Inbar Klang – Hochschild invariants and duality.

Factorization homology – "higher dimensional" generalization of Hochschild homology.

Input: n-mfld M, E_n-algebra A
↖ (space, spectra, chain complex ...)

[An E_1-algebra is something like an associative algebra.
E_∞-algebra $\approx$ commutative algebra.
E_n's interpolate between assoc. and commutative.]

Output: $\int_M A$ a space, spectra, chain complex ...

Properties. (1) homology theory for manifolds (Ayala–Francis)
(has functoriality, Mayer-Vietoris, ..)

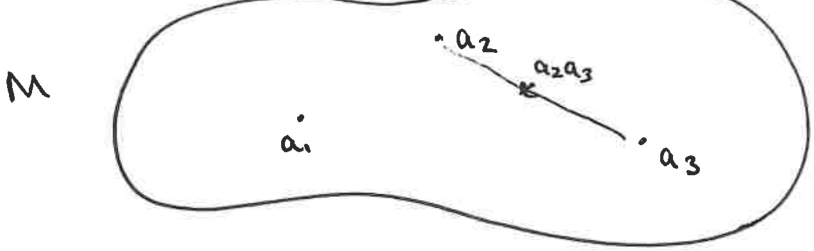
(2) If A is a dga, then $\int_{S^1} A \simeq HC_*(A)$
↖ Hochschild chains on A

If A is a ring spectrum $\int_{S^1} A \simeq THH(A)$

(3) gives an n-dimensional topological field theory $Z(N) \simeq \int_N A$

Picture: (A a topological ab. group.)

$\int_M A$: configurations of pts in M labeled w/ elts of A
when pts collide, multiply labels.



Hochschild cohomology

Def. If A is a dga, $HC^*(A) := R\mathrm{hom}_{A\otimes A^{op}}(A, A)$

right derived homomorphisms.

We can think of this as the "derived center" of A.

If A is a ring spectrum: can do the same thing with $\otimes$ ~~replacing~~ replaced by $\wedge$.

Rmk. If A is an E_n-algebra, E_n-Hochschild cohomology makes sense:

replace $A\otimes A^{op}$ with $\int_{S^{n-1}\times\mathbb{R}} A$.

Can check that for $n=1$ we recover previous def.

Inbar's thesis: factorization homology $\int_M$, THC, THC_{E_n} for ring spectra that arise as Thom spectra.

Today: duality between HH_* and HH^*.

Example. M closed, oriented manifold.

$$\left.\begin{aligned} HH_*(C_*(\Omega M)) &\simeq H_*(\mathcal{L}M) \leftarrow \text{free loop space} \\ HH^*(C_*(\Omega M)) &\simeq H_{*+d}(\mathcal{L}M) \end{aligned}\right\} \text{"Poincaré duality" for Hochschild (co)homology.}$$

This duality between $HH_*(A)$, $HH^*(A)$ is part of the Calabi-Yau condition.

CY condition $\Rightarrow$ 2 dimensional topological field theory Z whose value on S^1 is $HC_*(A)$.

Ex: get operations on $H_*(\mathcal{L}M)$. A product.

Jt w/ Ralph Cohen: generalized notion of Calabi-Yau algebra to ring spectra.

Examples: • $\Sigma^\infty_+\Omega M$ suspension spectra of based loop space; "why" comes from duality discussed above

• Thom spectra $\Omega M^{\Omega f}$ of bundles over ΩM.

Thm (Klang) duality between $\int_{S^n}$ and $HC^*_{E_n}$ for some E_n-algebras

Q: is this part of an E_n Calabi-Yau condition?