

NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: KARL KOZLO Email/Phone: kkozol@ualberta.ca

Speaker's Name: SOPHIE MOREL

Talk Title: COTHOLOGY OF MODULI OF SHUKAS

Date: 4/9/19 Time: 9:30 (am) / pm (circle one)

Please summarize the lecture in 5 or fewer sentences: THE SPEAKER DISCUSSED
HOW TO DEFINE A LOCAL MODEL OF THE SPACE OF
SHUKAS, AND DEFINED SPACES WHICH GIVE SPACES
OF AUTOMORPHIC FORMS, ALONG WITH THEIR PROPERTIES

CHECK LIST

(This is **NOT** optional, we will **not** pay for incomplete forms)

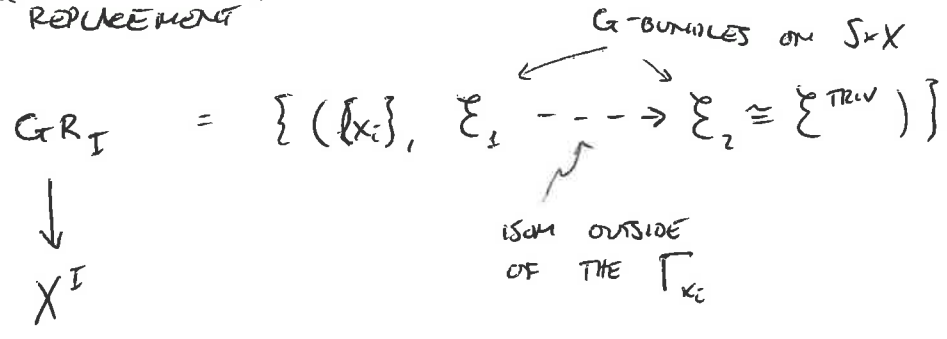
- Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
 - **Computer Presentations:** Obtain a copy of their presentation
 - **Overhead:** Obtain a copy or use the originals and scan them
 - **Blackboard:** Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
 - **Handouts:** Obtain copies of and scan all handouts
- For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list.
(YYYY.MM.DD.TIME.SpeakerLastName)
- Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

COTOPHOLOGY OF MODULI OF SHIMURA - MOREL

LOCAL MODELS

$\text{Sh}_{I,W}$ NOT SMOOTH IN GENERAL

WANT "REPLACEMENT"



