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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Justin Hilburn **Email/Phone:** jhilburn@uoregon.edu

Speaker's Name: Geordie Williamson

Talk Title: Modular representation theory and the Hecke category

Date: 11 / 20 / 2014 **Time:** 11 : 00 am pm **(circle one)**

List 6-12 key words for the talk: Perverse sheaf, Parity sheaf, decomposition theorem, intersection form, Hecke category, Hodge Theory

Please summarize the lecture in 5 or fewer sentences: The Hecke category is a monoidal category generated by shifts of summands of Bott-Samelson sheaves. In characteristic zero it is semi-simple due to the decompostition theorem, but in positive characteristic things are more interesting. In particular one gets new analogues Kazhdan-Lusztig poynomials that are ubiquitious in modular representation theory.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

☑ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.

☑ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.

- **Computer Presentations**: Obtain a copy of their presentation
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Geordie Williamson

Basic setting in rep theory

V standard module   often: a) $V/\mathrm{rad}\langle,\rangle$

$\langle,\rangle$ invariant form 0 or simple

b) we get all simples this way.

Parallel in geometry

$f: \tilde{X} \to X$ resolution of variety / $\mathbb{C}$ semismall

(i.e. $X = X_\lambda$ s.t. $\mathrm{codim}\, X_\lambda \geq \frac{1}{2} \pi^{-1}(x)$ $\forall x \in X_\lambda$)

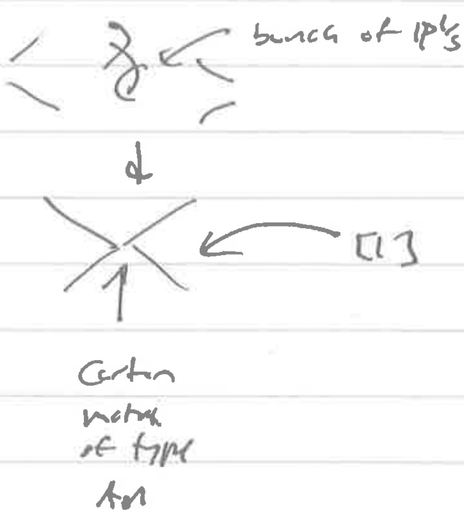
Observation: k field coefficients

$f_* \underline{k}_{\tilde{X}} [\dim_{\mathbb{C}} \tilde{X}] \cong \bigoplus IC(\overline{X}_\lambda, \mathcal{L}_\lambda)$

$\Longleftrightarrow$

certain intersection forms associated to strata are nondegenerate.

Ex



A_n-singularities

sometimes these structures are literally the same

E.g. Springer resolutions
Geometric Satake
Nakajima Q-varieties

Sometimes can zoom in

$G \supset B \supset T$ Kac-Moody group $/\mathbb{C}$ (W,S) Weyl group

char $K = p \geq 0$

$$\left(D^b(B\backslash G/B : K), \overset{\text{convolution}}{*} \right)$$

$\mathcal{H}_K$ = full additive monoidal Karoubian subcat generated by $\underline{K}_{P_s}[1] = B_s$

= All sums of shifts of summands of Bott Samelson sheaves.

$f_* K_{BS(\underline{w})}[?]$

where $BS(\underline{w}) = P_{s_1} \times_B \times \cdots \times_B P_{s_m}/B \to G/B$

If $p=0$ $\mathcal{H}_K$ is full subcategory of semi-simple complexes. If $p > 0$ it is more interesting.

Thm (Juteau-Mautner-W)

$\{$indec objects in $\mathcal{H}_K\}/\cong(n) \simeq W$

$\mathcal{E}(w) \longleftarrow w$

"parity sheaf"

a) for fixed ω and $p \gg 0$

$$\mathcal{E}(\omega) = IC(\omega)$$

To understand $\mathcal{H}_K$ need to understand failure of decomposition theorem

b) (imprecise) $IH^*(X, \mathbb{Z})$

$$\begin{matrix} IH^*(X,\mathbb{Z}) \\ \| \\ \overline{BwB} \end{matrix}$$

distance between intersection cohomology complex and parity sheaf measures failure of P.D. of $IH(\overline{BwB}, \mathbb{Z})$

(more precise) Achar – Riche

<u>Corollary</u>

$$\underset{\substack{\nearrow \\ \text{split} \\ \text{orthendieck} \\ \text{gr}}}{[\mathcal{H}_K]} = \underset{\substack{\nwarrow \\ \text{Hecke} \\ \text{algebra}}}{H} = \bigoplus_{x \in W} \mathbb{Z}[v^{\pm 1}] H_x$$

$$[\mathcal{E}(\omega)] \longmapsto {}^p\underline{H}_x \quad \text{“}p\text{-canonical” basis}$$

$$\begin{matrix} {}^p\underline{H}_x \\ \| \\ \sum {}^p h_{y,x} H_y \end{matrix}$$

$\uparrow$ ~~[illegible]~~ p-Kazhdan Lusztig polynomial.

