1. Kinetic transport of quasicrystals

Joint work with Marklof.

Lorentz gas model: P locally finite point sets in $\mathbb{R}^d$, asymptotic density 1, a fixed ball of radius r centered at each point, and non-interacting particles reflect on the surface of these scatterers.

Limits:

$T \to \infty$: central limit theorem in dimension 2.

$r \to 0$: Boltzman-Gradt limit.

Let $K_r = \mathbb{R}^d - Balls$, $T^1 K_r$ is the phase space. When $r \to 0$, we need to scale it by r^{d-1} ("macroscoptic coordinates"), hence the flow ϕ_r is on $T^1(r^{d-1}K_r)$. Extend it to $T^1(\mathbb{R}^d)$.

Let $f \in L^1(T^1(\mathbb{R}^d))$ be the density, $L_r^t f$ be the evolution of density, Question: is there a limit of L_r^t as $r \to 0$? If there is, does it satisfy linear Boltzmann equation $(\partial_t + V\nabla_Q)f = \int_{S_1^{d-1}}(f_t(Q,V') - f_t(Q,V)\sigma(V',V)dV'$?

Proved for random P by Gallavotti, Spahn. etc.

How about for a fixed P?

For integer lattice, linear Boltzmann can not hold. (Golse, 06)

Theorem A (M-S) In the lattice case, the limit exists, but satisfies generalized linear Boltzmann equation:
Use extended phase space: $X = T^1(\mathbb{R}^d \times \mathbb{R}_{>0} \times S_1^{d-1}$. For $f \in L^1(T^1\mathbb{R}^d)$, $\tilde{f}_t$ on X is the solution of:
$f_0(Q,V,\xi,V_+) = f(Q,V)\rho(V,\xi,V_+)$, $(\partial_t + V\nabla_Q - \partial\xi)\tilde{f}_t(Q,V,\xi,V_+) = \int_{S_1^{d-1}} \tilde{f}(Q,V_0,0,V)\rho_0(V_0,V,\xi,V_+)dV_0$,
then $L^t(f)$ can be obtained by integrating over the last 2 parameters of $\tilde{f}$.

Quasicrystal setting: (e.g. Penrose tiling)

Consider P obtained by cut-and-project method. $\mathbb{R}^n = \mathbb{R}^d \times \mathbb{R}^m$, the two projections are π, π_{int}. Let L be a lattice in $\mathbb{R}^n$, $A = \overline{\pi_{int}(L)}$, $W \subset L$ a regular "window" set, $P = \{\pi(x) : x \in L, \pi_{int}(x) \in W\}$.

Theorem A can be extended to quasicrystal with cut and project case. X has another factor W.

The proof is based on:

Theorem B: Starting at (Q_0, V_0), the i-th impact has parameters ξ_i (distance), V_i, w_i (internal parameter). Let (Q_0, V_0) be random with absolutely continuous probability measure Λ, then $(Q_0, V_0, \xi_i, w_i, V_i)$ has a limit distribution as $r \to 0$ with density $\Lambda(Q_0, V_0)\rho(V_0, \xi_1, w_1, V_1) \prod_{j \geq 2}(V_{j-2}, W_{j-1}, V_{j-1}, \xi_i, w_i, V_i)$.

Let the lattice be $L = \mathbb{Z}^n g$, consider the orbit of $\Gamma g \begin{pmatrix} Rotation \cdot diag(r^{d-1}, \dots r^{-1}) & 0 \\ 0 & I_m \end{pmatrix}$, and let $r \to 0$, using the theorems of Ratner and Shah.