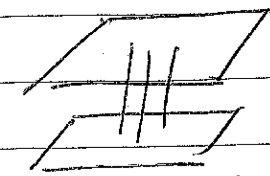
If you don't have the tetrahedral property



in a torus

each slice is a circle

$\partial(\bigcirc) = \emptyset$

Yi Wang - Finite total Q-curvature on a locally conformally flat manifold
joint work with Zhiqin Lu.

In conformal geometry, there is a notion of Q-curvature (Branson 1980's)

In dim 4

just a scalar that is paired w/ a conformal covariant operator

$$Q_g := \frac{1}{12}\Big\{ \underbrace{-\Delta R + \tfrac{1}{4}R^2}_{\text{leading terms}} - 3|E|^2 \Big\}$$

E - traceless part of Ric

In other dimensions it has other expression

Paneitz op

$$P_g := \Delta^2 + \delta\left(\tfrac{2}{3} R g - 2\,\mathrm{Ric}\right) d$$

$\delta$ - divergence

$d$ - differential

(lost her)

conformal change of metric

$g_u = e^{2u} g_0$

Nice PDE $\Rightarrow \quad P_{g_0} u + 2 Q_{g_0} = 2 Q_{g_u} e^{4u}$

analogous to what happens for Gaussian curvature K

$-\Delta g_0 u + K_{g_0} = K_{g_u} e^{2u}$

Q also shows up in the Chern-Gauss-Bonnet formula for a closed 4-manifold.

$(M^n, g) = (\mathbb{R}^n, e^{2u}|dx|^2)$ w/ $\int_M |Q| dv_g < \infty$ is a conformally flat manifold w/ finite total Q-~~me~~ curvature

Mentions thm by Chang Qing Yang 200

normal metric - nonnegative at $\infty$.

Comment: Q-curvature controls the geometry not just at $\infty$, but inside as well

$\downarrow$

result ^(sic) on isoperimetric ratio at $\infty$.

THEOREM (W '11)

$(M^n, g) = (\mathbb{R}^n, e^{2u}|dx|^2)$

noncompact, complete, Rie man w/ nor metric

If $\int_{M^n} |Q_g| dv < \infty$

and $\int_{M^n} Q_g \, dv_g < C_n$

$\Rightarrow |\Omega|_g \leq C(g) \, |\partial\Omega|_g^{n/(n-1)}$

$C_n$ = integral of Q curvature on a semi sphere.

$C_2 = 2\pi \quad C_4 = 4\pi^2$, etc.

Proof: construction of quasiconformal mappings.

THEOREM (W '13)

avoiding too much $Q^+$ v.s $Q^-$ so that they don't cancel out a lot.

If $\alpha := \int_{M^n} Q_g^+ \, dv_g < C_n$

$\beta := \int_{M^n} Q_g^- \, dv_g < \infty$

$\Rightarrow |\Omega|_g \leq \underbrace{C(\alpha, \beta, n)}_{\text{stuff}} |\partial\Omega|_g^{n/(n-1)}$

Counterexample

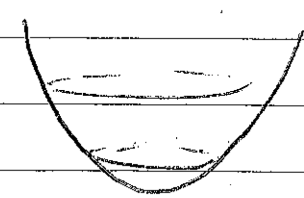
$M^2$ = Half cylinder

$\int_{M^n} k^+(x)\,dy = 2\pi$ and doesn't satisfy the isoperimetric inequality

In higher dimension there exists "natural" conformally flat structures & as in $n=2$ but we can assume it

THEOREM (Lu, W '15) even dim locally con flat complete man w/ fin total $Q$-curvature and finitely many con flat-simple ends, w/ normal metric (lost her) and $\int_{M^n} |L|^n dv_g < \infty$

b/c we want to avoid




where

$L$ - 2nd fundamental form

$$\Rightarrow \int_{M^n} Q_g\, dv_g \in 2C_n\mathbb{Z}$$

$C_n$ - intg of $Q$-cur on std $n$-hemisphere $S^n_+$



can prove previous thm for All global conformal invariant

$\int_M Q(g)\,dv_g$ is invariant under conformal changes of metric

Any such global conf inv satisfies

(Aleksakis '09) $Q(g) = W(g) + \mathrm{div}_i T^i(g) + A \cdot \mathrm{Pfaff}(R)$

$W(g)$: local conf inv of weigh $-n$

$T^i(g)$: intrinsic vector field w/ weigh $-n+1$ = derivatives of the metric up to $(n-1)$ order (or smt like this).

$A$: const

Weyl's classical thm

For an intrinsic vector field

$$T^i(g) = \sum_{q\in Q} a_q\, C^{q,i}(g)$$

$C^{q,i}(g)$: partical contraction w/ one free index $i$

$$= \mathrm{p\,contr}\left(\nabla^{(m_1)}_{r_1\cdots r_{m_1}} R_{i_1 j_1 k_1 l_1} \otimes \cdots \otimes \nabla^{(m_a)}_{t_1\cdots t_{m_a}} R_{i_a j_a k_a l_a}\right)$$

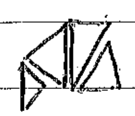
w/ $\sum_{t=1}^{a} (m_t+2) = n-1$

We do not have the cut off function b/c we don't have lower Ricci curvature bound.

Open Questions

* Uniqueness of tangent cone at $\infty$. (should be there if Q-curvature is int or $Q^+$ is int)
* Geodesic distance estimate for $n \neq 2$
* Volume growth of geodesic balls.

Look up ( Gauss map $M \longrightarrow S^n$ )



Raquel Perales Aguilar - Convergence of Manifolds and Metric Spaces with boundary

I. Gromov-Hausdorff convergence

DEFINITION $(X_1, d_1)$, $(X_2, d_2)$ compact metric spaces

check $d_{GH}((X_1,d_1),(X_2,d_2)) = \inf_Z \{ d_H^Z(\phi(X_1), \phi(X_2)) \}$

inf is over where $\phi_i : X_i \rightarrow Z$

where $d_H^Z(A,B) = \inf_\epsilon \{ \epsilon \mid T_\epsilon A \supset B, T_\epsilon B \supset A \}$

THEOREM Gromov (compactness theorem)

Let $\mathcal{X}$ denote the class of compact metric spaces s.t.

1) $\exists\, D > 0$ s.t. $\mathrm{diam}(X) \leq D \ \forall X \in \mathcal{X}$
2) $\exists\, N( \ , \mathcal{X}) : (0,\infty) \longrightarrow \mathbb{N}$
~~where~~ $\forall X \in \mathcal{X} \ \exists \{ B(x_i, r) \}_{i=1}^{N(r)}$ cover of $X$ for all $X \in \mathcal{X}$

then each sequence of $\mathcal{X}$ has a GH convergent subsequence

THEOREM (for manifolds w/ no boundary)

$(M_j, g_j)$ of Riemann manifold

$Ric(M_j) \geq K \quad Vol(M_j) \leq V \quad diam(M_j) \leq D$

then $\exists$ a subsequence of $\{M_j\}$ that GH-converges.