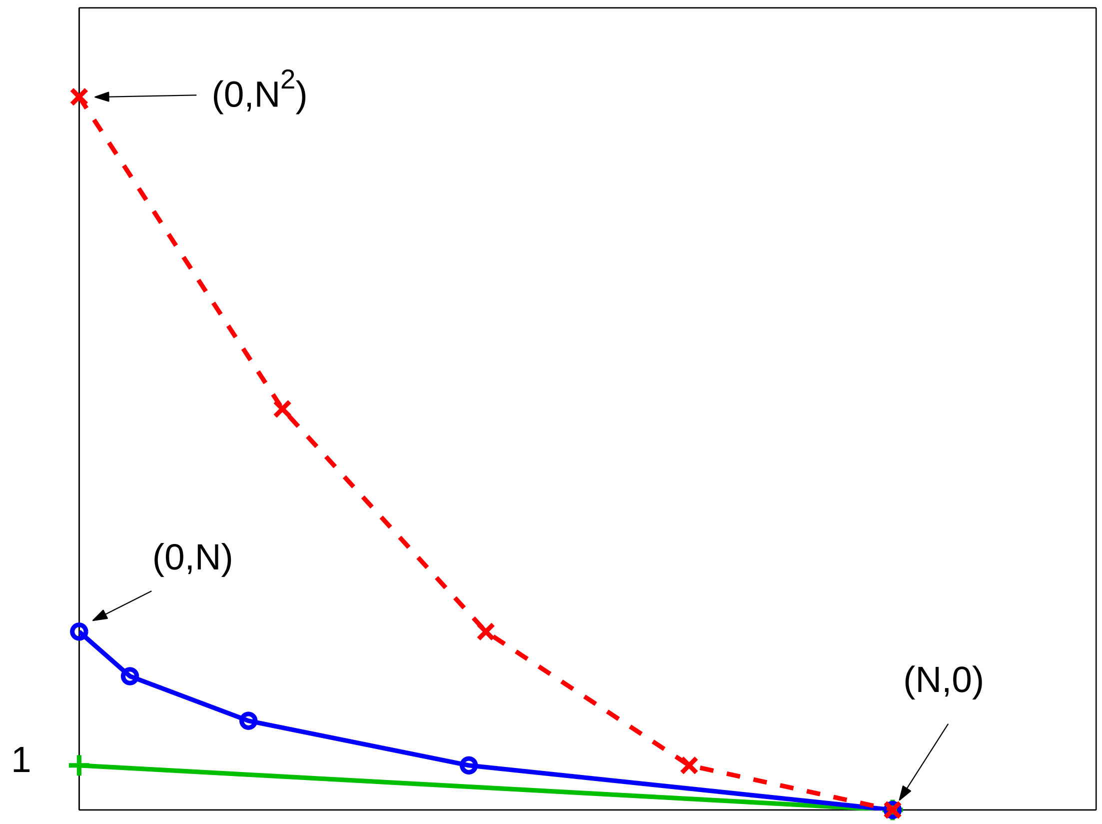
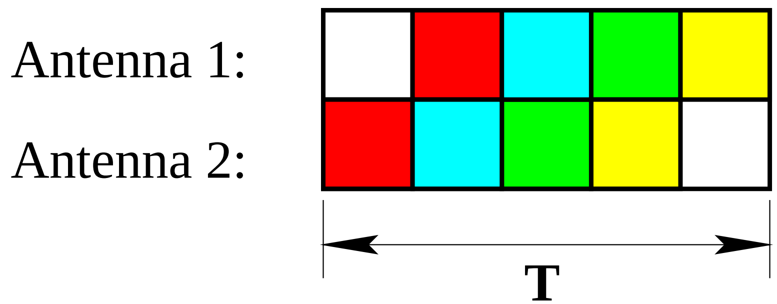
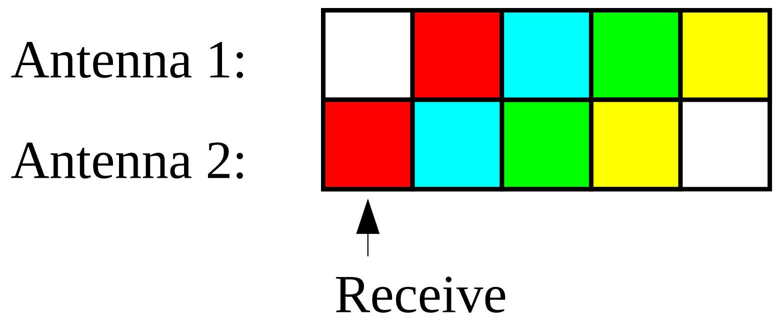


