

NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

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Speaker's Name: Hiraku Nakajima

Talk Title: Coulomb Branches of 3d N=4 Supersymmetric quantum field theories

Date: 11 / 18 / 2014 Time: 11:00 **am** pm (circle one)

List 6-12 key words for the talk: Supersymmetry, Higgs Branch, Coulomb Branch, Hyperkahler, Monopole Formula, Quantum Field Theory

Please summarize the lecture in 5 or fewer sentences:
From a 3d N=4 supersymmetric gauge theory physicists can construct two hyperkahler manifolds: the Higgs and Coulomb branches. Unfortunately until this time only the Higgs branch has a mathematically satisfactory definition. Nakajima gave a conjectural description of the coordinate ring of the coulomb branch in terms of the cohomology of a moduli space of monopoles.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
 - **Computer Presentations:** Obtain a copy of their presentation
 - **Overhead:** Obtain a copy or use the originals and scan them
 - **Blackboard:** Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
 - **Handouts:** Obtain copies of and scan all handouts
- For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list.
(YYYY.MM.DD.TIME.SpeakerLastName)
- Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

Nakajima - Coulomb branches of 3d $N=4$ gauge theories
 and Donaldson-Thomas invariants
 (Groverman - Fukaya)

G reductive group $\subset GL$
 M symplectic representation of G

\implies
 3D $N=4$ SCFT gauge theory

often
 $N \oplus N^*$
 where
 N is a rep
 $\subset GL$.

2 Hyperkähler manifolds

- M_H Higgs branch
- M_C Coulomb branch

M_H

$$M_H := M // G = M^{-1}(0) // G = \text{Spec}(\mathbb{C}[M^{-1}(0)]^G)$$

\uparrow symplectic quotient

$$M^{-1}(0)^{\text{free}} / G \subset M // G$$

\uparrow holomorphic symplectic manifold

Properties

① $\mathbb{C}^* \curvearrowright M // G$ symplectic form $\omega_t > 0$
 \uparrow induced by scalar multiplication on M

② If G has nontrivial character $\chi: G \rightarrow \mathbb{C}^*$
 can consider $M //_{\chi} G = M^{-1}(0) //_{\chi} G = \text{Proj}(\bigoplus_{n \geq 0} \mathbb{C}[M^{-1}(0)]^{\chi^n})$
 \uparrow G.I.T quotient

$M //_{\alpha} G \rightarrow M // G$ is often a symplectic resolution.

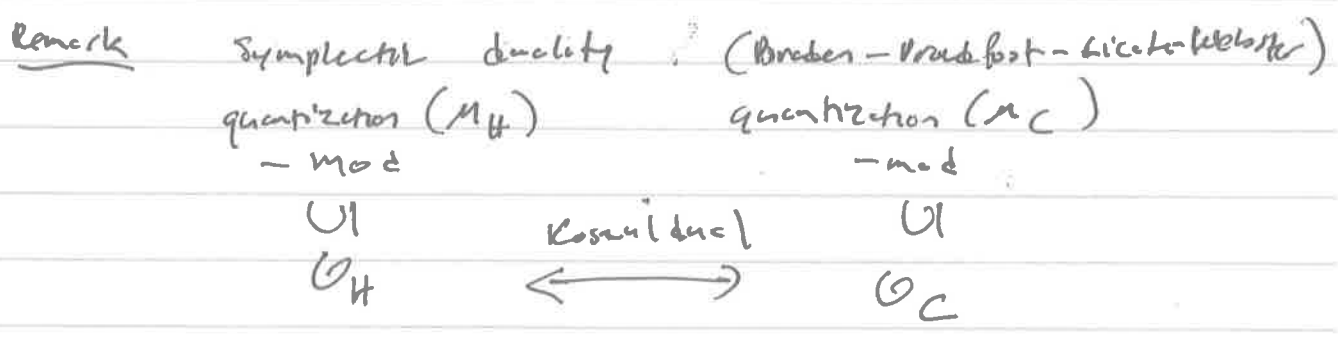
③ Suppose M is a representation of a larger group $\tilde{G} \supset G$. Then $G_F = \tilde{G}/G$ acts on $M // G$.
 ↑ Flavor symmetry

④ $M = N \oplus N^*$ quantization of $M // G$ by quantum Hamiltonians of $D(N)$ by G
 ↑ differential operators on N

Goal: Give a definition of Coulomb branch.

Note $\dim_{\mathbb{C}} M_{\mathbb{C}} = 2 \cdot \text{rank } G$.

Issue: quantum corrections of metric on $M_{\mathbb{C}}$.



