MSRI
Mathematical Sciences Research Institute

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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Pengcheng Xu Email/Phone: pengcheng.xu@okstate.edu

Speaker's Name: Jeremy Kahn

Talk Title: Surface subgroups of isometries of hyperbolic 3-space

Date: 03/21/2013 Time: 9:30 am / pm (circle one)

List 6-12 key words for the talk: Surface subgroup; Kleinian group; mixing; geodesic flow; pair of pants; cubulation

Please summarize the lecture in 5 or fewer sentances: The speaker explained his work with Markovic about building nearly geodesic immersed closed surfaces in a closed hyperbolic 3-manifold out of immersed pair of pants.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

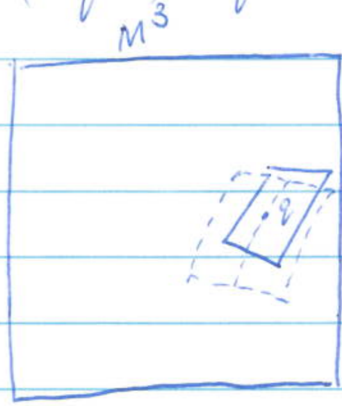
- ☐ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☐ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
    - **Computer Presentations**: Obtain a copy of their presentation
    - **Overhead**: Obtain a copy or use the originals and scan them
    - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
    - **Handouts**: Obtain copies of and scan all handouts
- ☐ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☐ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☐ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

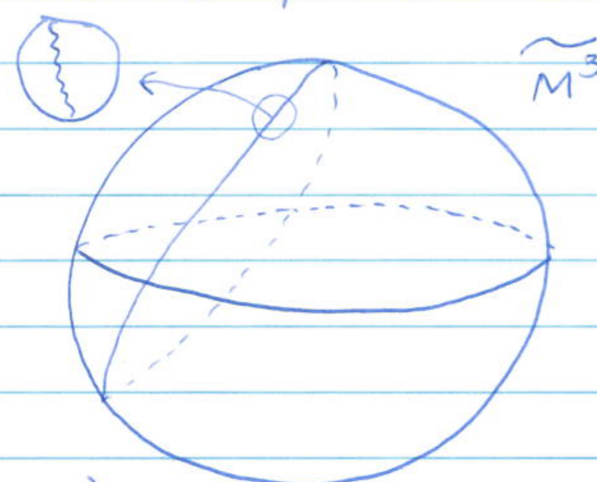
Thursday
1^{st} talk

Surface Subgroups of Kleinian groups

speaker : Jeremy Kahn.

For every closed hyperbolic M^3 and $\varepsilon > 0$ we can find a finite set $\{f_i : S_i \to M\}_{i \in I}$ of immersed ε-almost geodesic hyperbolic surfaces, such that for any $p \in M$ and $P^2 \subseteq T_pM$, there is $i \in I$, $q \in S_i$, s.t
$(d_q f_i)(T_q S_i)$ is ε-close to (p, P)



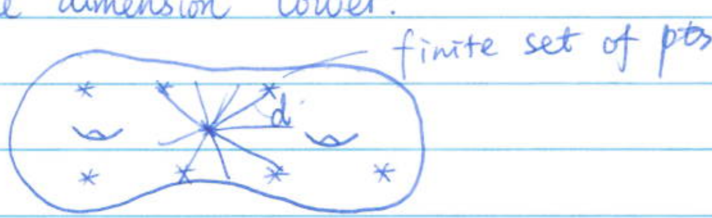


(Sageev) (Bergeron-Wise)
Corollary : ~~M is~~ $\pi_1(M)$ is cubulated

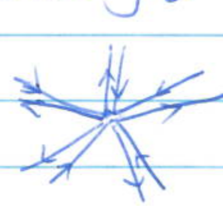
Theorem (Agol-Wise) Every finite npc cube complex with hyperbolic π_1 is virtually special.

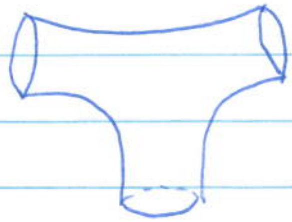
Corollary : M is virtually Haken (& virtually fibred)

One dimension lower.