Low diversity due to lack of coding over space

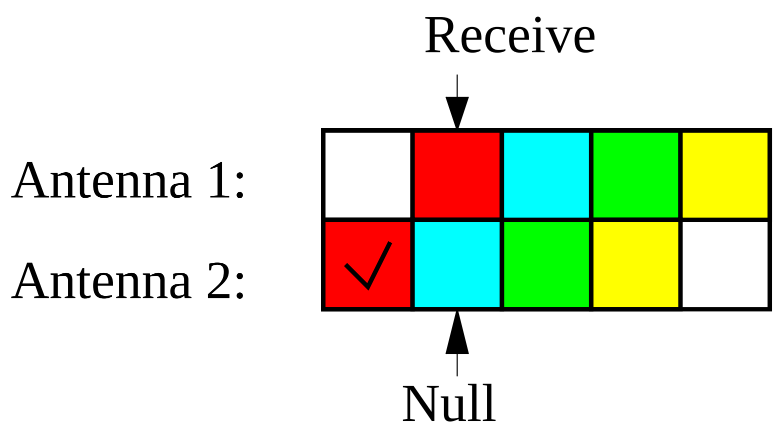
D-BLAST



D-BLAST



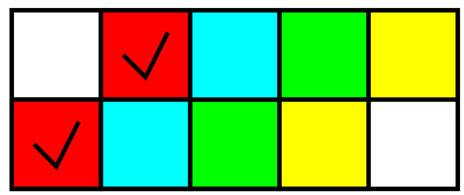
D-BLAST



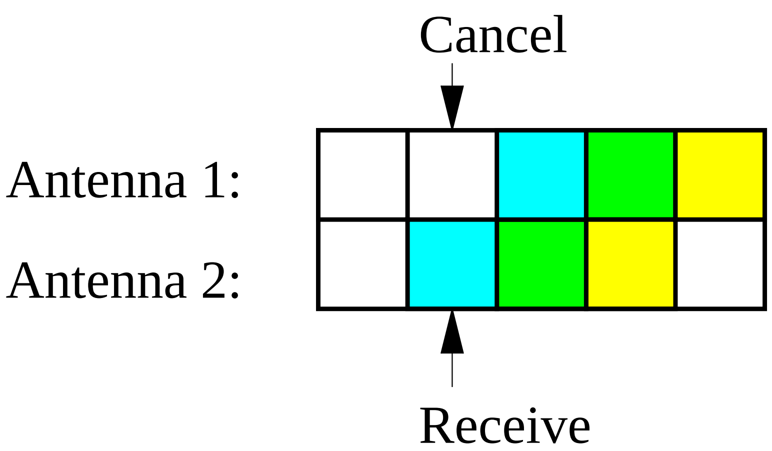
D-BLAST

Antenna 1:

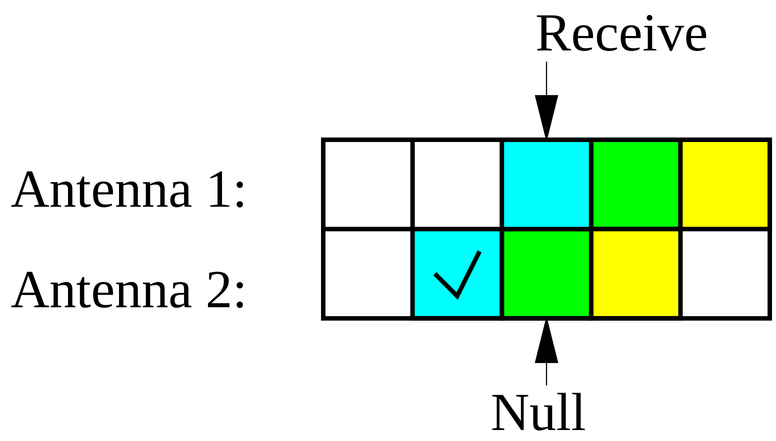
Antenna 2:



