MSRI
Mathematical Sciences Research Institute

17 Gauss Way Berkeley, CA 94720-5070 p: 510.642.0143 f: 510.642.8609 www.msri.org

NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: KAROL KOZIOL Email/Phone: (315)569-6968

Speaker's Name: PEI-YU TSAI

Talk Title: NEWFORMS FOR ODD ORTHOGONAL GROUPS

Date: 8/15/14 Time: 2:00 am / pm (circle one)

List 6-12 key words for the talk: ORTHOGONAL GROUPS, NEWFORMS, SYMPLECTIC MOTIVE

Please summarize the lecture in 5 or fewer sentences: IN THIS TALK, AN ANALOG OF NEWFORMS FOR ORTHOGONAL GROUPS IS INVESTIGATED. MOREOVER, AN ~~[illegible]~~ ANALOG OF CASSELMAN'S RESULT ON NEWFORMS FOR $PGL_2(\mathbb{Q}_p)$ IS PROVEN.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

☒ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.

☒ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.

- **Computer Presentations**: Obtain a copy of their presentation
- **Overhead**: Obtain a copy or use the originals and scan them
- **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
- **Handouts**: Obtain copies of and scan all handouts

☒ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.

☒ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)

☒ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

NEWFORMS FOR ODD ORTHOGONAL GROUPS

– P.-Y. TSAI

NEWFORMS (CLASSICALLY)

- HOL'C CUSPFORMS, HECKE EIGENFORMS, AND L-FN ADMIT EULER PRODUCTS
- CUSP FORMS WHICH DO NOT COME FROM LEVEL-RAISING (COMPLEMENTARY TO OLDFORMS)
- MODULARITY:

$E/\mathbb{Q}$ ELL CURVE OF COND N $\longrightarrow$ NEWFORM OF WT 2 f LEVEL $\Gamma_0(N)$

$$L(E,s) = L(f,s) \quad \text{w/}$$
$$\Lambda(f,s) = \pm N^{-s}\Lambda(f, 2-s)$$

GALOIS ACTION ON TATE MODULE

$$\xrightarrow[\text{LANGLANDS}]{} \quad \pi = \bigotimes_v{}' \pi_v \quad \text{AUT REP OF } PGL_2(\mathbb{A})$$

2010: GROSS' LETTER TO SERRE
CAN WE ASSOCIATE

$$\{\text{MOTIVE}\} \longrightarrow \{\text{AUTOMORPHIC FORM}\} \ ?$$

MSRI
Mathematical Sciences Research Institute
Shiing-Shen Chern Hall 17 Gauss Way Berkeley, CA 94720-5070 p: 510.642.0143 f: 510.642.8609 www.msri.org

* MOTIVE M IS A SYMPLECTIC PURE MOTIVE

1) $M = H_1(A)$ A ABELIAN VARIETY OF DIM n

WT -1, RK $2n$ PURE

$M \times M \longrightarrow \mathbb{Q}(1)$ WEIL PAIRING

2) $M = H^d(X)$ X SMOOTH PROJ VAR. OF DIM d $/\mathbb{Q}$

WT d

$M \times M \xrightarrow{\text{CUP}} \mathbb{Q}(-d)$ SYMPLECTIC PAIRING IF d ODD

3) $M = H_1(X)$ X CURVE

SYMPLECTIC PURE MOTIVE WT -1, SAY RK $2n$

WANT TO GET

M SYMPLECTIC PURE MOTIVE $\rightsquigarrow$ $\mathbb{C}f$ $\subset \pi = \bigotimes_v' \pi_v$ AUT REP CUSPIDAL

$\hookrightarrow \mathcal{F}(G(\mathbb{Q})\backslash G(\mathbb{A}))$ (CUSP FORMS)

$WD_{\mathbb{Q}_p} \longrightarrow GSp_{2n}$ ℓ-ADIC IF $\hat{G} = Sp_{2n}$, $G = SO_{2n+1}$

OR

$WD_{\mathbb{R}} \longrightarrow GSp_{2n}$ DE RHAM

FOR EACH FINITE v, π_v SHOULD BE IRRED, SMOOTH, GENERIC REP OF $SO_{2n+1}(\mathbb{Q}_v)$

MSRI
Mathematical Sciences Research Institute
Shiing-Shen Chern Hall 17 Gauss Way Berkeley, CA 94720-5070 p: 510.642.0143 f: 510.642.8609 www.msri.org