$\text{GR}_{I,W} \subset \text{GR}_I$ PARAMS V.B. W/ MOD 'N BDD BY W_i AT X_i

$W \in \text{REP}(\hat{G}^I)$

$\text{GR}_{I,W} \ni G_{\infty D_I} = \text{RELATIVE VERSION OF } L^*G = "G[t]"$

HAVE $\Gamma_{\sum u_i X_i} \subset X = X^I$

\uparrow LOCALLY HAS EQN $\prod t_i^{u_i}$

$t_i = \text{EQN OF DIVISOR } X = X_i$

$\rightsquigarrow \Gamma_{\sum \infty X_i} = \varprojlim \Gamma_{\sum u_i X_i}$

ALSO HAVE $G_{\sum u_i X_i} = \text{RES}_{\Gamma_{\sum u_i X_i} / X^I} G$

\uparrow REL. VERSION OF $G[t]/t^n$

$\rightsquigarrow G_{\infty D_I} = \varprojlim G_{\sum u_i X_i}$

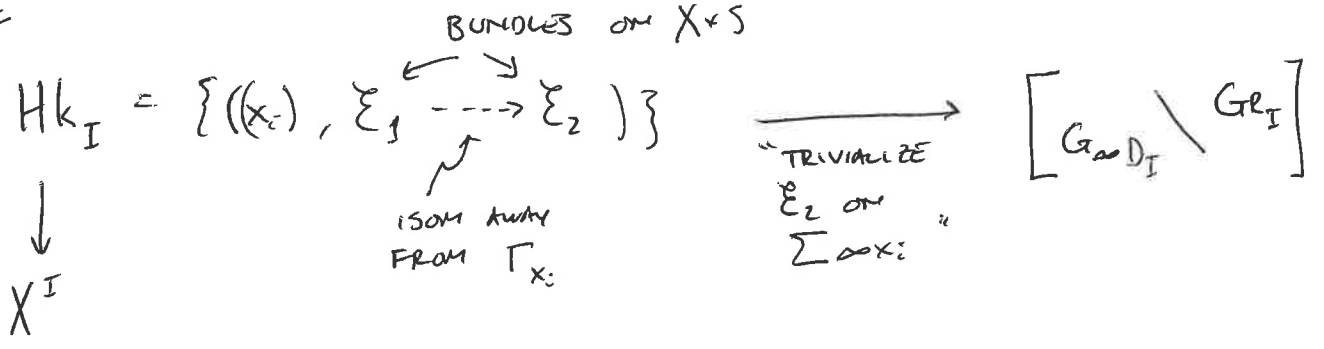
EX $I = \{1\}, X = \mathbb{A}^1$

$G_{\infty D_I} = L^* G \times X$

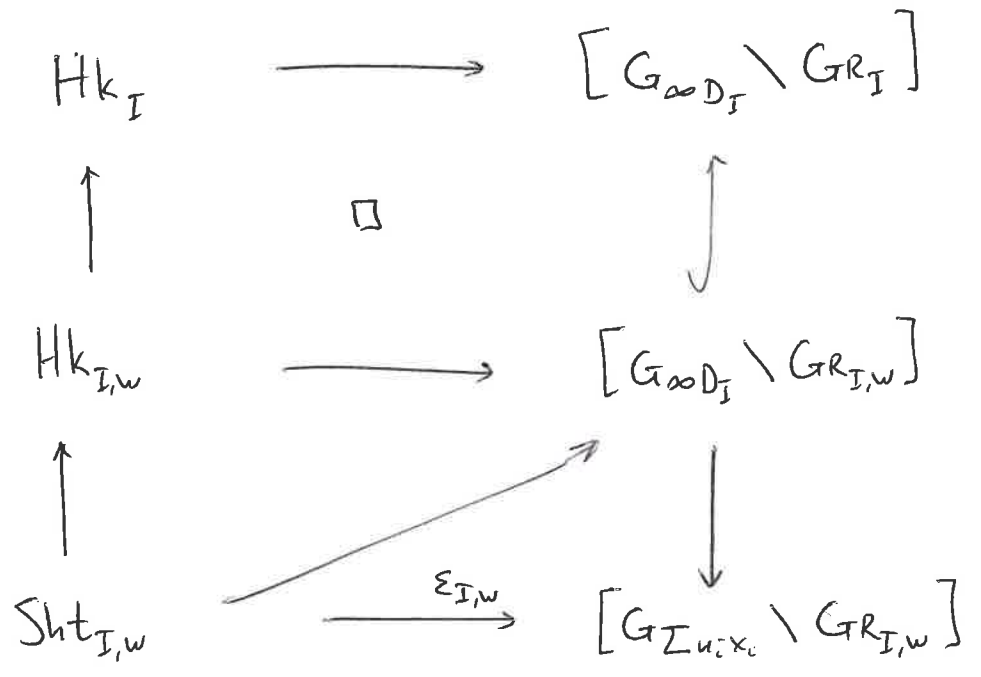
$I = \{1, 2\}, X = \mathbb{A}^1$

$G_{\infty D_I} |_{X^2 - \Delta(X)} = L^* G \times L^* G \times X^2$

$G_{\infty D_I} |_{\Delta(X)} = L^* G \times X$



CAN ALSO DEFINE $Hk_{I,W}$ AND $[G_{\infty D_I} \setminus Gr_{I,W}]$

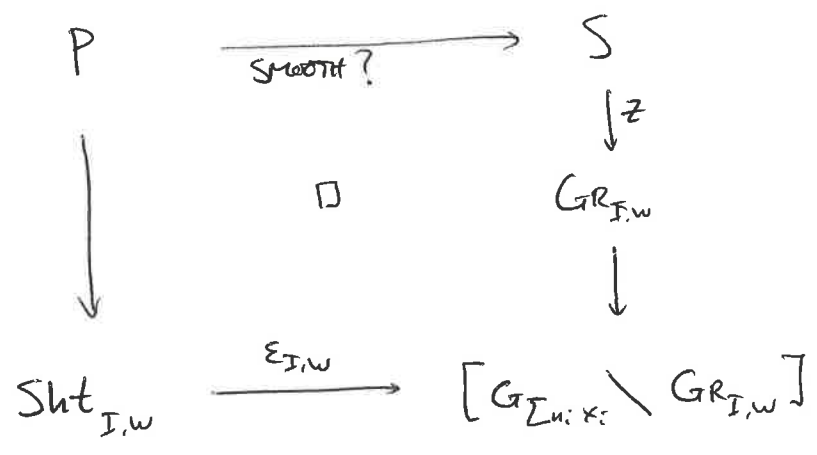


ON $Gr_{I,W}$, THE ACTION OF $G_{\infty D_I}$ FACTORS THROUGH SOME $G_{\Sigma u_i x_i}$

$\varepsilon_{I,W}$ IS LOCAL MODEL MAP

PROP $\Sigma_{I,W}$ IS SMOOTH OF REL. DIM. $(\sum n_i) \dim(G)$

WHY?



$$z = \left(\Sigma_1 \xrightarrow{\varphi} \Sigma_2 \cong \sum^{TRW} (X_i) \right)$$

\nwarrow
 OVER $\sum \infty X_i$

$$P : \mathcal{F}_1 \xrightarrow{\gamma} \mathcal{F}_2 \cong (id_X * \text{FROB}_S)^* \mathcal{F}_1$$