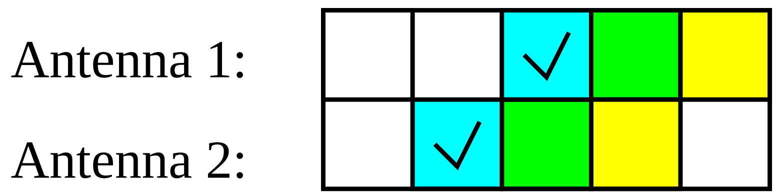
D-BLAST



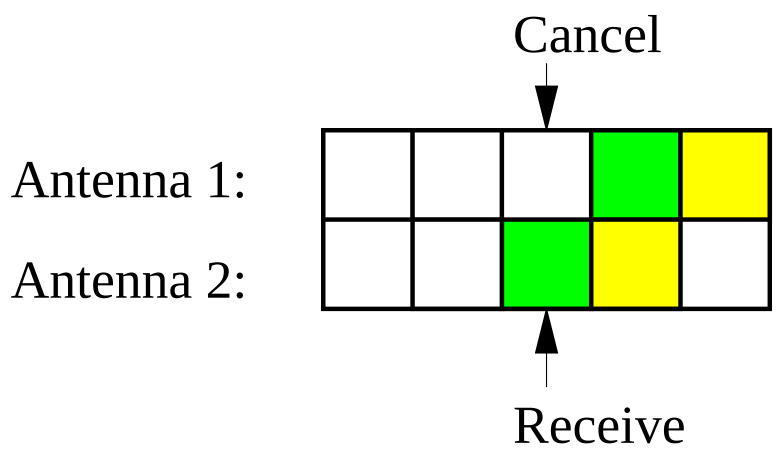
D-BLAST



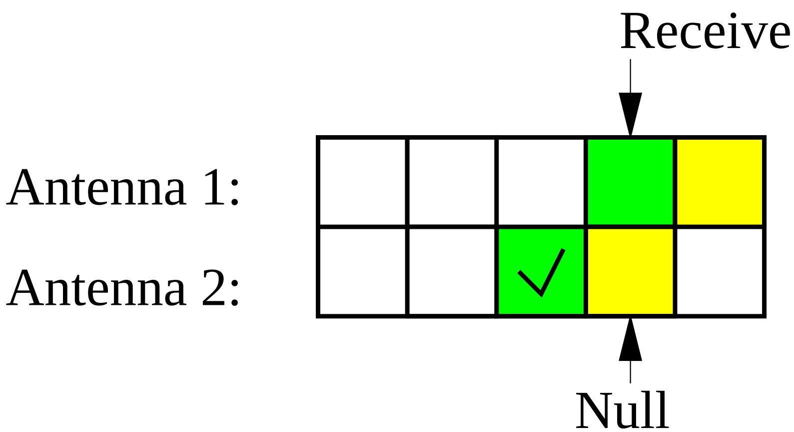
D-BLAST