For Type A flag variety this recovers classical Koszul duality on $\mathcal{O}_{\mathbb{P}^n}$

Properties (expected):

① M_C affine algebraic variety $\subseteq \mathbb{C}^x$
 (symplectic on open part)

Examples

a) Abelian gauge theory

$$1 \rightarrow G = T^d \xrightarrow{\text{gauge}} T^n \xrightarrow{\text{Flavor}} T_F^{n-d} \rightarrow 1$$

\uparrow gauge \parallel $(\mathbb{C}^x)^n$ \hookrightarrow Flavor

$M_H = M // T^d \subseteq T_F^{n-d}$ hypertoric manifold
 (Bielawski - Dancer)

Have dual form

$$1 \rightarrow (T_F^{n-d})^\vee \rightarrow (T^n)^\vee \rightarrow (T^d)^\vee \rightarrow 1$$

$M_C = M // (T_F^{n-d})^\vee$ canonically self dual

b) $G = GL(k)$, $M = 0$

$M_H = 0$

$M_C = \begin{cases} \text{Seiberg-Witten} & \text{for } k=2 \\ \text{Chowring-Hervey} & \text{for } k \geq 2 \end{cases}$

\Rightarrow moduli space of charge k BPS $SU(2)$ monopole

\hookrightarrow Donaldson $\Rightarrow \left\{ \begin{array}{l} \text{based maps } \mathbb{P}^1 \rightarrow \mathbb{P}^1 \\ \text{deg } k \end{array} \right\}$

use Nahm's trick

understand how classification of vector bundles on manifold with large symmetry

D-branes

Białowski

= symplectic reduction $T^*GL(k)$
 by $N_- \times N_-$ at (γ, γ)

where (γ, γ) is an sl₂ triple
 for regular nilpotent x .

c) quiver gauge theory (finite type)

$\Gamma = \text{APG}$ dynkin diagram

$Q = (Q_0, Q_1)$ quiver

$$V = \bigoplus_{i \in Q_0} V_i$$

$$W = \bigoplus_{i \in Q_0} w_i$$

$$G = \prod_i GL(V_i)$$

$$G_W = \prod_i GL(w_i)$$

$$M = \text{NON}^+$$

$$\bigoplus_{i \in Q_1} \text{Hom}(V_{s(i)}, V_{t(i)})$$

$$\bigoplus_i \text{Hom}(w_i, V_i)$$

\downarrow
 dim W dominant
 integral wt

$$u = \dim V - \sum \dim V_i$$

$$M_W = M_0(V, W)$$

quiver
 variety
 of finite type

expected answer:

$$\cong M_C \subset \text{Gr}_W$$



Affine Grassmannian
 $H = (\text{adjoint})$ group of
 type Γ

$\overline{H(0)} \in \Lambda$
 \uparrow
 transverse
 slice to
 $H(0)$ in

② $1 \rightarrow G \rightarrow \hat{G} \rightarrow G_F \rightarrow 1$

Br λ_F cover of G_F

$\mapsto \int M_C^{\lambda_F} \rightarrow M_C$ \mathcal{L}_{λ_F} line bundle

~~...~~

③ $\text{Hom}(\pi_1(G), \mathbb{C}^*) \rightarrow M_C$

can be extended to non-abelian group action

~~...~~

In action
can
these
are the
G.T.
quants
 $M //_{\lambda_F} T_F$

	$M //_{\lambda_F}$	M_C
G_F	action	line bundle
G	line bundle	action

Proposal of the definition of the Coulomb branch M_C

Need $M = N \oplus N^*$

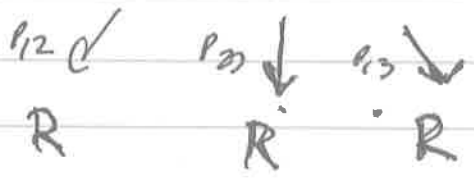
$G_F = \text{affine Grassmannian of } G$
 $= G(\mathbb{C}[z]) / G(\mathbb{C}[z])$

$G(\mathbb{C}[z]) \backslash G_F = \left\{ \begin{array}{l} G\text{-bundles on } D = \text{Spec } \mathbb{C}[z] \\ + \text{ trivialization on } \hat{D} = \text{Spec } \mathbb{C}((z)) \end{array} \right\}$

$$R = \left\{ (P_1, P_2, \varphi, s_1, s_2) \mid \begin{array}{l} s_i \in H^0(P_i; X_G N) \\ \varphi(s_1) = s_2 \end{array} \right\}$$

convolution diagrams

$$\left\{ (P_1, P_2, P_3, \varphi_{12}, \varphi_{23}, s_{12}, s_{23}) \mid \varphi_{12}(s_{12}) = s_{23} \right\}$$



$$(P_{13}) \leftarrow (P_{12} \circ P_{23})$$

$$\rightarrow H_x^{BM}(R) \otimes H_x^{on}(R) \rightarrow H_x^{BM}(R)$$

Proposed $\mathbb{C}^k \curvearrowright \mathbb{C}[M_C] = H_0^{BM}(R)$

equivariant parameter = deformed parameter

★ Natural generalization

$$\mathbb{C}^k \curvearrowright \mathbb{C}[R] \quad \mathbb{C}^k\text{-equiv homology } D = H_{\mathbb{C}^k, 0}^{BM}(R)$$

★ $M \cong O \quad G = GL(k) \quad (k \geq 1)$

$H_x(GR)$ bezukommutativ
Finkelberg
Mirkovic

Structure of GR

\rightarrow " of R

\rightarrow LES for $H_0^{BM}(R)$

get formula for dimensions called monopole formula Hanany