Copyright © 1975
The National Council of Teachers of Mathematics
1906 Association Drive, Reston, Virginia 22091

APPLICATIONS OF GROUPS AND ISOMORPHIC GROUPS TO TOPICS IN THE STANDARD CURRICULUM, GRADES 9–11: PART II

Many relationships between groups and topics of secondary school mathematics are shown by the author, who proposes that the study of groups be included as standard fare in the mathematics curriculum of the average college-bound student.

By ZALMAN USISKIN

The University of Chicago
Chicago, Illinois

IT IS possible to form a set in an infinite additive group by beginning with any nonzero real a and adding it to itself over and over again, then including zero and the opposites of all numbers in the set. We call such a set the set of integral multiples of a. If $a = 3$, then here is such a set.

$$\{0, 3, -3, 6, -6, 9, -9, \cdots\}$$

With addition, this set forms a group, the group of *integral multiples* of 3.

In Part I of this article, the set of integral powers of 2 was seen to form a group with multiplication. This set can be thought of as beginning with 2, multiplying 2 by itself over and over again, then including 1 and the reciprocals of the numbers in the set. Thus the sets of integral powers and integral multiples are formed by analogous means, with the only differences being the number begun with (the *generator*) and the operation used. This hints at the existence of isomorphic groups. Listing a possible correspondence between the sets shows that this is the case.

⟨set of integral multiples of 3, +⟩	⟨set of integral powers of 2, ·⟩
0	1
3	2
−3	.5
6	4
−6	.25
9	8
−9	.125
·	·
·	·
·	·

Part I of this article, which appeared in the February issue of the MATHEMATICS TEACHER, contains applications of groups to sentence solving, systems, and real- and complex-number operations. Definitions and examples of groups and isomorphic groups are given there.