<u>Hypothesis</u>

$\mathcal{H}_K$ and p-canonical basis should control much mod p rep theory.

Remark $^\vee\mathcal{H}_X$ is algorithmically computable by work with Elias.

Examples

(1) Jantzen: G finite dim

$\check{G}_k$ dual gp over k

(assume $p > h$)

$$[\Delta((p-1)\rho + x\rho) : L((p-1)\rho + y s)]$$

$$\overset{\shortparallel}{{}^p h_{xy}(1)}$$

Lusztig's character formula predicts $\longrightarrow$ $\parallel$ "torsion free situation"

$h_{xy}(1)$

(2) Finkelberg - Mirković conjecture

$$(\operatorname{Rep} G^\vee_k)_0 \equiv \operatorname{Perv}_{I\text{-constructible}}(G((t))/G[[t]])$$

block of HW module

$\cup$

Z closed I-stable subvariety

(3) Relation: $\mathcal{E}(\omega) = \mathcal{I}\mathcal{C}(\omega)$ correct on Z then Lusztig's conjecture.

(4) Simon Riche: Numerical version of (*)

Application 1

Kahn–He–W: formulas for entries in intersection forms via Schubert cells.

<u>Consequence</u> - "Torsion explosion"

~~Thm Suppose~~

Focus on (!) for GL_n

mult by x_1, x_n $\circlearrowright H^*(GL_n(\mathbb{C})/B;\mathbb{Z}) = \mathbb{Z}[x_1,\dots,x_n]/(e_i(x))$

$\circlearrowleft$ Demazure operators ∂_w

<u>Thm</u> suppose that

$$0 \neq C = \partial_{w_n}(\cdots \partial_{w_2}(x_1^{a_2} x_n^{b_2} \partial_{w_1}(x_1^{a_1} x_n^{b_1}))) \in \mathbb{Z}$$

Then there exists an ~~explicit~~ explicit Bott-Samelson resolution

$f: BS(\underline{w}) \to GL_{n+N}(\mathbb{C})/B$

such that decomposition theorem fails with coefficients in $\mathbb{Z}/C\mathbb{Z}$

$N = \sum a_i + \sum b_i$

Idea of proof: Given w_i, a_i, b_i produce

$BS(\underline{w}) \to GL_{n+N}(\mathbb{C})/B$

miracle: critical fiber is irreducible and smooth with self intersection $\pm C$.

Using this result

<u>Thm</u> (Bourgain - Kontorovich) prime divisors of entries in the semi group $\langle \begin{pmatrix}1&1\\1&0\end{pmatrix}, \begin{pmatrix}0&1\\1&1\end{pmatrix} \rangle$ grow exponentially in word length.

$\Rightarrow$ torsion in $IC(\overline{BwB}/B;\mathbb{Z})$ in GL_n grows exponentially in n.

$\Rightarrow \nexists\ p \in \mathbb{Z}[r]$ such that Lusztig's character formula holds for $GL_r(\overline{\mathbb{F}}_p)$ for $p > p(r)$.

$$\begin{array}{ccc} \mathrm{Rep}\ G^\vee_K & \supset & \mathrm{Tilt} = \langle \Delta\text{-filtered} \rangle \cap \langle \nabla\text{-filtered} \rangle \\ \cup & & \cup \\ (\mathrm{Rep}\ G^\vee_K)_0 & \supset & \mathrm{Tilt}_0 \end{array}$$

Rmk
0) indecomposable tilting modules have a nice classification $T(\lambda)$
1) $\{\mathrm{char}\ T(\lambda)\} \longrightarrow$ simple chars
but $\mathrm{char}\ T(\lambda)$ is more diff
i.e. answer unknown for sl_3
2) Tilt is closed under $\otimes$-product
$\mathrm{Tilt}_0 \circlearrowleft$ Translation functors
Θ_s = translation through small
$[\mathrm{Tilt}_0] \cong \mathrm{sgn}\ \mathbb{Z}W \underset{\mathbb{Z}W_f}{\otimes} \mathbb{Z}W \ \circlearrowleft\ [\Theta_s]$ $\underset{\text{ves}}{\shortparallel}$

Conjecture ($p > h$) after forgetting grading
$\overline{\mathcal{H}}_K \cong \mathrm{Tilt}_0$ with $\Theta_s = B_s = \underline{K}_{B_s}(1)$

$\langle \xi_w \rangle_{w \notin f_W}$

$\uparrow$ image of $\mathcal{H}_K$ in $D^b(G((t))/I)$

Consequence $(T(x \cdot_p 0) : \Delta(y \cdot_p 0)) = h_{x,y}(1)$

observation conjecture follows if $\mathrm{Tilt}_0 \circlearrowleft \mathcal{H}_K$ with $\Theta_s = b_s$