MSRI
Mathematical Sciences Research Institute

17 Gauss Way Berkeley, CA 94720-5070 p: 510.642.0143 f: 510.642.8609 www.msri.org

NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Kara Kozia Email/Phone: kkozwol@ualberta.ca

Speaker's Name: Sophie Morel

Talk Title: Cohomology of moduli of Shtukas

Date: 4/9/19 Time: 9:30 am / pm (circle one)

Please summarize the lecture in 5 or fewer sentences: The speaker discussed how to define a local model of the space of Shtukas, and defined sheaves which give spaces of automorphic forms, along with their properties

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☒ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☒ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - **Computer Presentations**: Obtain a copy of their presentation
  - **Overhead**: Obtain a copy or use the originals and scan them
  - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - **Handouts**: Obtain copies of and scan all handouts
- ☒ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☒ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☒ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

COHOMOLOGY OF MODULI OF SHTUKAS - MOREL

LOCAL MODELS

$Sht_{I,W}$ NOT SMOOTH IN GENERAL

WANT "REPLACEMENT"

G-BUNDLES ON $S \times X$

$$Gr_I = \{ (\{x_i\}, \mathcal{E}_1 \dashrightarrow \mathcal{E}_2 \cong \mathcal{E}^{triv}) \}$$

ISOM OUTSIDE OF THE Γ_{x_i}

$$\begin{array}{c} Gr_I \\ \downarrow \\ X^I \end{array}$$

$Gr_{I,W} \subset Gr_I$ PARAMS V.B. W/ MOD'N BDD BY W_i AT x_i

$W \in Rep(\widehat{G}^I)$

$Gr_{I,W} \circlearrowleft G_\infty D_I$ = RELATIVE VERSION OF L^+G = "$G[\![t]\!]$"

HAVE $\Gamma_{\Sigma n_i x_i} \subset X \times X^I$

LOCALLY HAS EQN $\prod t_i^{n_i}$,

t_i = EQN OF DIVISOR $x = x_i$

$$\rightsquigarrow \quad \Gamma_{\Sigma \infty x_i} = \varinjlim \Gamma_{\Sigma n_i x_i}$$

ALSO HAVE $G_{\Sigma n_i x_i} = RES_{\Gamma_{\Sigma n_i x_i}/X^I}\, G$

REL. VERSION OF $G[\![t]\!]/t^n$

$$\rightsquigarrow \quad G_\infty D_I = \varprojlim G_{\Sigma n_i x_i}$$

<u>EX</u> $I = \{1\}$, $X = \mathbb{A}^1$ $\qquad G_{\infty D_I} = L^+G \times X$

$I = \{1,2\}$, $X = \mathbb{A}^1$ $\qquad G_{\infty D_I}\big|_{X^2 \setminus \Delta(X)} = L^+G \times L^+G \times X^2$

$$G_{\infty D_I}\big|_{\Delta(X)} = L^+G \times X$$

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$$\begin{tikzcd} Hk_I = \{((x_i), \mathcal{E}_1 \dashrightarrow \mathcal{E}_2)\} \arrow[r, "\text{``TRIVIALIZE } \mathcal{E}_2 \text{ ON } \sum \infty x_i\text{''}"'] \arrow[d] & {[G_{\infty D_I} \backslash Gr_I]} \\ X^I \end{tikzcd}$$

BUNDLES ON $X \times S$

ISOM AWAY FROM Γ_{x_i}

CAN ALSO DEFINE $Hk_{I,w}$ AND $[G_{\infty D_I} \backslash Gr_{I,w}]$

$$\begin{tikzcd} Hk_I \arrow[r] & {[G_{\infty D_I} \backslash Gr_I]} \\ Hk_{I,w} \arrow[u] \arrow[r] \arrow[ur, phantom, "\square"] & {[G_{\infty D_I} \backslash Gr_{I,w}]} \arrow[u, hook] \arrow[d] \\ Sht_{I,w} \arrow[u] \arrow[ur] \arrow[r, "\varepsilon_{I,w}"] & {[G_{\sum n_i x_i} \backslash Gr_{I,w}]} \end{tikzcd}$$

ON $Gr_{I,w}$, THE ACTION OF $G_{\infty D_I}$ FACTORS THROUGH SOME $G_{\sum n_i x_i}$