π SHOULD BE CUSPIDAL :

$$\pi \hookrightarrow \mathcal{F}(G(\mathbb{Q})\backslash G(\mathbb{A})) \longrightarrow \mathbb{C}$$

$$F \longmapsto \int_{U(\mathbb{Q})\backslash U(\mathbb{A})} F(u)\,\bar{\Theta}(u)\,du$$

U MAXL UNIP. OF G, $\Theta : U(\mathbb{Q})\backslash U(\mathbb{A}) \longrightarrow S^1$ GENERIC

(NON TRIV. ON ALL SIMPLE ROOTS)

$$\mathrm{HOM}_{U(\mathbb{A})}(\pi, \Theta) = \bigotimes_v \underbrace{\mathrm{HOM}_{U(\mathbb{Q}_v)}(\pi_v, \Theta_v)}_{\text{SPACE OF WHITTAKER FUNCTIONALS}} \leftarrow 1\text{-DIML}$$

WHEN $v = \infty$, $M = H_1(X)$ X CURVE OF GENUS n

WT -1, RK $2n$

M_{dR} HODGE NUMBER $(p_i, q_i) = (0,1)$

$\rightsquigarrow$ π_∞ HAS HARISH-CHANDRA PARAM $\alpha = (\frac{1}{2}, \ldots, \frac{1}{2})$

$K = S(O_{n+1} \times O_n)$

MOREOVER, THE MINIMAL K-TYPE OF $\pi_\infty|_K$ IS

$\lambda = (1, \ldots, 1)$ (HIGHEST WT)

$\rightsquigarrow$ GENERIC LIMIT DISC. SERIES FOR $G(\mathbb{R}) = SO_{n+1,n}$

MSRI
Mathematical Sciences Research Institute
Shiing-Shen Chern Hall 17 Gauss Way Berkeley, CA 94720-5070 p: 510.642.0143 f: 510.642.8609 www.msri.org

$\pi_\infty|_K = V_\lambda + \cdots$

MIN. K-TYPE

$V_\lambda = V_{\cancel{\#} v} \otimes V_\mu$

SO_{n+1} SO_n

USING BRANCHING LAWS FOR ORTHOGONAL GRPS, GROSS SHOWED IF $H_\infty \trianglelefteq O_n \hookrightarrow K$, THEN $V_\lambda^{H_\infty}$ IS 1-DIM'L

<u>EX</u>. $n = 1$

$(SO_3(\mathbb{Q}_p) \cong) PGL_2(\mathbb{Q}_p) > \Gamma_0(p^m)$

CASSELMAN: GIVEN π GENERIC, $\pi^{\Gamma_0(p^m)}$ IS 1 DIML FOR SMALLEST m ST $\pi^{\Gamma_0(p^m)} \neq 0$

USES

- JACQUET, PIATETSKI-SHAPIRO, SHALIKA: $GL_n(\mathbb{Q}_p)$ — $GL_{n-1}(\mathbb{Q}_p)$
- ROBERTS - SCHMIDT: $PGSp_4(\mathbb{Q}_p)$ $(\cong SO_5(\mathbb{Q}_p))$ — $SO_4(\mathbb{Q}_p)$
- MIYAUCHI: $U_{2,1}$ OF UNRAM'D LOCAL FIELDS — $U_{1,1}$

FOR $SO_{2n+1}(\mathbb{Q}_p)$, ONE WOULD LIKE TO COMPARE W/ $SO_{2n}(\mathbb{Q}_p)$

MSRI
Mathematical Sciences Research Institute
Shiing-Shen Chern Hall 17 Gauss Way Berkeley, CA 94720-5070 p: 510.642.0143 f: 510.642.8609 www.msri.org

V : $2n+1$ DIM'L VECTOR SPACE $/\mathbb{Q}$, W/ BASIS $\{e_1, \dots, e_n, v_0, f_n, \dots, f_1\}$

W/ GRAM MATRIX $\begin{pmatrix} & & & & & 1 \\ & & & & 1 & \\ & & & 2 & & \\ & & 1 & & & \\ & ⋰ & & & & \\ 1 & & & & & \end{pmatrix}$ IE $\langle e_i, f_j \rangle = \delta_{ij}$

$\langle v_0, v_0 \rangle = 2$

$\rightsquigarrow$ QUADRATIC FORM ON V W/ DISC 2

LET $G = SO(V)$, AND SET $L = \left(\bigoplus_{i=1}^{n} \mathbb{Z} e_i\right) \oplus \mathbb{Z} v_0 \oplus \left(\bigoplus_{i=1}^{n} \mathbb{Z} f_i\right)$