D-BLAST

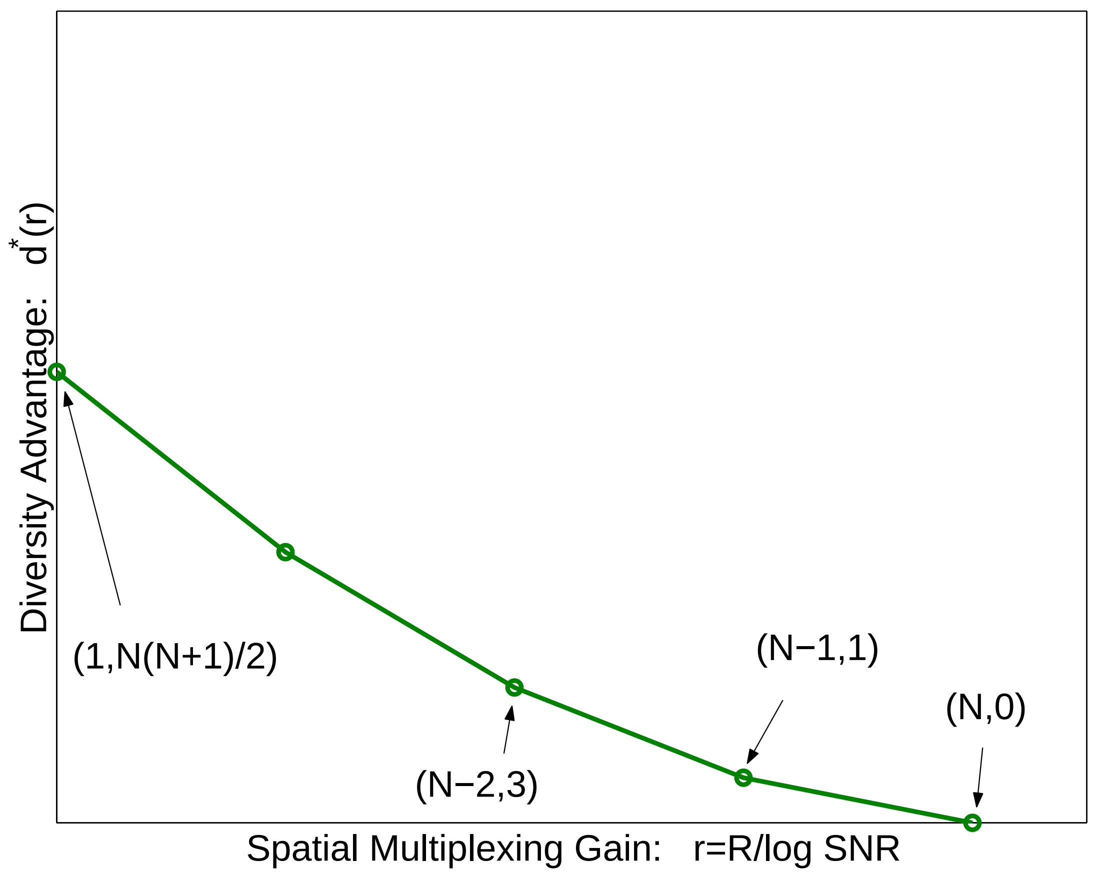


D-BLAST

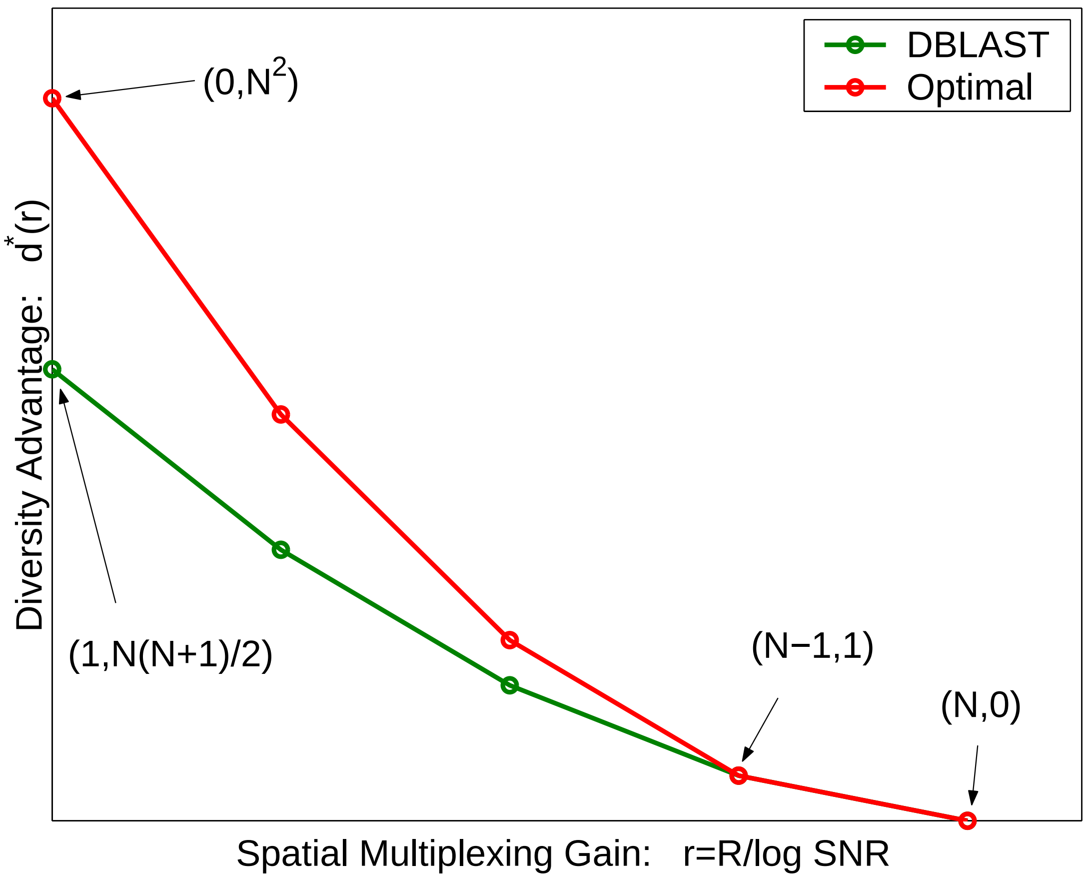


Ignore the overhead for now.

D-BLAST: Square System