$\varepsilon_{I,w}$ IS LOCAL MODEL MAP

PROP $\mathcal{E}_{I,W}$ IS SMOOTH OF REL. DIM. $\left(\sum n_i\right) \dim(G)$

WHY?

$$\begin{tikzcd} P \arrow[r, "\text{SMOOTH ?}"'] \arrow[d] \arrow[dr, phantom, "\square"] & S \arrow[d, "z"] \\ & Gr_{I,W} \arrow[d] \\ Sht_{I,W} \arrow[r, "\mathcal{E}_{I,W}"] & \left[G_{\sum n_i x_i} \backslash Gr_{I,W}\right] \end{tikzcd}$$

$$z = \left(\mathcal{E}_1 \overset{\varphi}{\dashrightarrow} \mathcal{E}_2 \cong \mathcal{E}^{TRIV}, (x_i) \right)$$

OVER $\sum \infty x_i$

P : $\mathcal{F}_1 \overset{\psi}{\dashrightarrow} \mathcal{F}_2 \cong (id_X \times FROB_S)^* \mathcal{F}_1$

ISOM OUTSIDE x_i

$+$ $\mathcal{F}_2\big|_{\sum n_i x_i} \overset{\alpha}{\cong} \mathcal{E}^{TRIV}$, EXTEND α TO $\tilde{\alpha}$ ON $\sum \infty x_i$

ST IF $\mathcal{F}_1'$ IS $\mathcal{F}_2$ "GLUED WITH" $\mathcal{E}_1$, THEN

$$\mathcal{F}_2 \cong (id_X \times FROB_S)^* \mathcal{F}_1'$$

IE, P IS THE EQUALIZER OF

$$Bun_{G, \sum n_i x_i} \times_{X^I} S \overset{b_1}{\underset{b_2}{\rightrightarrows}} Bun_G \times S$$

$$b_1(\mathcal{F}_2, \alpha) = \mathcal{F}_2$$

$$b_2(\mathcal{F}_2, \alpha) = \underbrace{(id_X \times FROB_S)^*}_{\text{FROB ON } Bun_G} \mathcal{F}_1'$$

b_1 IS SMOOTH

b_2 HAS 0 DIFFERENTIAL

$\Longrightarrow$ $P \to S$ SMOOTH $\square$

DEF OF $\mathcal{H}_I$

$$\mathrm{SAT}_I : \mathrm{REP}(\widehat{G}^I) \longrightarrow P_{G_{\infty D_I}}(\mathrm{GR}_I)$$

$\Big($COEFF $\Lambda = E, \mathcal{O}_E, \mathcal{O}_E/\varpi_E^m \Big)$

$$\begin{array}{ccc} \mathrm{Sht}_{I,W,N} & \xrightarrow{\varepsilon_{I,W}} & [G_{\sum n_i x_i} \backslash \mathrm{GR}_I] \\ \downarrow p_{I,W} & & \\ (X \smallsetminus N)^I & & \end{array}$$

$$\boxed{\text{DEF} \quad \mathcal{H}_I(W) := H^0\left(p_{I,W\,!}\ \varepsilon_{I,W}^*\ \mathrm{SAT}_I(W)\right) \quad \text{IND-CONSTRUCTIBLE SHEAF ON } (X \smallsetminus N)^I}$$

THIS IS FUNCTORIAL IN W

PROPERTIES

- $\mathcal{H}_\varphi(\mathbb{1}) = H^0_c(\mathrm{Sht}_{\varphi,N}(\bar{k}))$
  $= C_c(G(F) \backslash G(\mathbb{A}) / K_N, \Lambda)$

- COMPATIBILITY WITH FUSION : $\varphi : I \longrightarrow J$

EX $\quad I = \{1,2\} \xrightarrow{\varphi} \{1\} = J$

$\widehat{G}^J \xrightarrow{\mathrm{RES}_\varphi} \widehat{G}^I \quad , \quad X^J \xrightarrow{\Delta_\varphi} X^I$ $\quad$ ($\Delta_\varphi = \Delta_X$ HERE)

$$G_{\{1,2\}} \times_{X^2, \Delta} X \cong \mathrm{GR}_{\{1\}}$$

$$\mathrm{SAT}_{\{1,2\}}(W_1 \boxtimes W_2)\big|_{\Delta(X)} \cong \mathrm{SAT}_{\{1\}}(W_1 \otimes W_2)$$

$$\Longrightarrow \mathcal{H}_{\{1,2\}}(W_1 \boxtimes W_2)\big|_{\Delta(X)} \cong \mathcal{H}_{\{1\}}(W_1 \otimes W_2)$$