THEN L IS AN EVEN LATTICE

SET $L(N) = \left(\bigoplus_{i=1}^{n} \mathbb{Z} e_i\right) \oplus N\mathbb{Z} v_0 \oplus \left(\bigoplus_{i=1}^{n} N\mathbb{Z} f_i\right)$ $\qquad L(1) = L$

THEN $N^{-1}\langle \,,\, \rangle : L(N) \times L(N) \longrightarrow \mathbb{Z}$

IS A BILINEAR FORM ON $L(N)$

WE CAN WRITE $G(\mathbb{Q}_p) = SO(V \otimes \mathbb{Q}_p)$

$\cup$

$SO(L \otimes \mathbb{Z}_p)$ HYPERSPECIAL COMPACT GRP

$$\boxed{\begin{array}{ll} \underline{\text{DEF}} & J(1) = K(1) = SO(L \otimes \mathbb{Z}_p) \\ \underline{m \geq 1} & J(p^m) = SO(L(p^m) \otimes \mathbb{Z}_p) \subset G(\mathbb{Q}_p) \end{array}}$$

$J(1)$ HAS REDUCTIVE QUOTIENT $SO_{2n+1}(\mathbb{F}_p)$

BUT,

MSRI
Mathematical Sciences Research Institute

Shiing-Shen Chern Hall 17 Gauss Way Berkeley, CA 94720-5070 p: 510.642.0143 f: 510.642.8609 www.msri.org

SET $K(p^m)$ TO BE

$$\begin{tikzcd} K(p^m) \arrow[d] \arrow[r, phantom, "\triangleleft_2"] & J(p^m) \arrow[d] & \\ SO_{2n}(\mathbb{F}_p) \arrow[r] & O_{2n}(\mathbb{F}_p) \arrow[r, "\det"] & \{\pm 1\} \end{tikzcd}$$

RMK $K(1)$, $K(p)$ MAX'L OPEN COMPACT NOT THE CASE FOR $m > 1$

$$\boxed{\begin{array}{l}\text{CONJ: ONE CAN ASSOCIATE TO A SYMPLECTIC MOTIVE } M \text{ A NEWFORM} \\ F : G(\mathbb{Q})\backslash G(\mathbb{A}) / (H_\infty \times \underbrace{\textstyle\prod_p K_p}_{K(N)}) \longrightarrow \mathbb{C} \\ \text{WHERE } K_p = K(p^{f(p)}) \text{, WHERE } f(p) \text{ IS THE ARTIN CONDUCTOR OF THE WD REP FOR THE STD REP OF } M\end{array}}$$

ITS RESTRICTION TO THE REAL GROUP IS

$$F_\infty : \Gamma_0(N) \backslash G(\mathbb{R}) / H_\infty \longrightarrow \mathbb{C}$$

$$\Gamma_0(N) = K(N) \cap G(\mathbb{R})$$

$$\boxed{\begin{array}{l}\text{THM (T.) IF } \pi \text{ IS IRRED, GENERIC SMOOTH COMPLEX REP OF } G(\mathbb{Q}_p) \text{, THEN } \exists\, f \in \mathbb{Z}_{\geq 0} \text{ ST} \\ \qquad \pi^{K(p^f)} \text{ IS 1 DIM'L} \\ \text{AND} \qquad \pi^{K(p^m)} = 0 \qquad \text{FOR } m < f.\end{array}}$$

MSRI
Mathematical Sciences Research Institute
Shiing-Shen Chern Hall 17 Gauss Way Berkeley, CA 94720-5070 p: 510.642.0143 f: 510.642.8609 www.msri.org

$\boxed{\text{WHEN } \pi = \pi(M)_v \text{ (GENERIC REP ASS'D TO } M\text{) THEN } f \text{ AGREES W/ THE ARTIN CONDUCTOR OF THE } WD_{\mathbb{Q}_p}\text{-REP OF } M}$

NOTE THAT FOR THE ANALYTIC INVT

$$\varepsilon(\pi, s, \psi) = \underset{\pm 1}{\underset{\parallel}{\varepsilon(\pi)}} \cdot p^{-a(s-1/2)}$$

WE HAVE $\underline{f = a}$, AND $J(p^f)/K(p^f)$ ACTS ON THE LINE $\pi^{K(p^f)}$ BY $\varepsilon(\pi)$. ADDITIONALLY, THE WHITTAKER FUNCTIONAL IS NONZERO ON $\pi^{K(p^f)}$