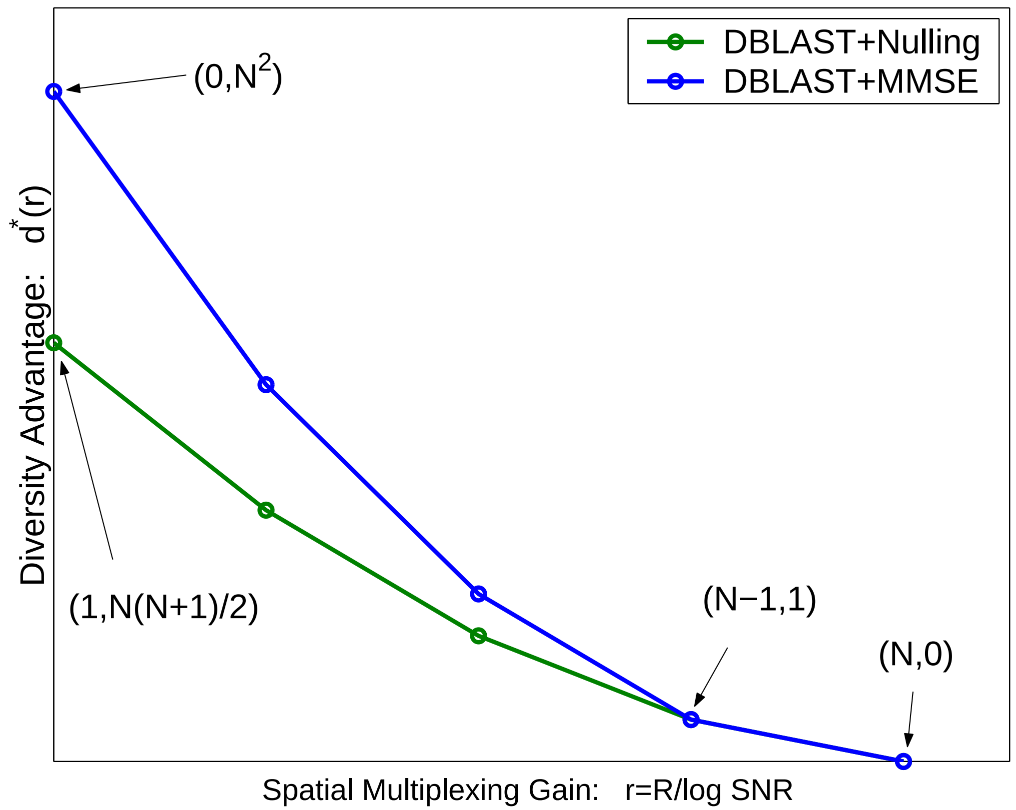
D-BLAST: Square System



- Can achieve full multiplexing gain
- Maximum diversity gain $d = \frac{N(N+1)}{2}$.