\nwarrow
 ISOM OUTSIDE X_i

$+ \mathcal{F}_2|_{\sum n_i X_i} \cong \sum^{TRW}$, EXTEND α TO $\tilde{\alpha}$ ON $\sum \infty X_i$
 ST IF \mathcal{F}_1' IS \mathcal{F}_2 "GLUED WITH" Σ_1 , THEN
 $\mathcal{F}_2 \cong (id_X * \text{FROB}_S)^* \mathcal{F}_1'$

IE, P IS THE EQUILIZER OF

$$\text{Bun}_{G, \sum n_i X_i} \times_{X^I} S \begin{array}{c} \xrightarrow{b_1} \\ \xrightarrow{b_2} \end{array} \text{Bun}_G \times S$$

$b_1(\mathcal{F}_2, \alpha) = \tilde{\mathcal{F}}_2$ b_1 IS SMOOTH
 $b_2(\mathcal{F}_2, \alpha) = \underbrace{(id_X * \text{FROB}_S)^* \mathcal{F}_1'}_{\text{FROB ON } \text{Bun}_G}$ b_2 HAS 0 DIFFERENTIAL
 $\implies P \rightarrow S$ SMOOTH □

DEF OF \mathcal{H}_I

$$\text{SAT}_I : \text{REP}(\widehat{G}^I) \longrightarrow \mathcal{P}_{G_{\infty D_I}}(G_{R_I}) \quad \left(\begin{array}{l} \text{COEFF} \\ \Lambda = E, \mathcal{O}_E, \\ \mathcal{O}_E / \mathcal{O}_E^m \end{array} \right)$$

$$\text{ShT}_{I,W,N} \xrightarrow{\Sigma_{I,W}} [G_{\Sigma_{N: X_i}} \setminus G_{R_I}]$$

$$\downarrow \text{P}_{I,W}$$

$$(X \setminus N)^I$$

DEF $\mathcal{H}_I(W) := H^0(\text{P}_{I,W}! \Sigma_{I,W}^* \text{SAT}_I(W))$

IND-CONSTRUCTIBLE SHEAF ON $(X \setminus N)^I$

THIS IS FUNCTORIAL IN W

PROPERTIES

- $\mathcal{H}_\varphi(\mathbb{1}) = H_c^0(\text{ShT}_{\varphi,N}(\mathbb{K}))$
 $= C_c(G(F) \setminus G(A) / K_N, \Lambda)$