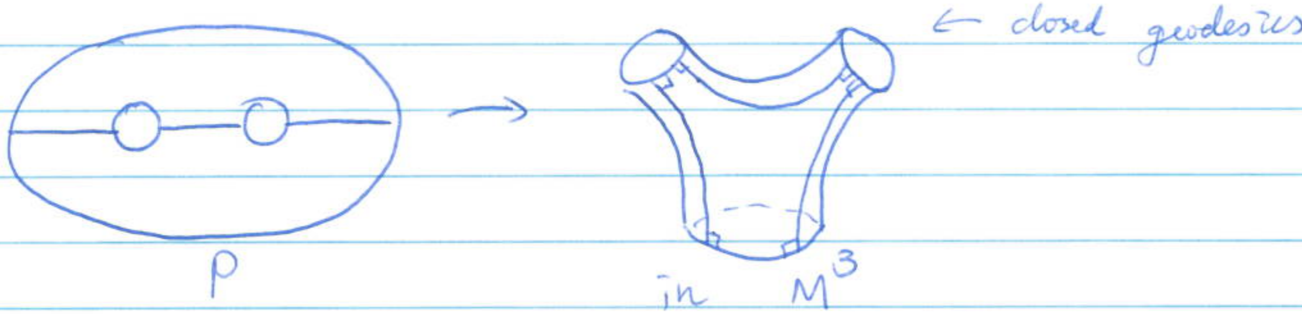
doubling trick

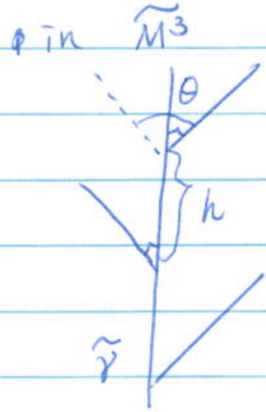




Finite P of paints (with repetition)
Doubling ~~them~~ to match them at each boundary geodesic.
Control bending angles of geometry of pants

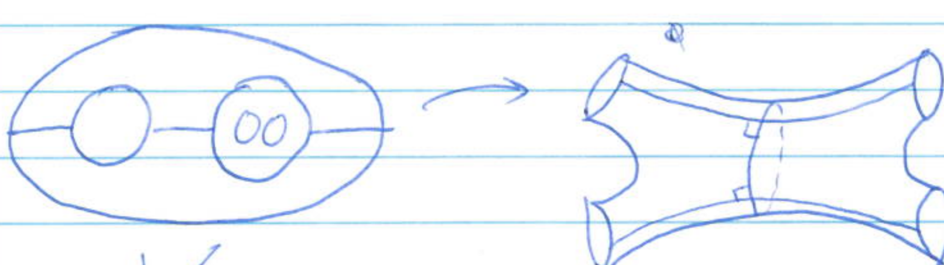
Wire and Cloth

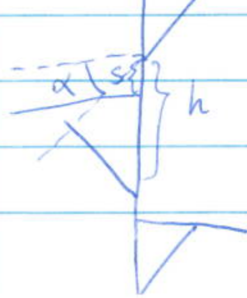




half length

$$hl(\gamma) = h + i\theta \in \mathbb{C}/_{2\pi i\mathbb{Z}}$$





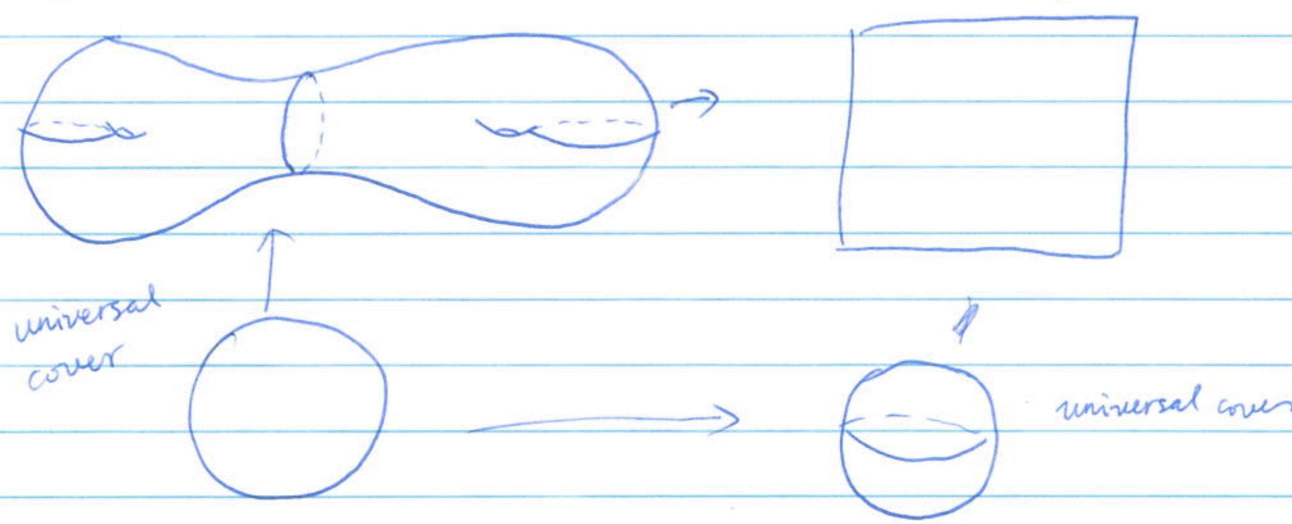
$$sh(\gamma) = s + i\alpha \in \mathbb{C}/_{hl(\gamma)\cdot\mathbb{Z} + 2\pi i\mathbb{Z}}$$

$\overset{R_0}{\exists} K_0$, $\forall M$, ~~$\exists R_0$~~ $\forall R > R_0$
$\varepsilon < \frac{1}{K_0}$.