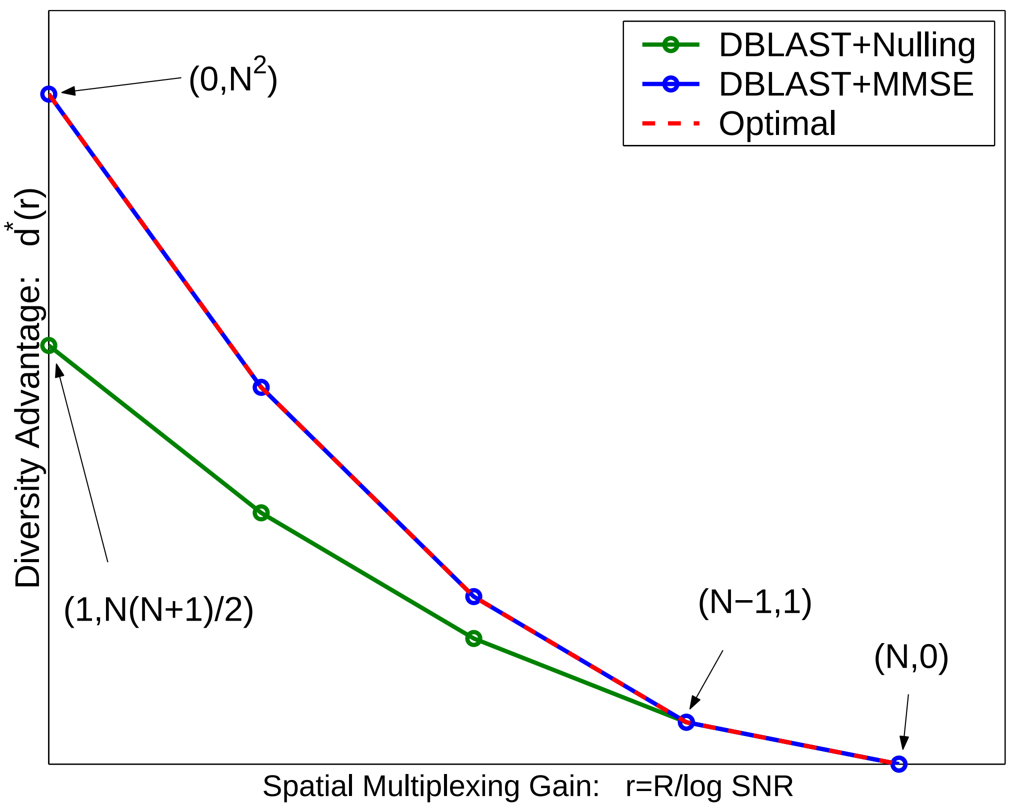
Replace Nulling by MMSE?

D-BLAST+MMSE



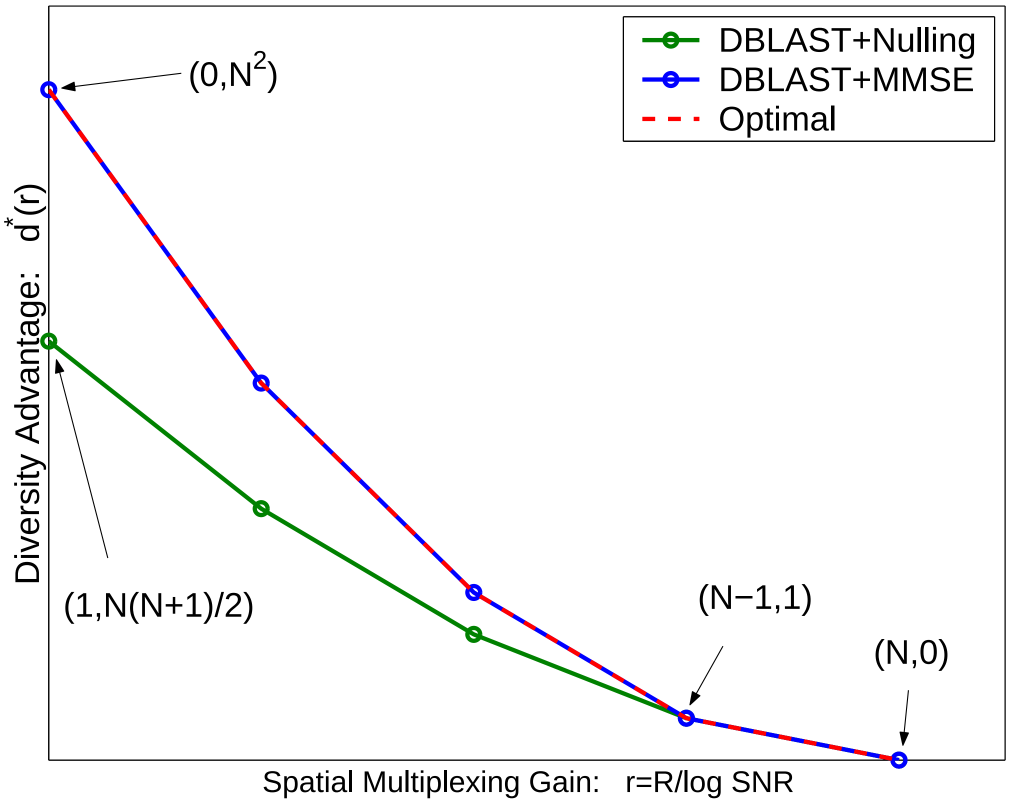
DBLAST+Nulling
DBLAST+MMSE
(0,N2)
(1,N(N+1)/2)
(N−1,1)
(N,0)
Diversity Advantage: d*(r)
Spatial Multiplexing Gain: r=R/log SNR

