Two Counting Problems in Geometry Solved

On Tuesday morning, February 18, 2003, the solutions to two old problems of geometry will be presented at the Mathematical Sciences Research Institute on the campus of the University of California in Berkeley.

Oleg Musin, a Russian mathematician now living in Los Angeles will present his solution of the "kissing number problem" in dimension 4, a problem first posed by Sir Isaac Newton in the 17th century. At the same session, Henry Cohn of Microsoft Research will describe spectacular progress made in the solution of the packing problem in 24 dimensions. Both problems are distinguished by the fact that for a long time mathematicians had in hand a likely candidate for the best solution; one with a high degree of symmetry. However, the possibility of doing a little better with a slight rearrangement could never be discounted until now.

This is a problem that got its name from billards: how many balls on a billiard table can just touch a given ball? The answer is exactly six; however, in three dimensions the question becomes more interesting and quite complicated. Newton and his colleague Gregory had a controversy about this in 1694 - Newton said that 12 should be the correct answer, while Gregory thought that by packing those balls more closely, a 13th ball could be squeezed in. Newton's solution is based on one of the platonic solids, the icosahedron is shown in the figure, where you can see the gaps Gregory hoped to use. But he was wrong, as was shown by Schutte and van der Waerden in 1953.

Mathematicians are excited now: after years of work, the Russian mathematician Oleg Musin solved the kissing number problem for dimension 4: The so-called *24-cell*, a four-dimensional "platonic solid" of remarkable beauty (FIG.~3), yields a configuration of 24 balls that would touch a given one in four-dimensional space.

Henry Cohn's work is about dense packing of balls in 8-dimensional and in 24-dimensional space. These dimensions are special since mathematicians know exceptionally good, "dense" packings for those dimensions,tied to extremely symmetric objects that are known as the *E8 lattice* and the *Leech lattice*. Indeed, Henry Cohn and his co-worker Abhinav Kumar (a mathematics graduate student at Harvard) get extremely close to proving that the known ball packings have optimal density --- they are off by at most 10^(-27) --- up to now . . .

Both Oleg Musin and Henry Cohn will present their work to an international audience of experts this week(November 17 to 21, 2003): the workshop "Combinatorial and Discrete Geometry" at the Mathematical Science Research Institute in Berkeley CA features both of them as principal speakers on Tuesday morning.

For further information, contact Jesus De Loera at 510 643-6057, jdeloera@msri.org
or Gunter Ziegler, 510 643-6045, ziegler@msri.org.