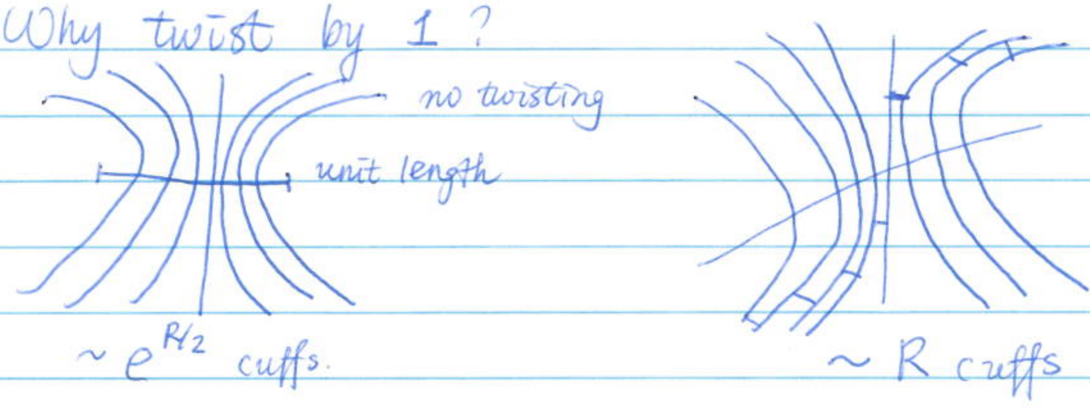
pants decomposition

Suppose $f : (S, \mathcal{C}) \to M$ is
such that for every $C \in \mathcal{C}$, (curve)
1. $|hl_f(C) - R| < \varepsilon$
2. $|Sh_f(C) - 1| < \varepsilon/R$

Then $f_* \pi_1(S) \to \pi_1(M)$ is injective, and, taking the "R-perfect" hyperbolic structure on S, we can extend
$\partial\tilde{f} : \partial\tilde{S} \to \partial\tilde{M}$ to
$\partial\hat{\tilde{f}} : \partial\mathbb{H}^3 \to \partial\mathbb{H}^3$ to be $1 + K_0\varepsilon$-quasiconformal.



Why twist by 1?

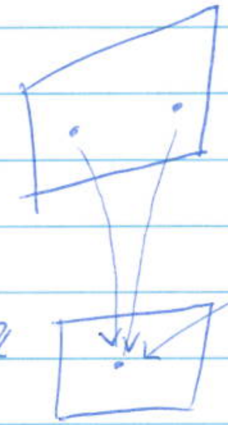


Finding Good Pants in M.

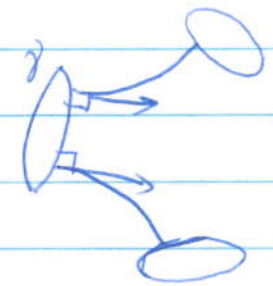
Thm: We can find a collection $\mathcal{P}$ of pants (non-empty multiset), such that $\mathcal{P}$ is evenly distributed every every $\gamma \in \partial P$, $P \in \mathcal{P}$

$N^1(\gamma)$

$\mathbb{C}/2\pi i\mathbb{Z}+\ell(\gamma)\mathbb{Z}$



$\xrightarrow{\sim} N^1(\gamma)$



foot

$f_\gamma(P)$

$\mathbb{C}/2\pi i\mathbb{Z}+h\ell(\gamma)\mathbb{Z} \cong N^1(\sqrt{\gamma})$

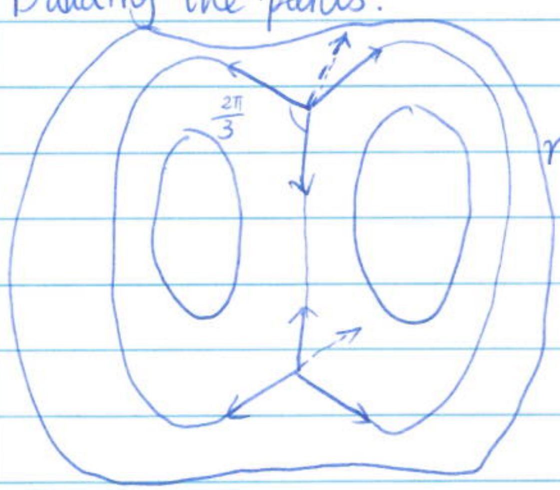
To build $\underset{\text{surface}}{S}$ from $\underset{\text{pants}}{\mathcal{P}}$ $\quad (\mathcal{P}_\gamma = \{P \mid \gamma\in\partial P\})$