D-BLAST+MMSE



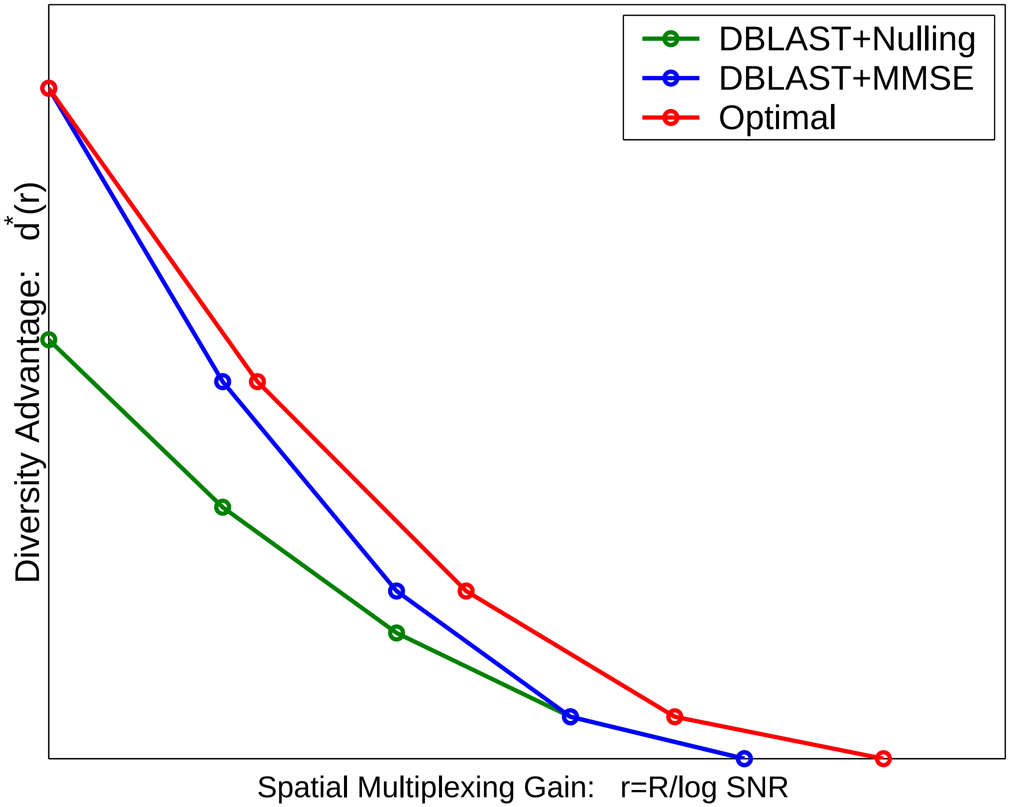
- Achieve the optimal: successive cancellation + MMSE has the optimal outage performance.

D-BLAST+MMSE



- Achieve the optimal: successive cancellation + MMSE has the optimal outage performance.
- Difference between MMSE and Nulling.

Penalty due to Overhead



Conclusion

- The diversity-multiplexing tradeoff is a fundamental way of looking at fading channels.
- Same framework can be applied to other scenarios: multiuser, non-coherent, more complex channel models, etc.