COMPATIBILITY WITH FUSION : $\varphi : I \rightarrow J$

EX $I = \{1, 2\} \xrightarrow{\varphi} \{1\} = J$

$$\widehat{G}^J \xrightarrow{\text{RES}_\varphi} \widehat{G}^I, \quad X^J \xrightarrow{\Delta_\varphi = \Delta_X \text{ HERE}} X^I$$

$$G_{\{1,2\}} \times_{X^2 \xrightarrow{\Delta}} X \cong G_{\{1\}}$$

$$\text{SAT}_{\{1,2\}} (W_1 \boxtimes W_2) |_{\Delta(X)} \cong \text{SAT}_{\{1\}} (W_1 \otimes W_2)$$

$$\Rightarrow \mathcal{H}_{\{1,2\}} (W_1 \boxtimes W_2) |_{\Delta(X)} \cong \mathcal{H}_{\{1\}} (W_1 \otimes W_2)$$

(CAN ALSO DO THIS w/ LEVEL STR)

- HECKE ACTION : CHOOSE $v \in |X|$
- GET AN ACTION

$$T_v : C_c(G(\mathcal{O}_v) \backslash G(F_v) / G(\mathcal{O}_v), \Lambda) \rightarrow \text{END}(\mathcal{H}_I(W) |_{(X-N(v))^{I^*}})$$

- PARTIAL FROBENIUS : $J \subset I$

$$\begin{array}{ccc} X^I & \xrightarrow{\text{FROB}_J} & X^I \\ & \parallel & \\ X^J \times X^{I-J} & & \text{FROB}_{X^J} \times \text{id}_{X^{I-J}} \end{array}$$

$$F_J : \text{FROB}_J^* \mathcal{H}_I(W) \xrightarrow{\sim} \mathcal{H}_I(W)$$



THE OPERATOR $S_{V,v}$ $v \in |X-N|$ $\text{DEG}(v) = 1$
 $V \in \text{REP}(\widehat{G})$

$$\begin{array}{ccc} \mathbb{1} \boxtimes W & \xrightarrow{\text{COEV}} & (V \otimes V^*) \boxtimes W \xrightarrow{\text{EV}} \mathbb{1} \boxtimes W \\ \nearrow & & \\ \text{REP}(\widehat{G}^{\text{ISSUI}}) & & \end{array}$$

$$\Lambda \boxtimes \mathcal{H}_I(W) \xrightarrow{S_{V,V}} \Lambda \boxtimes \mathcal{H}_I(W)$$

$$\text{on } \{V\} \times (X-N)^I = (X-N)^{\{1\} \sqcup I}$$

$$\mathcal{H}_{\{1\} \sqcup I}(\mathbb{1} \boxtimes W)$$

$$\mathcal{H}_{\{1\} \sqcup I}(\mathbb{1} \boxtimes W)$$

↓ COEV

↑ EV

$$\mathcal{H}_{\{1\} \sqcup I}((V \otimes V^*) \boxtimes W)$$

$$\mathcal{H}_{\{1\} \sqcup I}((V \otimes V^*) \boxtimes W)$$

|| FUSION

|| FUSION

$$\mathcal{H}_{\{1,2\} \sqcup I}(V \boxtimes V^* \boxtimes W) \xrightarrow{F_{12}} \mathcal{H}_{\{1,2\} \sqcup I}(V \boxtimes V^* \boxtimes W)$$

GET $S_{V,V} : \mathcal{H}_I(W) \longrightarrow \mathcal{H}_I(W)$ "EXCURSION"

THM 1) (S=T)

$$S_{V,V} |_{(X-N) \cup \{1\}^I} = T_V(h_{V,V})$$

↑
IMAGE OF V BY
CLASSICAL SAITAKE

2) CONGRUENCE RELN (EICHLER - SHIMURA)

$$\sum_{i=0}^{\dim(V)} (-1)^i (F_{12}^{\text{DEG}(V)})^i \circ \int_{\Lambda^{\dim(V)-i} V, V} = 0$$

on $\mathcal{H}_{\{1\} \sqcup I}(V \boxtimes W)$, on $\{V\} \times (X-N)^I$