(CAN ALSO DO THIS w/ LEVEL STR)

- HECKE ACTION : CHOOSE $v \in |X|$

  GET AN ACTION

$$T_v : C_c(G(\mathcal{O}_v)\backslash G(F_v)/G(\mathcal{O}_v), \Lambda) \longrightarrow \mathrm{END}\left(\mathcal{H}_I(W)\big|_{(X \setminus N \cup v)^I}\right)$$

- PARTIAL FROBENIUS : $J \subset I$

$$X^I \xrightarrow{\mathrm{FROB}_J} X^I$$

$$X^I = X^J \times X^{I \setminus J} \qquad \mathrm{FROB}_J = \mathrm{FROB}_{X^J} \times \mathrm{id}_{X^{I \setminus J}}$$

$$F_J : \mathrm{FROB}_J^* \mathcal{H}_I(W) \xrightarrow{\sim} \mathcal{H}_I(W)$$

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THE OPERATOR $S_{V,v}$ $\quad v \in |X \setminus N| \quad \deg(v) = 1$

$V \in \mathrm{REP}(\widehat{G})$

$$\mathbb{1} \boxtimes W \xrightarrow{\mathrm{COEV}} (V \otimes V^*) \boxtimes W \xrightarrow{\mathrm{EV}} \mathbb{1} \boxtimes W$$

$\rotatebox{90}{$\in$}$ $\mathrm{REP}(\widehat{G}^{\{1\} \sqcup I})$

$$\begin{tikzcd}
\Lambda \boxtimes \mathcal{H}_I(W) \arrow[rr, "S_{V,v}"] \arrow[d, phantom, sloped, "\cong"] & & \Lambda \boxtimes \mathcal{H}_I(W) \\
\mathcal{H}_{\{1\}\sqcup I}(\mathbb{1}\boxtimes W) \arrow[d, "\mathrm{coev}"] & & \mathcal{H}_{\{1\}\sqcup I}(\mathbb{1}\boxtimes W) \arrow[u, phantom, sloped, "\cong"] \\
\mathcal{H}_{\{1\}\sqcup I}((V\otimes V^*)\boxtimes W) \arrow[d, phantom, sloped, "\cong"] \arrow[d, phantom, "\text{FUSION}"'] & & \mathcal{H}_{\{1\}\sqcup I}((V\otimes V^*)\boxtimes W) \arrow[u, "\mathrm{ev}"] \\
\mathcal{H}_{\{1,2\}\sqcup I}(V\boxtimes V^*\boxtimes W) \arrow[rr, "F_{\{1\}}"'] & & \mathcal{H}_{\{1,2\}\sqcup I}(V\boxtimes V^*\boxtimes W) \arrow[u, phantom, sloped, "\cong"] \arrow[u, phantom, "\text{FUSION}"']
\end{tikzcd}$$

ON $\{v\}\times(X\setminus N)^I = (X\setminus N)^{\{1\}\sqcup I}$

GET $\quad S_{V,v} : \mathcal{H}_I(W) \longrightarrow \mathcal{H}_I(W)$ $\quad$ "EXCURSION"

<u>THM</u> 1) $(S=T)$

$$S_{V,v}\big|_{(X\setminus N\cup\{v\})^I} = T_v(h_{V,v})$$

IMAGE OF V BY CLASSICAL SATAKE

2) CONGRUENCE RELN (EICHLER – SHIMURA)

$$\sum_{i=0}^{\dim(V)} (-1)^i \left(F_{\{1\}}^{\deg(v)}\right)^i \circ S_{\Lambda^{\dim(V)-i}V,\, v} = 0$$

ON $\mathcal{H}_{\{1\}\sqcup I}(V\boxtimes W)$, $\quad$ ON $\{v\}\times(X\setminus N)^I$