We find $\sigma : \mathcal{P}_\gamma \to \mathcal{P}_\gamma$ $\quad$ (~~for $\gamma\in\partial P,\ P\in\mathcal{P}$~~)

s.t. $|f_\gamma(P) - f_\gamma(\sigma(P)) - i\pi - 1| < \frac{\varepsilon}{R}$

and take $\mathcal{P}_\gamma^+ \sqcup \mathcal{P}_\gamma^- \to \mathcal{P}_\gamma^+ \sqcup \mathcal{P}_\gamma^-$ (σ, σ^{-1})

Building the pants.



$r = R + \log\sqrt{3}$

$h\ell(p) = R$

Affinity between V & W.

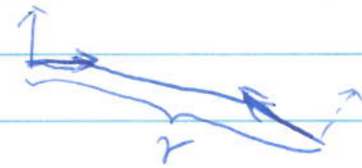
$a(v,w) =$

in $\mathbb{H}^3$ $\to$

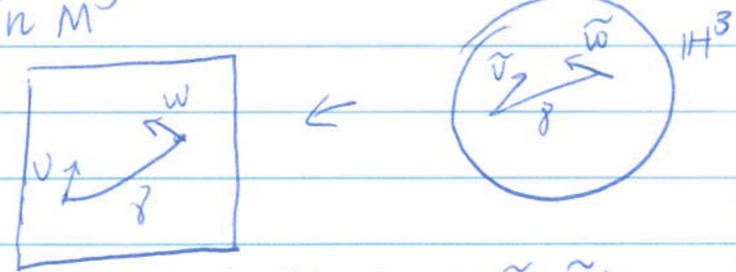
$$\int_{F\mathbb{H}^3} ((g_r)_*\psi_v^\varepsilon)(\psi_{-w}^\varepsilon)$$

1. $a(\gamma\cdot v, \gamma\cdot w) = a(v,w)$ $\gamma \in \mathrm{Isom}(\mathbb{H}^3)$

2. $a(v,w) > 0 \Rightarrow$ nearly



in M^3

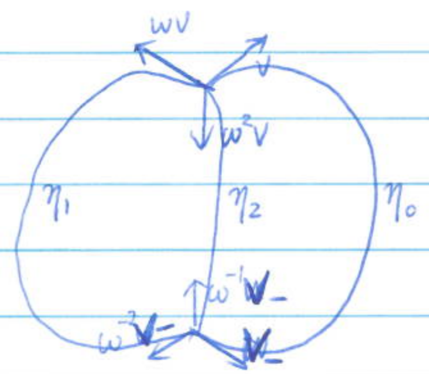


$$\sum_\gamma a_\gamma(v,w) = \int_{FM^3} \left((g_r)_* \psi_v^\varepsilon\right) \psi_{-w}^\varepsilon$$

$$\to \left(\int \psi_v^\varepsilon\right)\left(\int \psi_w^\varepsilon\right) \Big/ \int 1 \quad \text{by Mixing}$$

$$a_\gamma(v,w) = a(\tilde v, \tilde w)$$

3. $\sum_\gamma a_\gamma(v,w) = 1 + O(e^{-qR})$



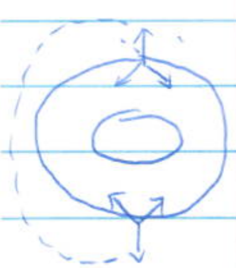
Good connected tripod

$$b(V_+, V_-, (\eta_i)) = \prod_{i=0}^{2} a_{\eta_i}(\omega^i V_+, \omega^{-i} V_-)$$

$$d\mu_3(\quad) = b(\quad)\, d\Lambda(\quad)$$

$\downarrow$

measure on good pants.



$$\begin{tikzcd}
\text{good connected tripods} \arrow[r] \arrow[d] \arrow[dr, phantom, "\doteq"] & \text{Pants} \arrow[d, "\text{feet}"] \\
\text{good connected bipods} \arrow[r, "\text{predicted feet}"'] & N^1(\sqrt{\gamma})
\end{tikzcd}$$

$$\text{forget}\, p_* \mu_3 = \underbrace{\sum_\gamma a_\gamma(\omega^2 V_+, \omega^{-2} V_-)}_{1 + O(e^{-qR})} \mu_2$$